The Model of Global Blood Circulation and Applications

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Abstract— **1D model of global blood circulation describes hemodynamics of healthy vascular system very well. Real applications demand considerations of vascular pathologies, implants and influence of external effects. In this paper we discuss how to do it on the basis of the model. The first approach is to update the state equation of the model. This equation describes elastic properties of the vessel wall. The second approach is to use a 3D model of the blood flow in the region of interest. The 3D model should be coupled with the 1D model of global blood circulation.**

Keywords— **Model of global blood circulation, 3D-1D model, vascular pathologies and implants, state equation, boundary conditions.**

I. Introduction

In many medical applications one needs to know the influence of medical treatment, pathologies or external effects on the blood flow. The 1D model of global blood circulation is an effective tool for it. This model allows us to calculate hemodynamics in the whole cardiovascular system. The model base is quite simple and verified and it can be easily elaborated. In this paper we summarize our experience in cardiovascular modeling and discuss developed approaches for considering pathologies, implants or effects on the flow by the model of global blood circulation. Two groups of techniques enable us to do it. The first group is based on manipulations with the state equation. Another group deals with multiscale modelling.

II. Materials and Methods

A. 1D Model of Global Blood Circulation

The model of global blood circulation is based on the set of differential equations [1, 2, 3]. It consists of mass and momentum conservation laws and the state equation:

$$\frac{\partial S}{\partial t}+\frac{\partial(S\bar{u})}{\partial x}=0, \tag{1}$$

$$\frac{\partial \bar{u}}{\partial t}+\frac{\partial(\bar{u}^2/2+\bar{p}/\rho)}{\partial x}=\psi(t,x,S,\bar{u}), \tag{2}$$

$$\bar{p}-p_{\text{ext}}=\rho c_0^2 f(S), \tag{3}$$

where $\bar{u}$, $\bar{p}$, S are unknown velocity, averaged in vessel cross section, pressure and the area of vessel cross section; ρ is density of blood; c_0 is the rate of small disturbance propagation in the vessel wall; p_{ext} is an external pressure of the vessel. The function ψ represents influence of external forces, for example, frictional force.

System of equations (1)– (3) is set in every vessel of the vascular network. Another system of equations is demanded in every node of the network to coordinate solutions from connecting vessels together. It consists of mass conservation law and Poiseuilleś pressure drop condition, combined with the appropriate compatibility condition for (1)– (2) [3, 4, 5].

Such 1D model of global blood circulation describes hemodynamics in the healthy vessel network very well. Most problems that are interesting for physicians deal with external effects, vascular pathologies or implants. Below we discuss how to take into account influences of these factors by the model of global blood circulation.

B. The State Equation

The state equation (3) describes elastic properties of the vessel wall. The empirical function $f(S)$ in (3) is known for healthy vessels [6]:

$$f(S)=\begin{cases}\exp(S\hat{S}^{-1}-1)-1, & \text{for } S>\hat{S}\\ \ln(S\hat{S}^{-1}), & \text{for } S\le\hat{S}\end{cases}, \tag{4}$$

where $\hat{S}$ is the cross section area of a vessel at rest. It is possible to take into account some effects on the cardiovascular system by the model of global blood circulation changing the state equation or its parameters.

We can change p_{ext} in (3) to include sceletal-muscle pump in the 1D model of global blood circulation [4]. In this case p_{ext} is considered to be a periodic function. The vein valves preventing the backward blood flow are important in this problem. Modified friction force can represent their function.

The effects of the enhanced external counterpulsation methods could be considered in the same way. Three pressure cuffs sequentially apply pressure to calves, things and

I. Lacković and D. Vasić (eds.), *6th European Conference of the International Federation for Medical and Biological Engineering*,
IFMBE Proceedings 45, DOI: 10.1007/978-3-319-11128-5_101

lower abdomen. Cuff pressure could be simulated by p_{ext} in the state equation (3).

Depending on the problem the state equation can be recovered. A more common technique demanding the model of the elastic vessel wall can be applied. This technique will be demonstrated in case of vascular pathology or implant.

