Coupled surface-subsurface flow modeling in GeRa software package

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GeRa code

GeRa (Geomigration of Radionuclides) – software package designed for 3D hydrogeologic modeling to assess radioactive waste disposal safety

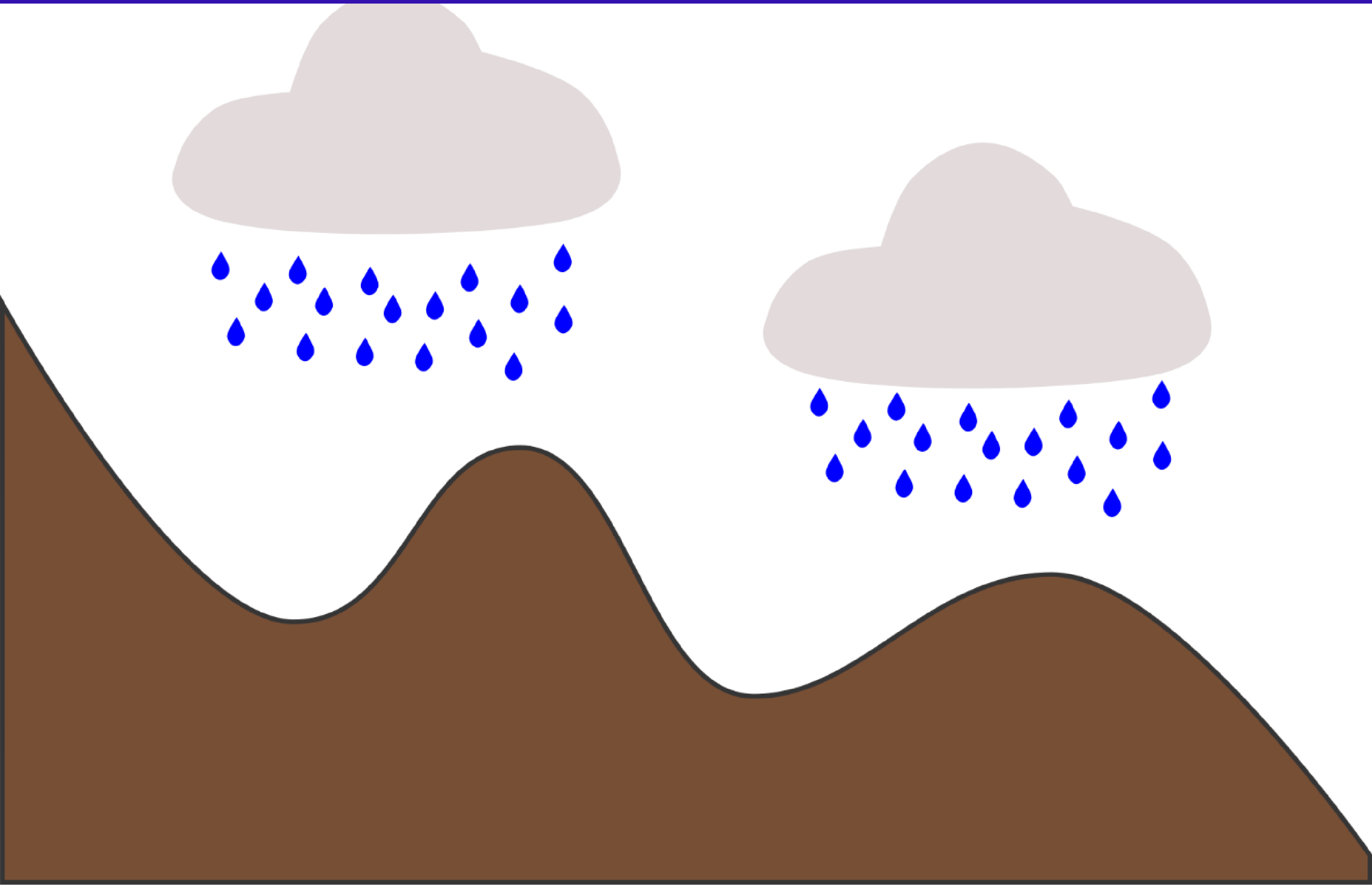
Nuclear Safety Institute of the Russian Academy of Sciences, Moscow

ИБРАЭ

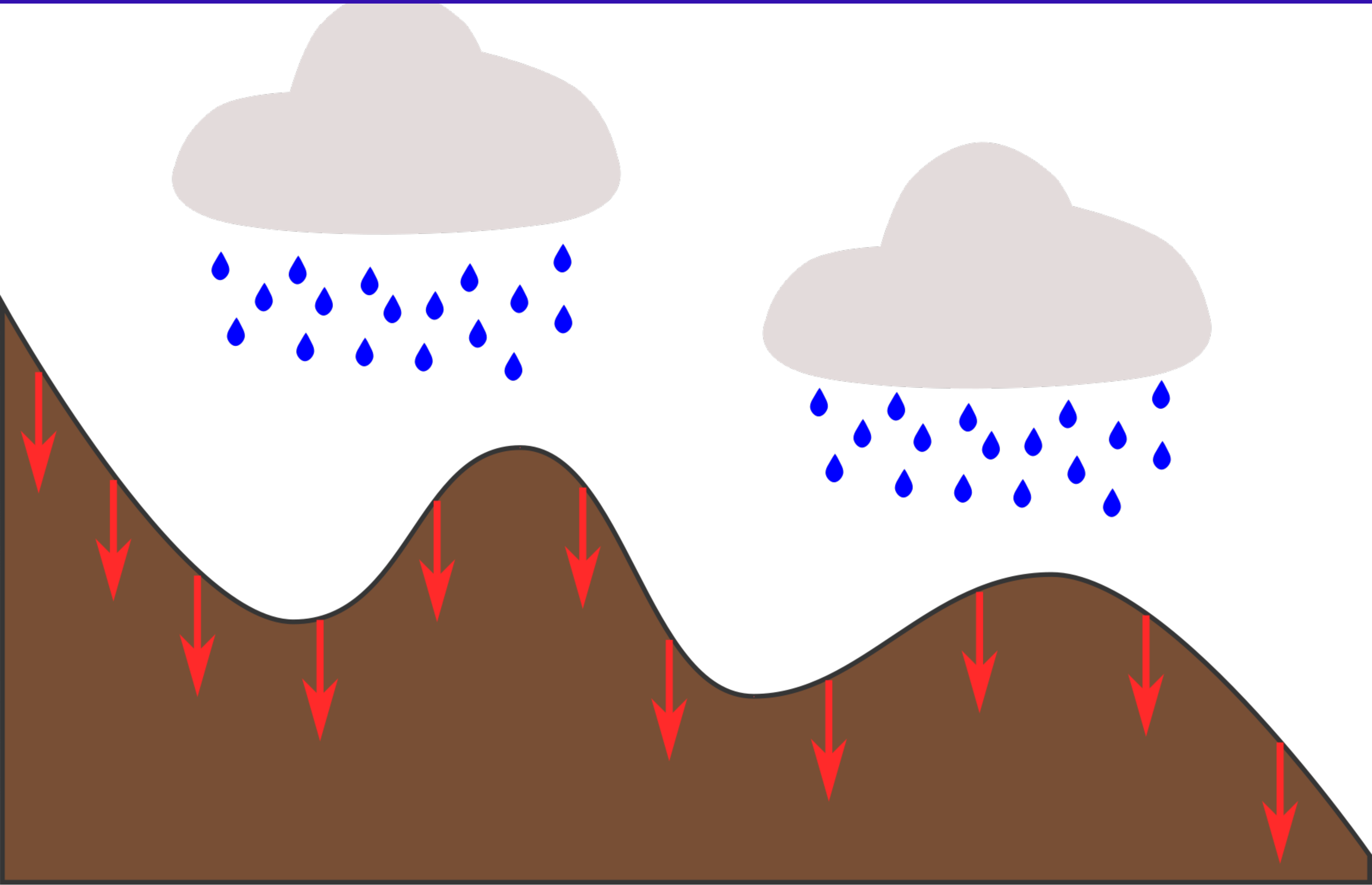
Marchuk Institute of Numerical Mathematics of the Russian Academy of Sciences, Moscow