Elastic properties of the vessel wall change if there is any pathology or implant in the vessel. Often the shape of the vessel changes too. In this case we need to calculate a new state equation. We suggest using the fiber model of elastic vessel wall for this purposes. The materials of the wall could be assumed both linear and nonlinear. The wall is represented by the set of fibers. This model allows us to calculate forces in the wall or any reaction of the wall on deformation. The local force density is given by the expression [5]

$$\vec{f} = \frac{\partial}{\partial s}(T\vec{\tau}), \vec{\tau} = \frac{\partial \vec{X}}{\partial s}\left|\frac{\partial \vec{X}}{\partial s}\right|^{-1}. \quad (5)$$

$\vec{X}(s,t)$ represents the position of the fiber points in space, where Lagrange coordinate s is the arc length of the fiber in the unstressed state; $\vec{\tau}$ is the unit tangent vector; $T(s,t)$ is a fiber tension. Expression for tension T could vary depending on the fiber properties and shape. In the case of Hookean wall material we use fiber tension in the form

$$T = E_*\left(\left|\frac{\partial X}{\partial s}\right| - 1\right), \quad (6)$$

where E_* is the elastic modulus of the fiber. Balancing forces on the wall we solve the following equation of static equilibrium:

$$p = (\vec{f}, \vec{n})h, \quad (7)$$

$\vec{n}$, h are surface normal and wall thickness; p is transmural pressure. If the pressure p is constant everywhere in the vessel we calculate the state equation for the healthy vessel solving the equation (7). In the vein with cava filter the additional pressure of implant appears in filter fixation points. This additional pressure should be taken into account by the left-hand side of the expression (7).

We elaborated the fiber model of the elastic vessel wall to calculate the state equation for the atherosclerotic arterial wall [5]. We consider the atherosclerotic vessel as a three-layer circular cylindrical shell inflated by internal pressure. The wall materials are considered to be linear-elastic, isotropic. The three layers are the fibrous cap, the lipid pool and the wall. The fibrous cap and the arterial wall are modelled as a set of fibers as it was described earlier. The lipid pool is represented by the set of radial springs. In order to estimate spring displacement, we use the solution of the deformation problem for the incompressible isotropic cylinder $(a \le r \le b)$ under internal pressure p_a and external pressure p_b. In this case the relation between the radial displacement $u(r)$ of cylinder points and given pressures is expressed as

$$p_a - p_b = \frac{2(b^2 - a^2)E_c r}{3a^2b^2}u(r), \quad (8)$$

where E_c is Young modulus of the cylinder. We balance forces of the atherosclerotic wall solving the system of equations (7) for fibrous cap and arterial wall and (8) for lipid pool. Such technic allows us to calculate the state equation for the vessels with different geometries of atherosclerotic plaques.

We presented several ways how to modify the state equation depending on the special problem. The new state equation is prescribed for targeted vessel/vessels in the model of global blood circulation.

C. 3D-1D Model of Blood Flow

Multiscale modeling also helps us to take into account additional physical effects by the model of global blood circulation. 3D-1D models are rather widespread nowadays. 1D part of our 3D-1D model of blood flow is described in subsection A.The 3D part is based on the Navier-Stokes equations. Vessel walls in 3D domain could be assumed rigid or elastic. The problem of fluid structure interaction (FSI) appears in the second case.

3D domain can be different in applications. 3D-1D models can account whirls that appear for example in bifurcations, or atherosclerotic plaques or aneurisms if they are represented in 3D domains.

The 1D part in such 3D-1D models can be used for the following purposes. The first reason is monitoring of blood flow at some distance from 3D domain. The second reason is to pose correct boundary conditions for problems where the data on the 3D boundary is unknown.

One of the most important features of 3D-1D models is coupling boundary conditions between domains of different dimensions. Fluid flux and normal stress continuation between 3D and 1D parts of the model is rather common choice. This choice does not guarantee correct energy balance for the coupled model. This point could be crucial for problems where the energy analysis takes place. For example energy losses are often investigated when optimization of graft angles or geometry takes place.

In case of rigid vessel wall in 3D domain we suggest using the following boundary coupling condition. This condition demands the continuity of the linear combination of the fluid flux and the energy flux between 3D boundary Γ_{out} and 1D boundary $x = d$ [7]:

$$\bar{p}\int\limits_{\Gamma_{out}} \mathbf{u}\cdot\mathbf{n}\mathbf{d}\mathbf{s}+\frac{\rho}{2}\int\limits_{\Gamma_{out}} |\mathbf{u}|^2(\mathbf{u}\cdot\mathbf{n})\mathbf{d}\mathbf{s}=(\bar{p}S\bar{u}+\frac{\rho}{2}S\bar{u}^3)|_{x=d}, \quad (9)$$

where $\mathbf{u}$ is the velocity of blood in 3D; $\mathbf{n}$ - outward normal vector. The condition (9) should be combined with the continuity of normal stress. The correct energy balance is valid for the coupled model with such boundary conditions. The unsteady problem can be easily decoupled with splitting methods into the separate 1D and 3D problems with usual inflow-outflow boundary conditions on every time step. The inequalities $\mathbf{u}\cdot\mathbf{n}<0$ or $\mathbf{u}\cdot\mathbf{n}>0$ are *not* necessarily pointwise satisfied on Γ_{in} or Γ_{out}, respectively ($\mathbf{n}$ is the outward normal vector). This is very important for modeling of blood flow in veins, where the values of x-component of velocity can be negative.

D. Patient-Specific Simulations

Besides external effects, pathologies or implants, patient-specific simulations are demanded in the model of global blood circulation. One can extract 3D or 1D geometry from MRI or CT data. We used the following stages to reconstruct the 1D vascular graph:

1. 3D volume segmentation of vascular structure;
2. Meshing;
3. Centerlines extraction;
4. Centerlines merging for graph reconstruction.

Numerical simulations for real patient allow us to predict results of surgical operations [8].

III. Results

Blood flow through the network of arteries and veins during skeletal-muscle pump work was simulated. The network of the vessels was fitted to represent systemic circulation of long-distance runner with 175 cm body's height [9]. Average blood flow through a specific leg artery was studied. The artery was chosen arbitrarily since the difference in results for two leg arteries is only quantitative. Results show that specific stride frequency exists that maximizes blood flow through the lower extremities (fig. 1). This stride frequency is compared to the running regime of a world class athlete (London Olympics, 5000m winner).

Blood flow in the vessel network with atherosclerotic plaque was modeled [5]. We assumed several types of plaque geometries (fig. 2). The state equation for every atherosclerotic artery was calculated. An example of the state equation for a plaque of Type 2 is presented in the figure 3. The lumen of the vessel is narrowing in the middle of the vessel and

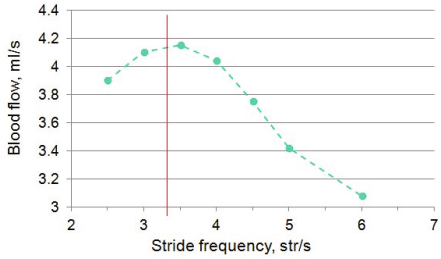

Fig. 1 Blood flow through the leg artery for a few stride frequencies. Vertical line is a stride frequency of an actual athlete during last third of a 5000m distance.

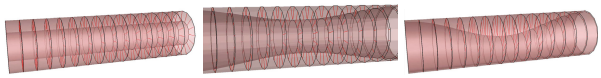

Fig. 2 Three types of plaque geometry with different inner surfaces: uniformly distributed over the vessel wall (Type 1), axial symmetric (Type 2) and asymmetric (Type 3) narrowing at the center of the vessel.

the curve of the state equation is unique in every point of the 1D vessel (the state equation depends on the x-coordinate). Modified state equations were used in the model of global

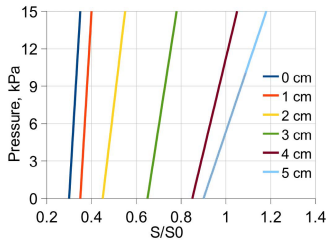

Fig. 3 The state equation for the artery with atherosclerotic plaque of Type 2. The index of each curve corresponds to the distance (in cm) to the minimal cross section. The minimal lumen is 30%.

blood circulation. This experiment allowed us to compare the blood flow in the vascular network with one or several atherosclerotic plaques with different geometries.