ИВМ

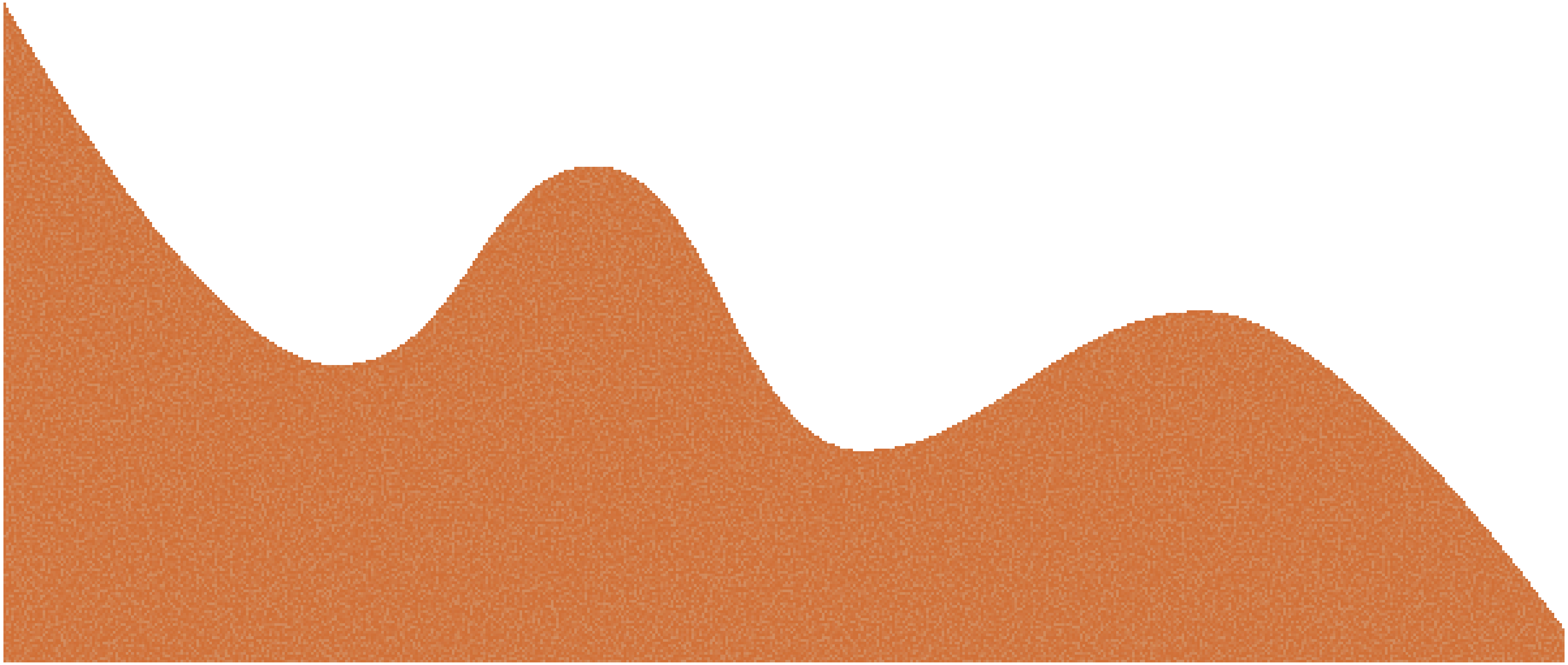
Without surface runoff



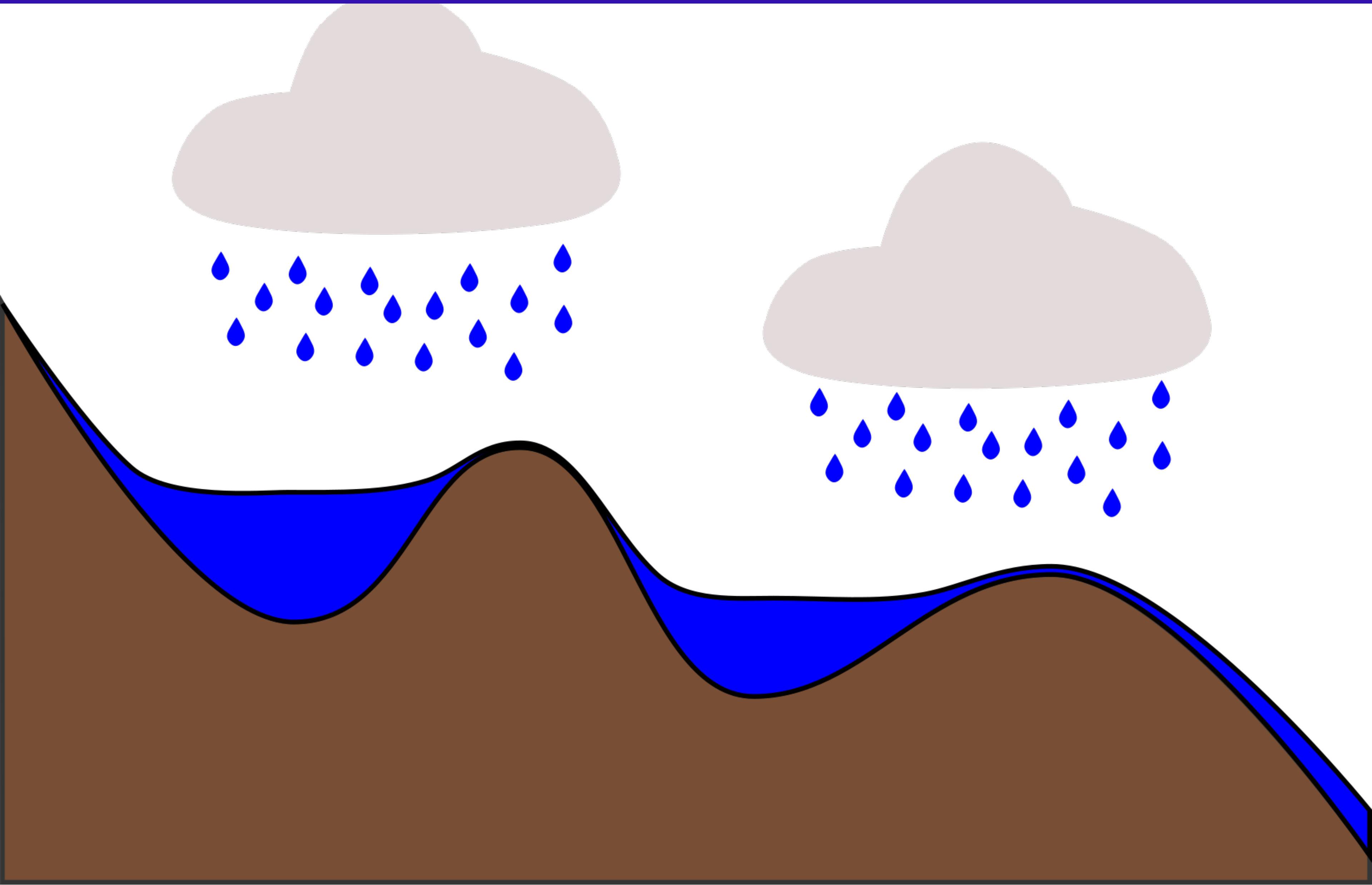
Without surface runoff



Without surface runoff



With surface runoff



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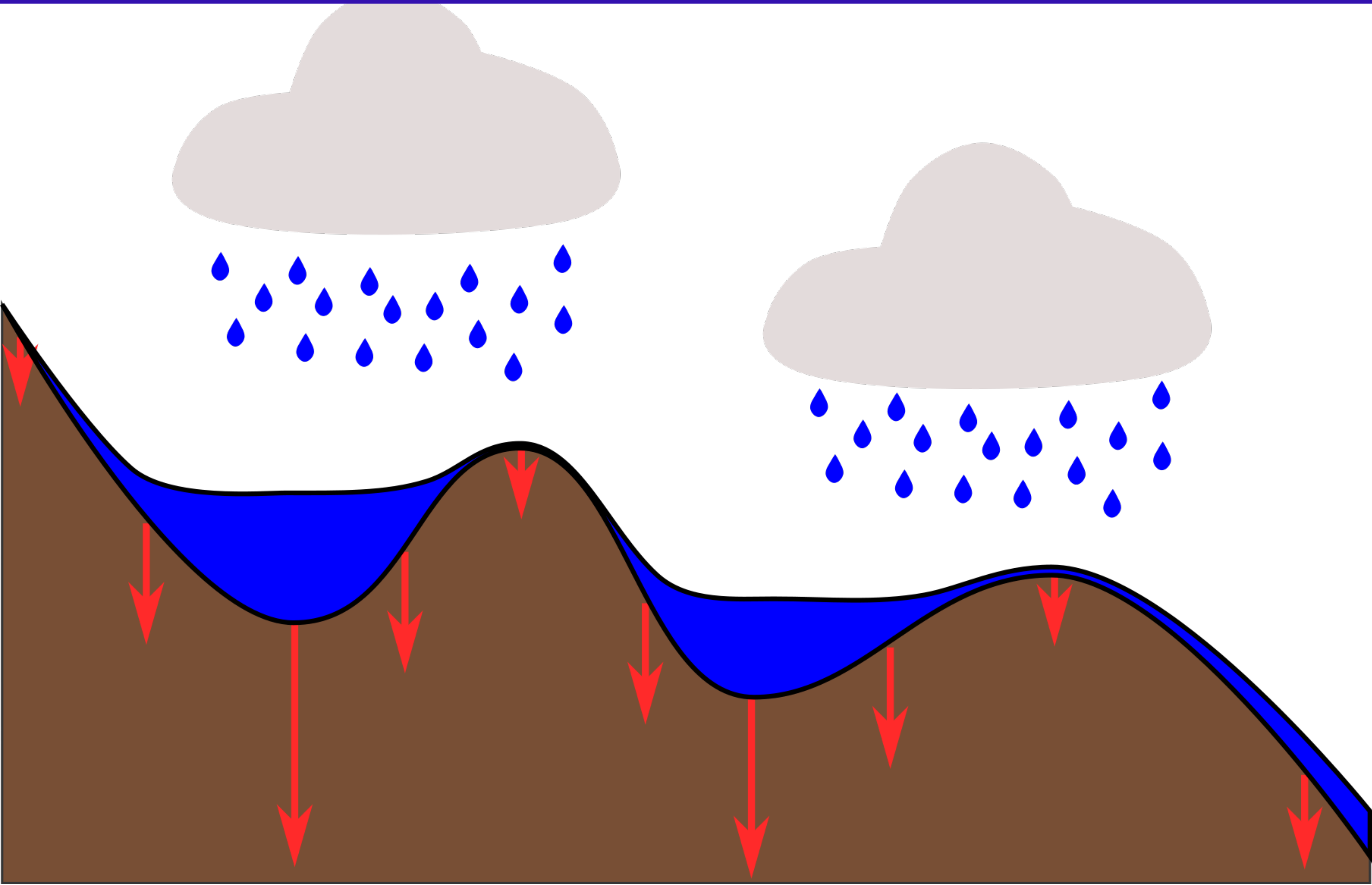
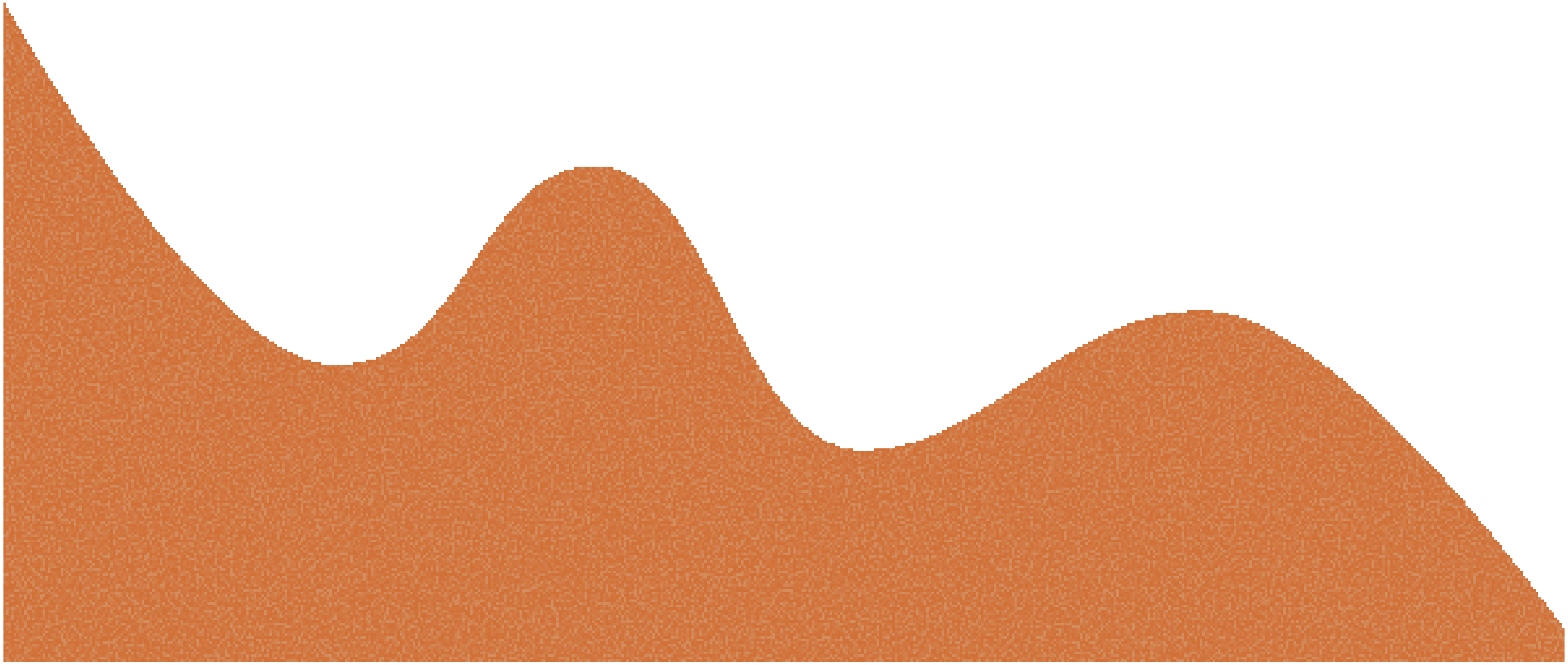
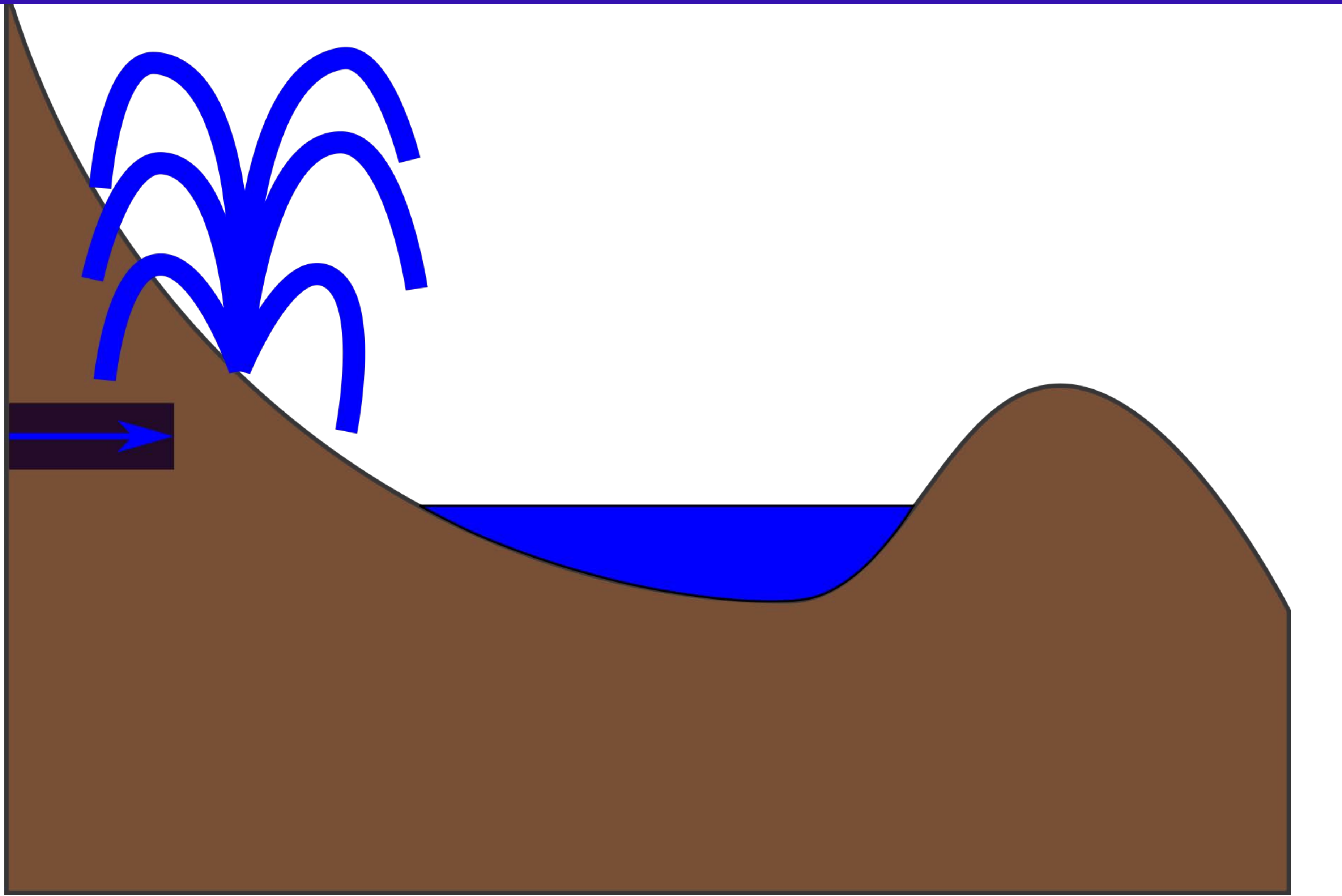