Blood flow in the vena cava with implanted cava filter was simulated by the 3D-1D model. The 3D domain represented the part of vena with the implant (fig. 4). The rest of the vessel was assumed 1D. This experiment allowed us to calculate such important statistics as the drag force acting on inclusions and the pressure drop.

The results of these numerical experiments conform medical data.

IV. Discussion

We address blood flow modeling in a vascular network with accounted pathologies, implants or external effects on the cardiovascular system. New techniques

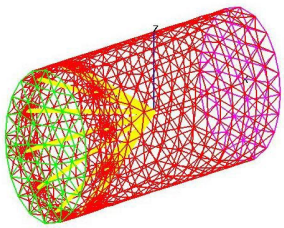

Fig. 4 The surface mesh for part of the vena cava with implanted cava filter.

make patient-specific simulations more realistic. However, it is still a big problem to do real time calculations. Elaboration of the model of global blood circulation leads to increase of the computing time. Small steps for space discretization are demanded in the vessels with x-coordinate dependent modified state equations. The solution of the 3D Navier-Stokes equations in 3D-1D models is also time-consuming.

Splitting algorithms are desirable for 3D-1D models. These algorithms need suitable boundary coupling conditions. In this work combining downstream coupling condition (9) with continuity of normal stress allows us to separate the 3D-1D problem for 3D and 1D problems.

Another issue is data mining for the models: it is impossible to obtain some parameters of the vascular network in vivo, e.g. some properties of the vessel wall.

V. Conclusions

In this paper we discussed how to take into account vascular pathologies or implants or other effects on hemodynamics by the model of global blood circulation. The first approach is based on update of the state equation. We can modify it by the fiber or fiber-spring model of the elastic vessel wall. Second approach uses 3D-1D models of blood flow. The domain of interest is considered to be 3D and is coupled with 1D model of global blood circulation in order to study both local flow and global hemodynamics. All these methods were verified: the results of numerical experiments correspond to real data. All described models can be adapted for the real patients.

Conflict of Interest

The authors declare that they have no conflict of interest.

Acknowledgements

We acknowledge the Moscow Institute of Physics and Technology (State University), The Department of Applied Mathematics, The International Translational Medicine and Biomodeling Research team. This work was partially supported by the RFBR grants 14-01- 00779 and 14-01-00830, Russian Federation President Grant for Government Support of Young Russian Scientists MK3675.2013.1, Federal programs Research and Development. The blood flow numerical model and the results of Sections A and B were supported by Russian Science Foundation grant 14-11-00766.

References

1. Quarteroni A, Formaggia L. *Mathematical Modelling and Numerical Simulation of the Cardiovascular System*. Amsterdam: Elsevier 2004.
2. Sherwin S, Franke V, Peiro J et al. One-dimensional modeling of vascular network in space-time variables *J. of Engineering Mathematics.* 2003;47:217 – 250.
3. Simakov S, Kholodov A. Computational study of oxygen concentration in human blood under low frequency disturbances *Mathematical Models and Computer Simulations.* 2009;1(2):283 - 295.
4. Simakov S, Gamilov T, Soe Y. Computational study of blood flow in lower extremities under intense physical load *Russian Journal of Numerical Analysis and Mathematical Modelling.* 2013;28(5):485–504.
5. Vassilevski Yu, Simakov S, Salamatova V et all. Vessel wall models for simulation of atherosclerotic vascular networks *Mathematical Modelling of Natural Phenomena.* 2011;6(7):82 - 99.
6. Caro C, Pedley T, Schroter R et al. *The Mechanics of Circulation*. Oxford: Oxford Univ. 1978.
7. Dobroserdova T, Olshanskii M. A finite element solver and energy stable coupling for 3D and 1D fluid models *Computer Methods in Applied Mechanics and Engineering.* 2013;259:166 - 176.
8. Simakov S, Gamilov T, Vassilevski Yu et all. Patient specific haemodynamics modeling after occlusion treatment in leg *Mathematical Modelling of Natural Phenomena.* 2014;to appear.
9. Otsuki T, Maeda S, Iemitsu M et all. Relationship Between Arterial Stiffness and Athletic Training Programs in Young Adult Men *Am J Hypertens.* 2003;146:168 – 174.

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