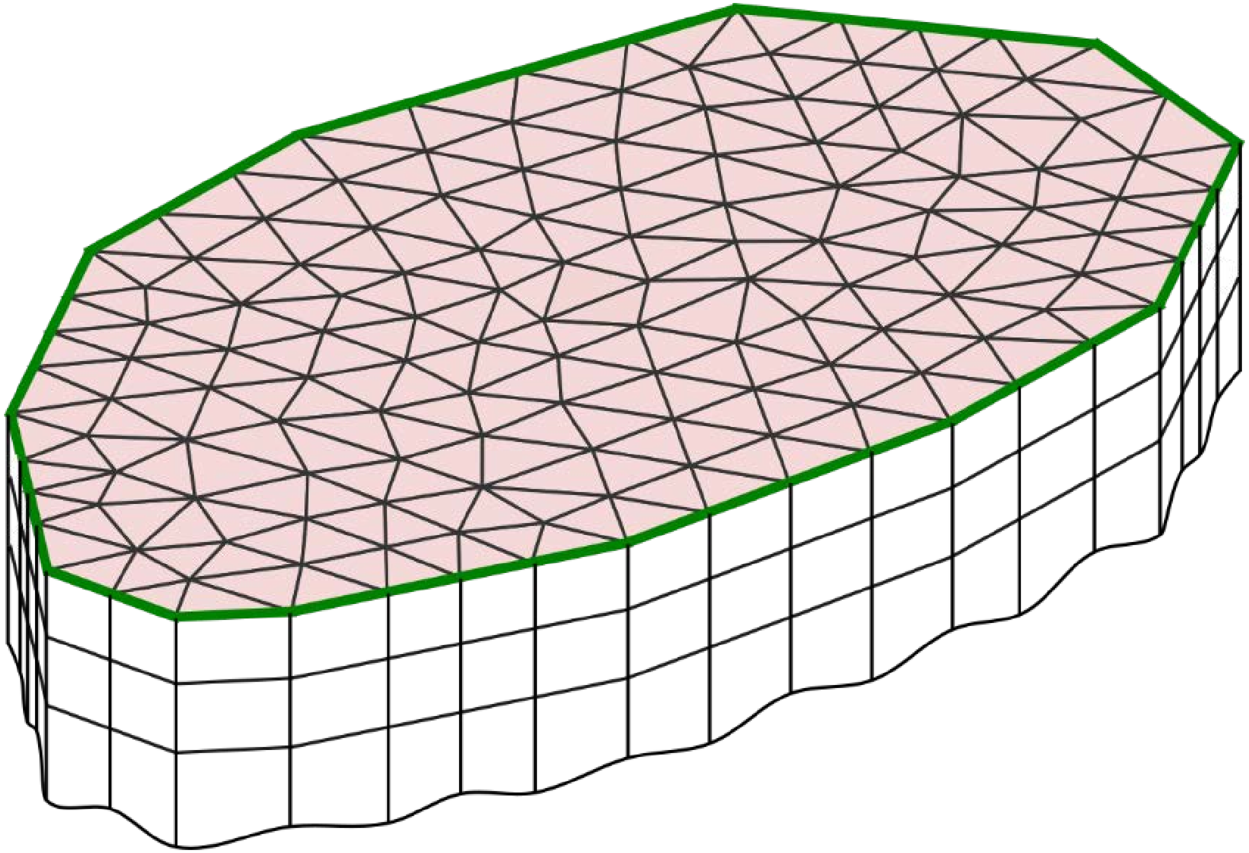
Without surface runoff



With surface runoff

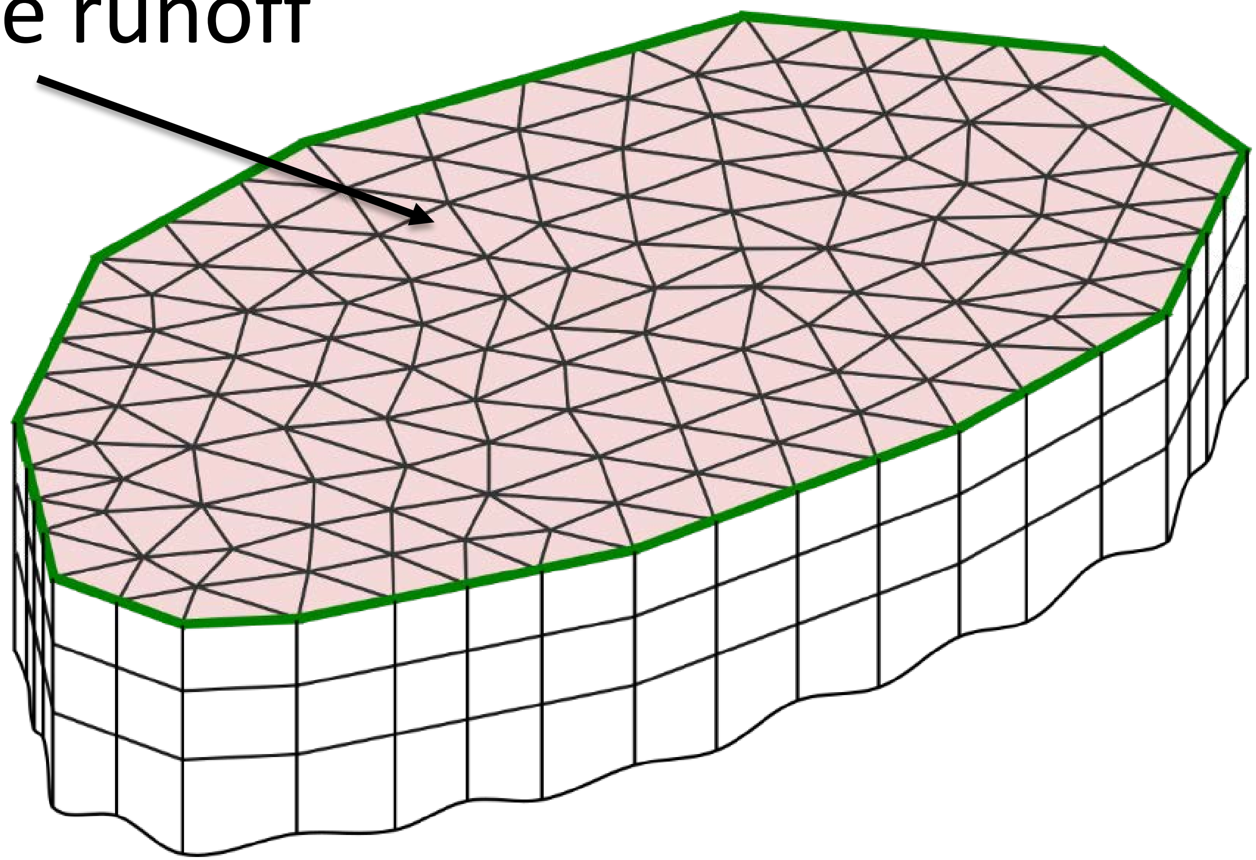


Coupled model mesh

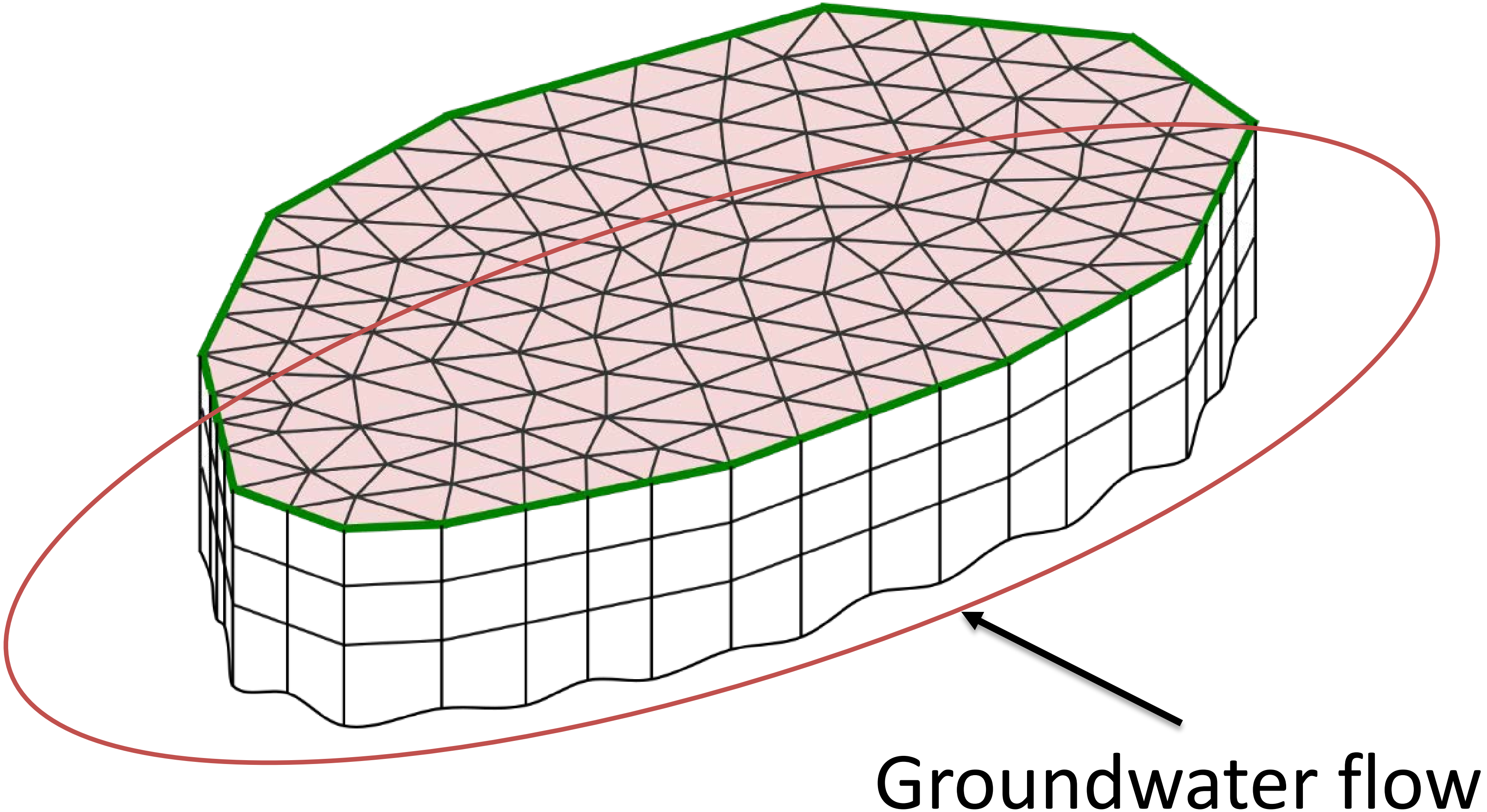


Coupled model mesh

Surface runoff



Coupled model mesh



Surface runoff model

Diffusive wave approximation of shallow water equations:

$$\frac{\partial h_s}{\partial t} - \frac{\partial}{\partial x}\left(\frac{h_s^{5/3}}{v\sqrt{|\nabla H_s|}} \frac{\partial H_s}{\partial x} \right) - \frac{\partial}{\partial y}\left(\frac{h_s^{5/3}}{v\sqrt{|\nabla H_s|}} \frac{\partial H_s}{\partial y} \right) = q - q_{ss}$$

- h_s – water depth
- $H_s = h_s + z$ – water elevation
- q – sources/sinks
- q_{ss} – surface-subsurface flux
- v – Manning's roughness coefficient

Groundwater flow model

Modified Richards equation:

$$\frac{\partial \theta(h_g)}{\partial t} + Ss_{stor}\frac{\partial h_g}{\partial t} - \nabla \cdot \left(K_g \nabla (h_g + z)\right) = 0$$

- θ – water content
- h_g – pressure head
- $S = S(h_g)$ – saturation
- s_{stor} – specific storage
- $K_g = K(h_g)$ – hydraulic conductivity

Groundwater flow model

Van Genuchten model:

$$\theta(h_g) = \begin{cases} \theta_r + \dfrac{\theta_s - \theta_r}{[1 + |\alpha h_g|^n]^m} & \text{for } h_g < 0 \\ \theta_s & \text{for } h_g \geq 0 \end{cases}$$

$$m = 1 - \frac{1}{n}$$

$$S_e = \frac{\theta - \theta_r}{\theta_s - \theta_r}$$

Mualem's model:

$$K(h) = K_s \cdot S_e^{0.5} [1 - (1 - S_e^{\frac{1}{m}})^m]^2,$$

$$S = \frac{\theta}{\theta_s}$$

- θ – water content
- θ_r, θ_s – residual and maximum water content
- α, n – medium parameters
- K_s – saturated conductivity
- S – saturation
- S_e – effective saturation

Coupling of the models

$$\frac{\partial h_s}{\partial t} - \frac{\partial}{\partial x}\left(\frac{h_s^{5/3}}{\nu\sqrt{|\nabla H_s|}}\frac{\partial H_s}{\partial x}\right) - \frac{\partial}{\partial y}\left(\frac{h_s^{5/3}}{\nu\sqrt{|\nabla H_s|}}\frac{\partial H_s}{\partial y}\right) = q - q_{ss}$$

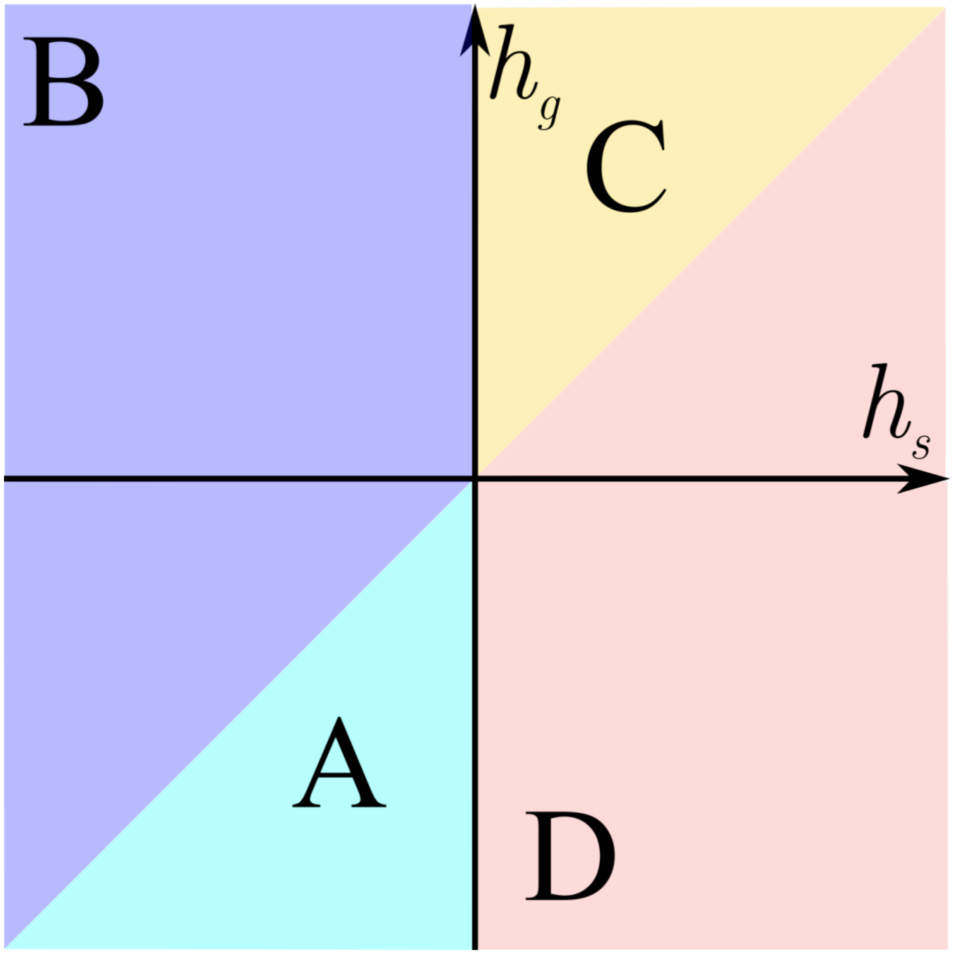
$$-K_g \nabla h_g = \begin{cases} q_{ss} & \text{if } x \in (surface) \\ 0 & \text{otherwise} \end{cases}$$

Coupling of the models

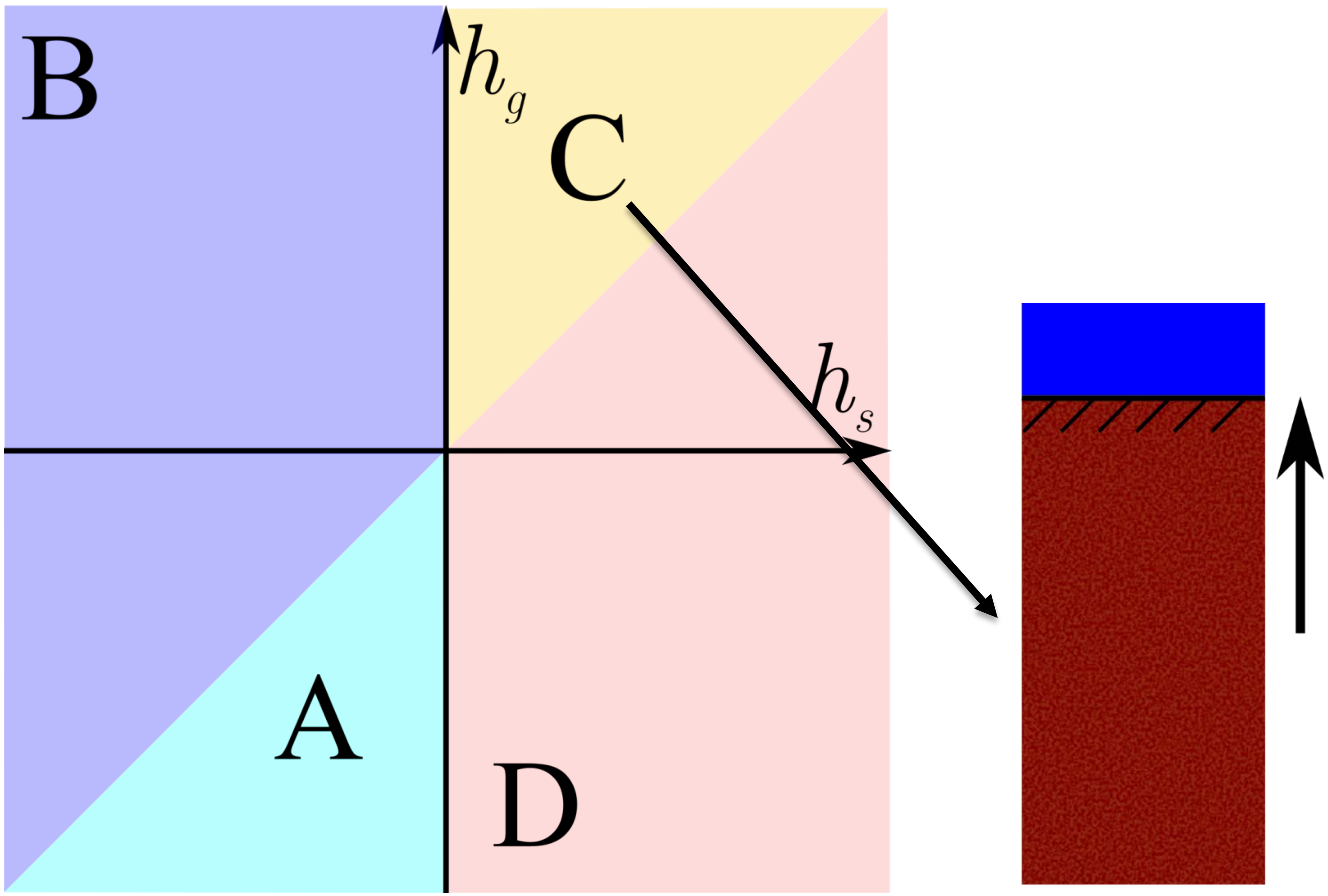
$$q_{ss} = \frac{K_{ss}}{d}(h_s - h_g)$$

- q_{ss} – surface-subsurface flux
- K_{ss} – near-bottom sediment conductivity
- d – near-bottom sediment layer width
- h_g – groundwater pressure head
- h_s – surface water depth

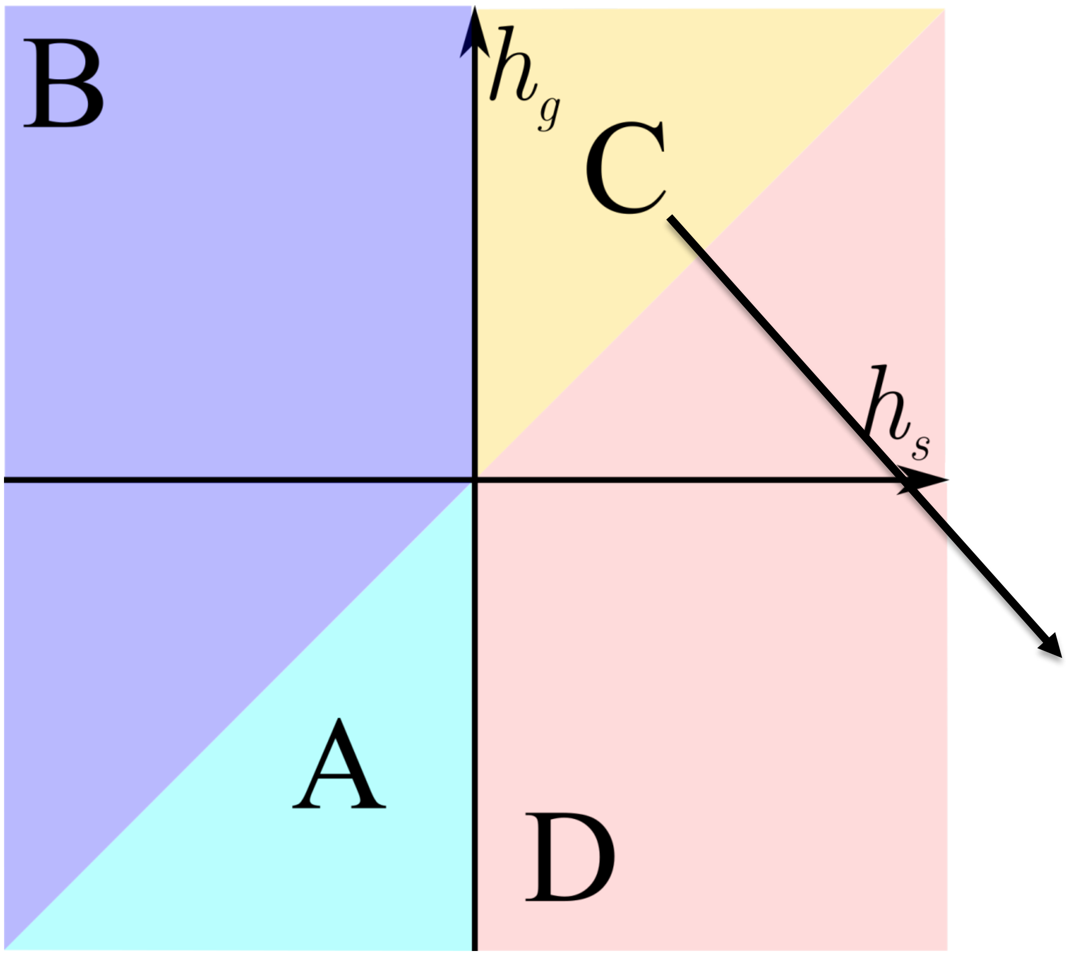
Coupling of the models



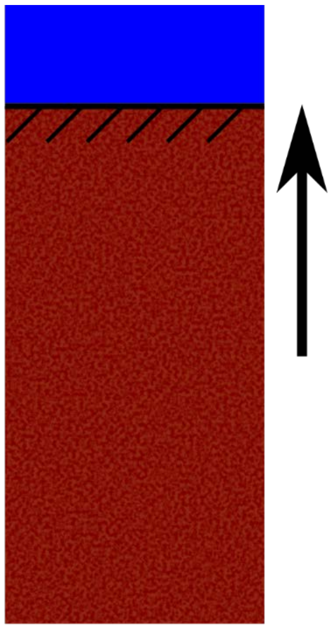
Coupling of the models



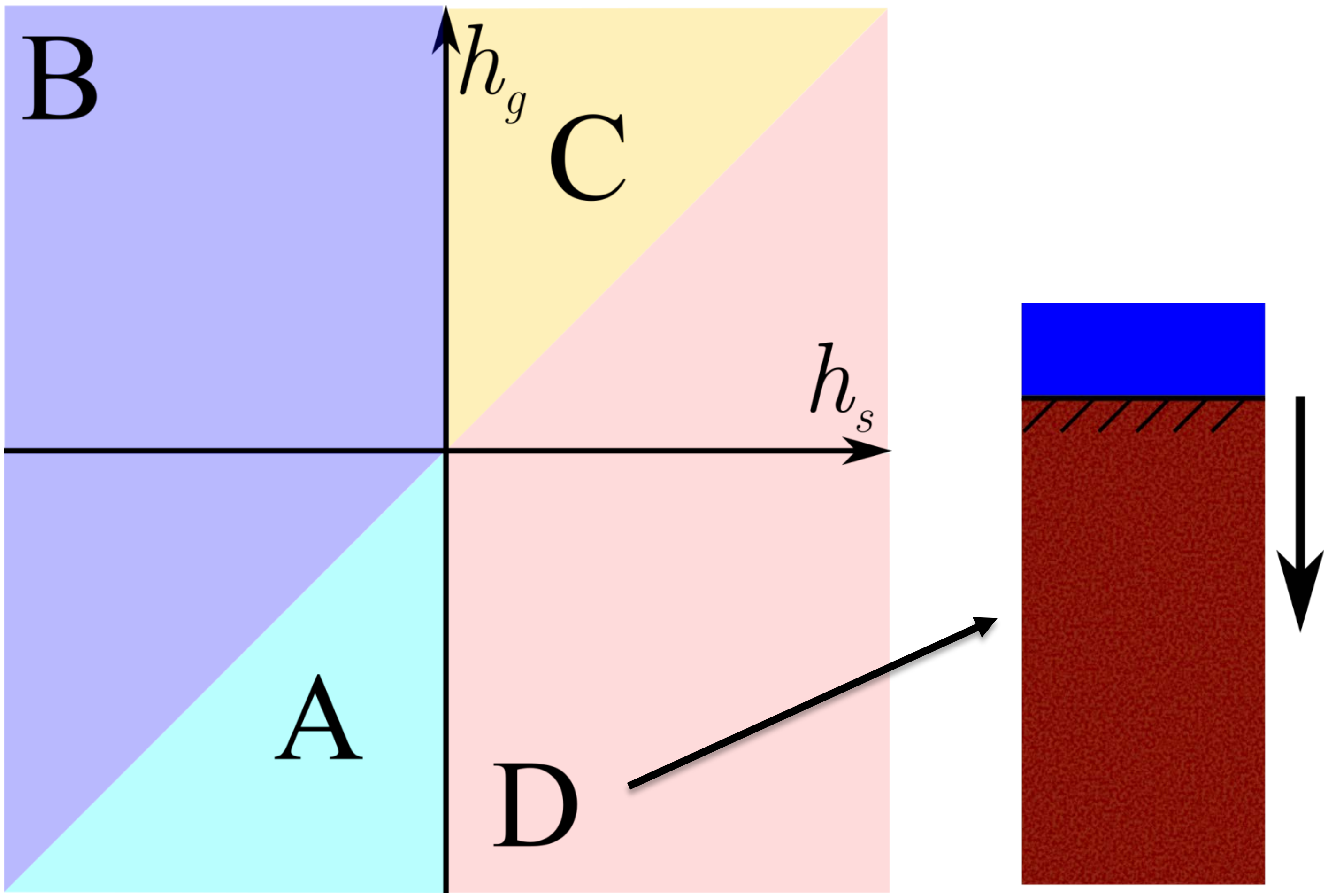
Coupling of the models



$$q_{ss} = \frac{K_{ss}}{d}(h_s - h_g)$$



Coupling of the models



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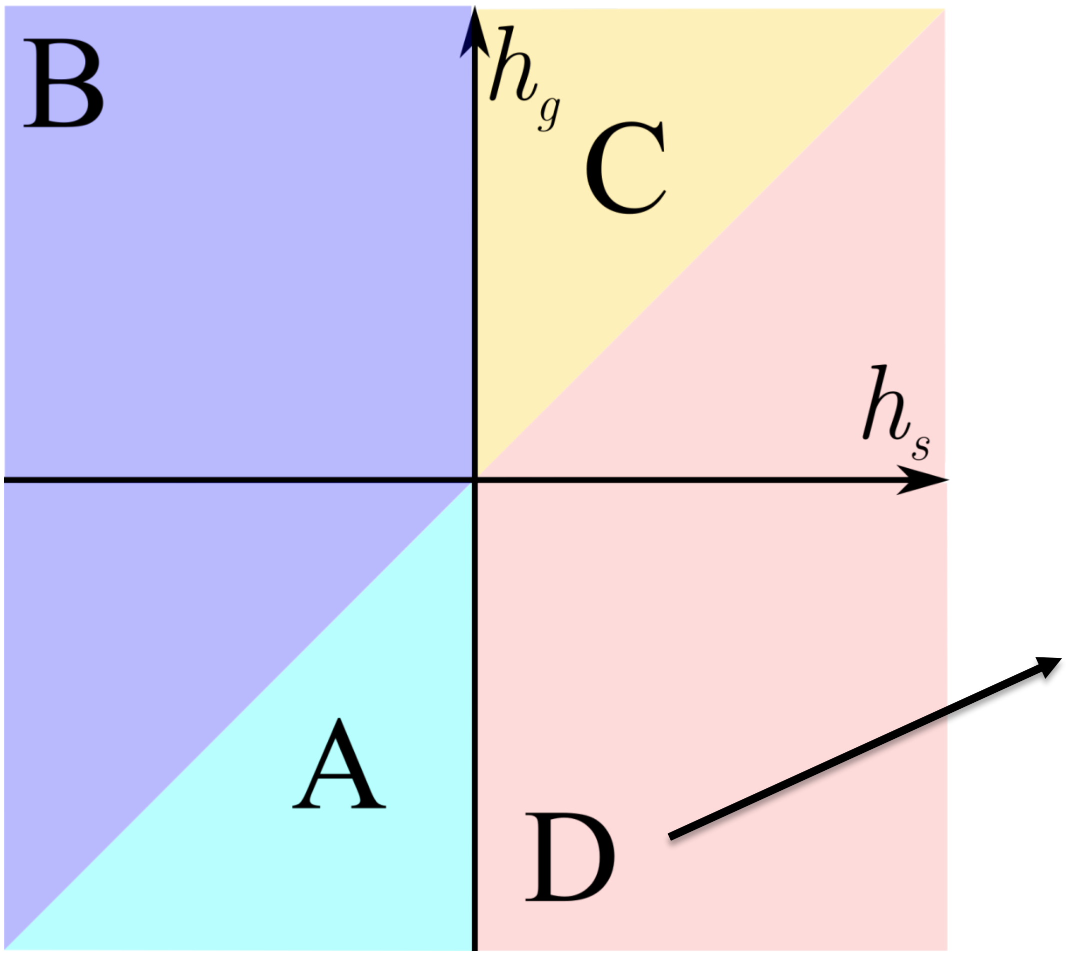
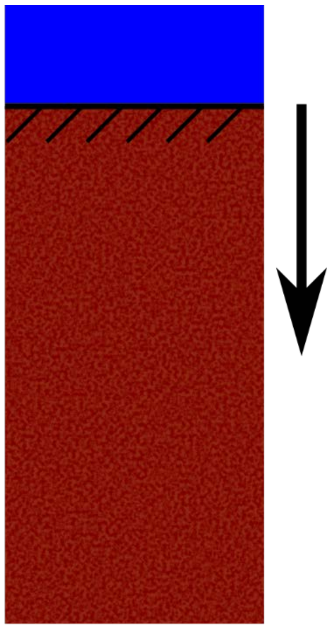


$$q_{ss} = \frac{K_{ss}}{d}(h_s - h_g)$$



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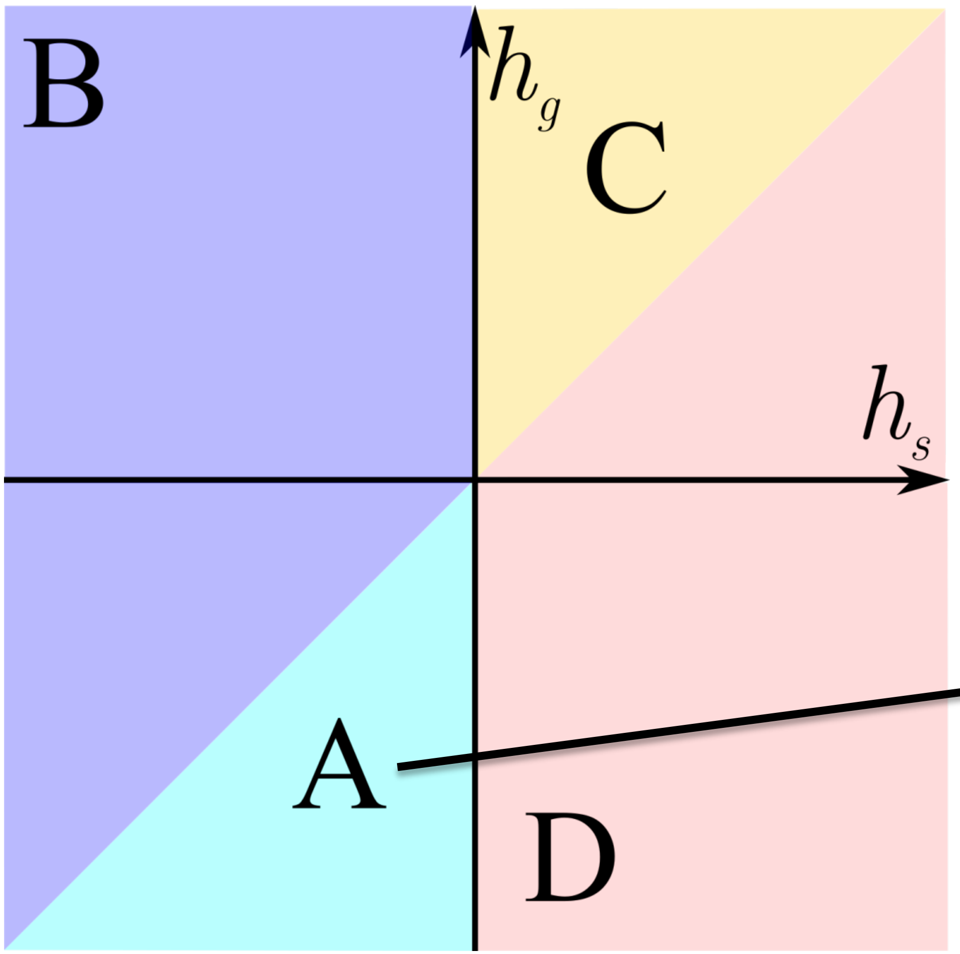
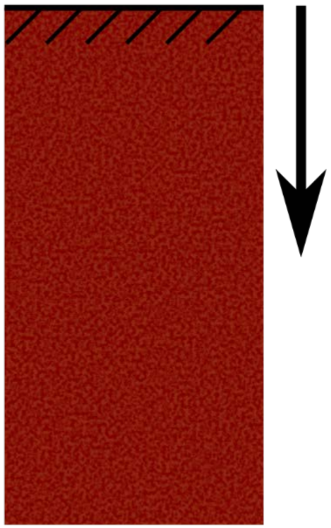
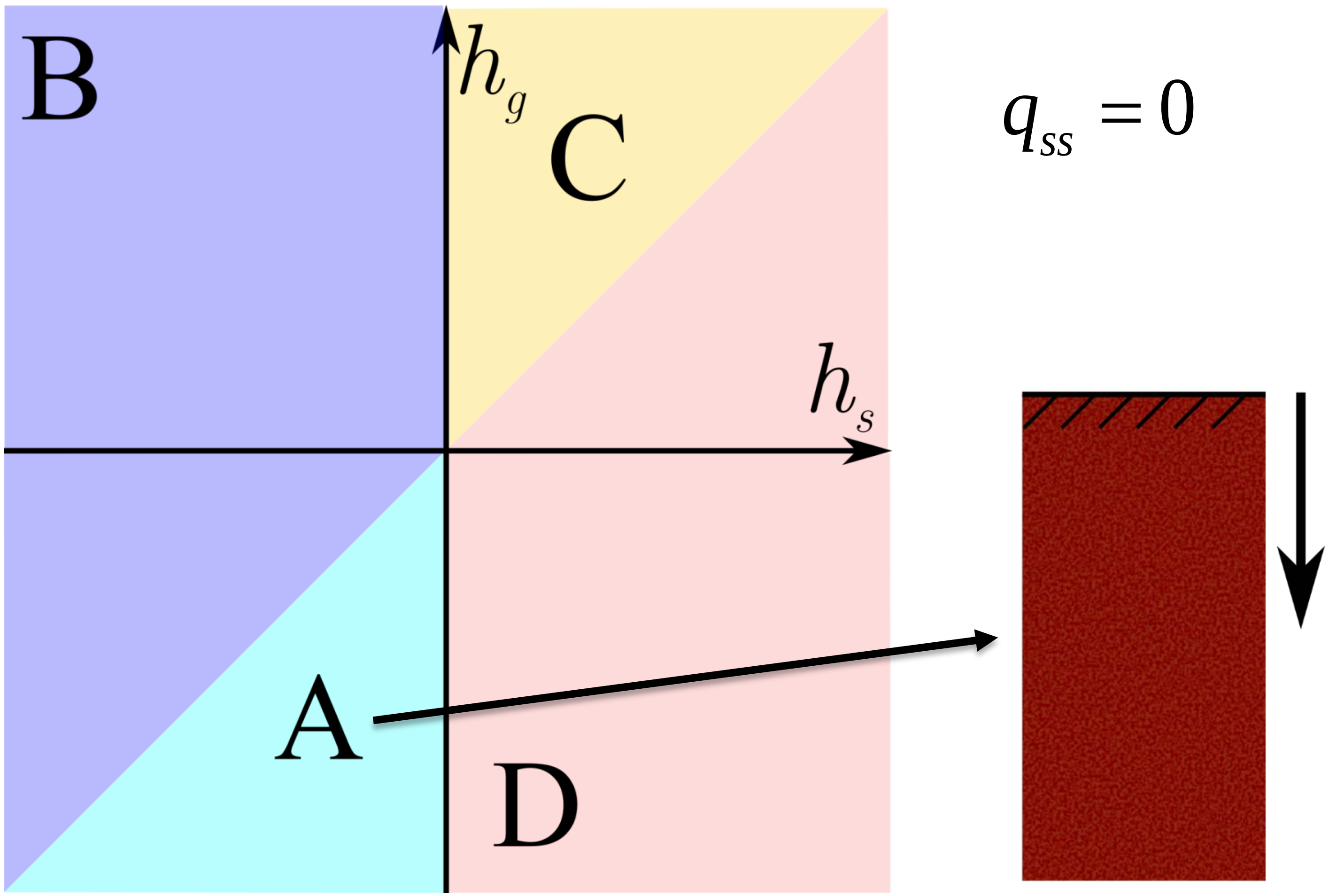


$$q_{ss} = \frac{K_{ss}}{d}(h_s - h_g) > 0$$

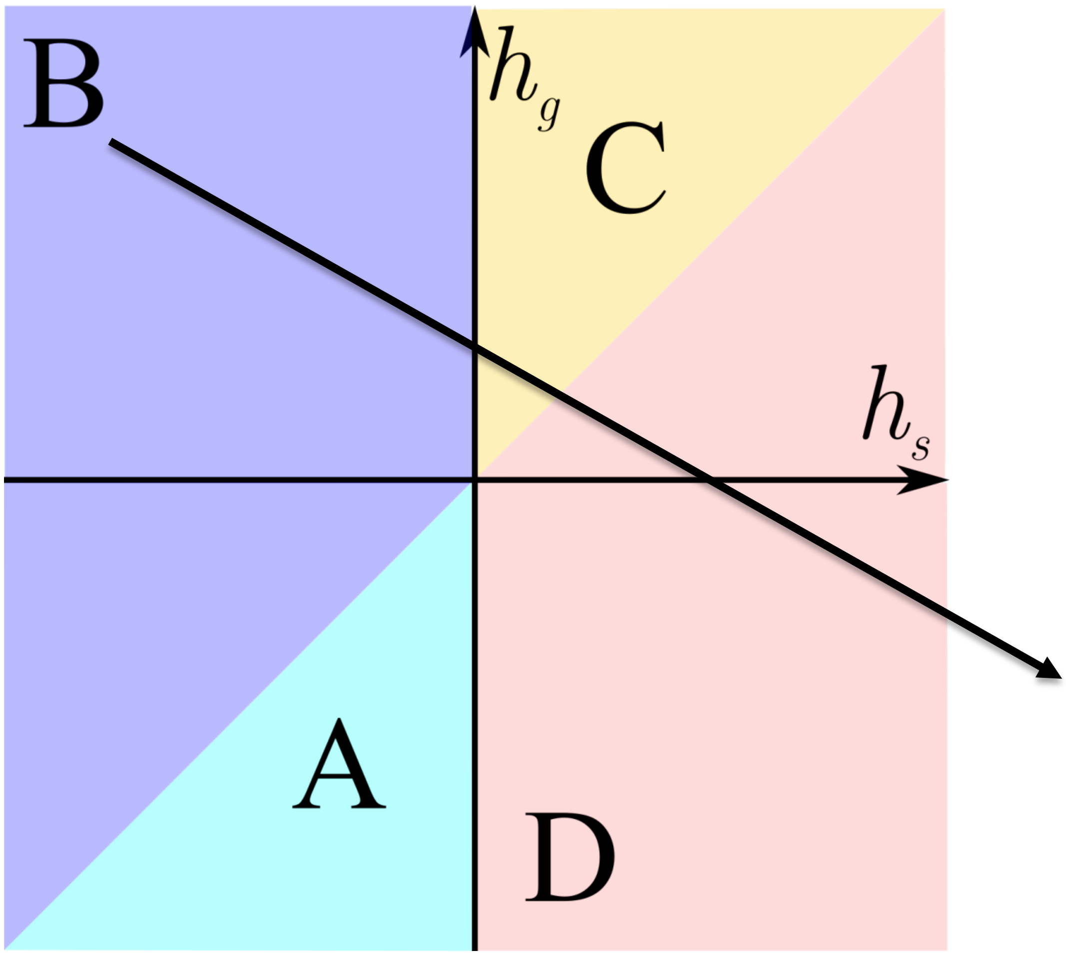


Coupling of the models

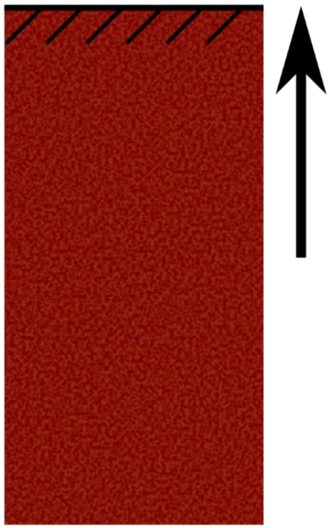


$$q_{ss} = 0$$

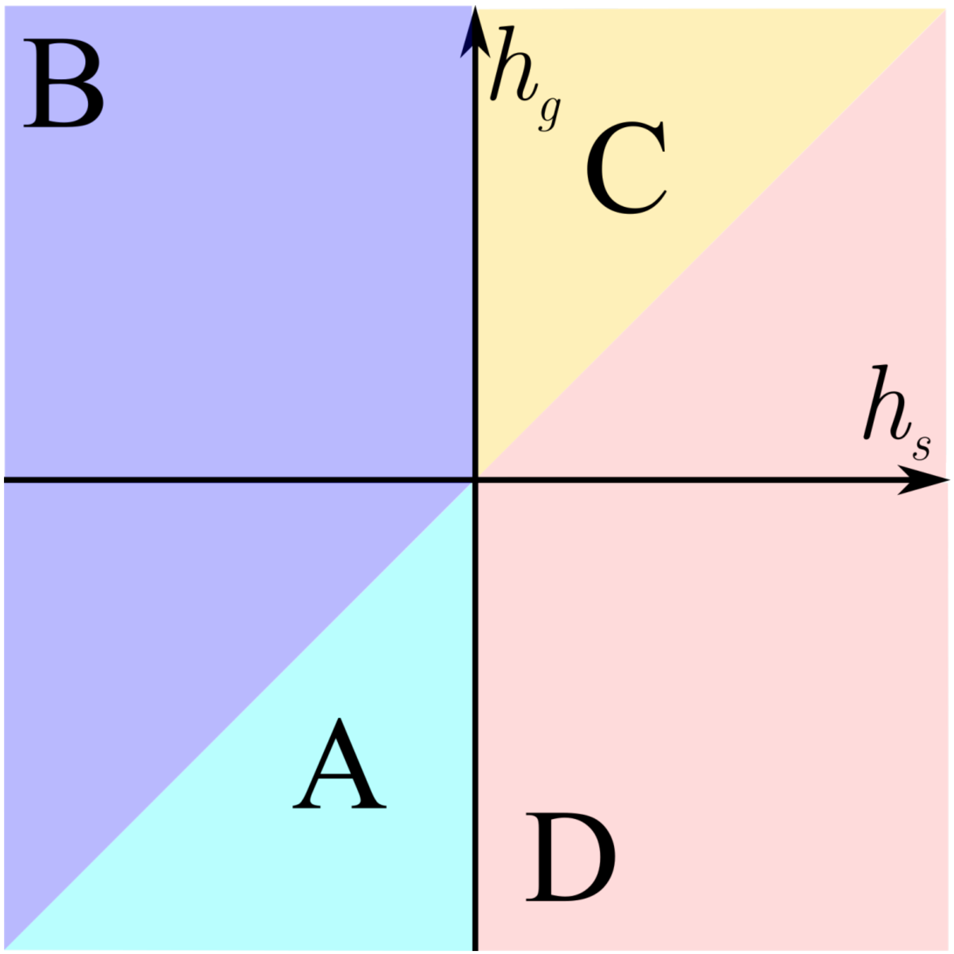
Coupling of the models



$$q_{ss} = \frac{K_{ss}}{d}(h_s - h_g) < 0$$



Coupling of the models



$$q_{ss} = \begin{cases} 0 \text{ in A} \\ \dfrac{K_{ss}}{d}(h_s - h_g) \text{ in B,C,D} \end{cases}$$

Surface runoff model discretization

$$\frac{\partial h_s}{\partial t} - \frac{\partial}{\partial x}\left(\frac{h_s^{5/3}}{v\sqrt{|\nabla H_s|}} \frac{\partial H_s}{\partial x} \right) - \frac{\partial}{\partial y}\left(\frac{h_s^{5/3}}{v\sqrt{|\nabla H_s|}} \frac{\partial H_s}{\partial y} \right) = q - q_{ss}$$

- h_s – water depth
- $H_s = h_s + z$ – water elevation
- q – sources/sinks
- q_{ss} – surface-subsurface flux
- v – Manning's roughness coefficient

Surface runoff model discretization

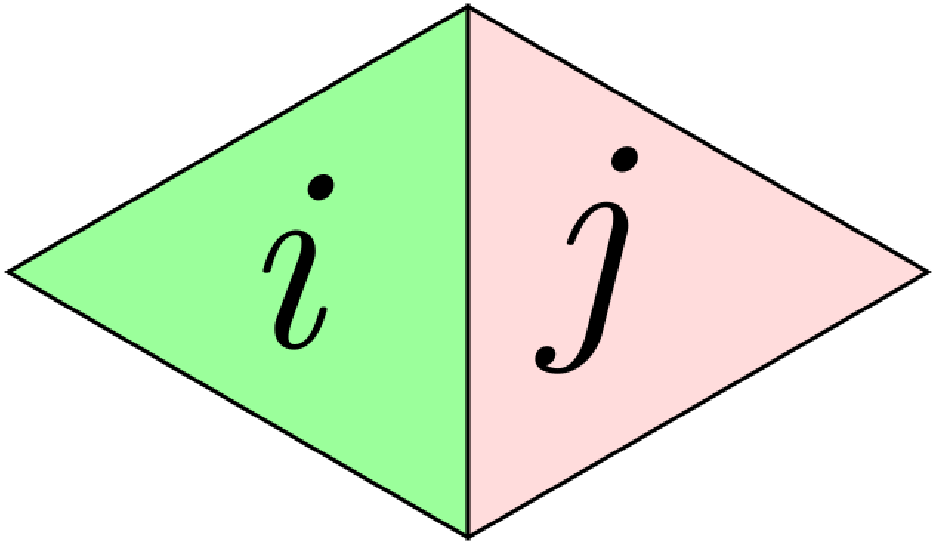
$$\frac{\partial h_s}{\partial t} - \nabla\left(K_s \nabla H_s\right) = q - q_{ss} \quad \Leftrightarrow \quad \frac{\partial H_s}{\partial t} - \nabla\left(K_s \nabla H_s\right) = q - q_{ss}$$

$$K_s = \frac{h_s^{5/3}}{\nu\sqrt{|\nabla H_s|}}$$

- h_s – water depth
- $H_s = h_s + z$ – water elevation
- q – sources/sinks
- q_{ss} – surface-subsurface flux
- ν – Manning's roughness coefficient

Discretization of diffusion coefficient of surface runoff model (numerator)

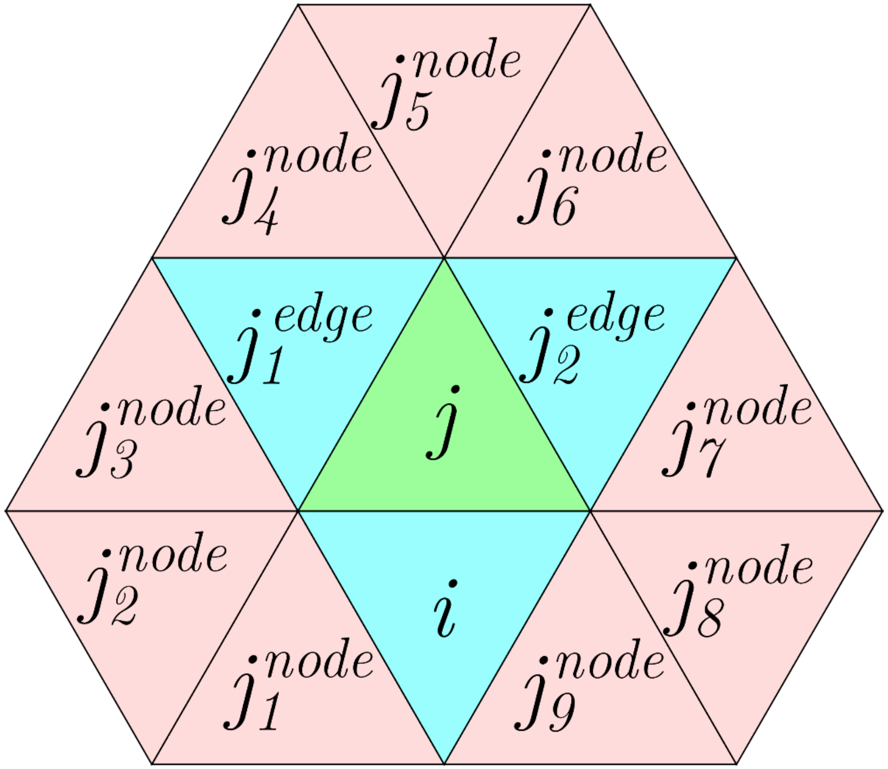
$$K_s = \frac{h_s^{5/3}}{\nu\sqrt{|\nabla H_s|}}$$



$$\left(h_s^{5/3}\right)_{ij} = (h_{s,ij})^{5/3} = \begin{cases} (h_{s,i})^{5/3}, H_{s,i} \geq H_{s,j}, \\ (h_{s,j})^{5/3}, H_{s,i} < H_{s,j}. \end{cases}$$

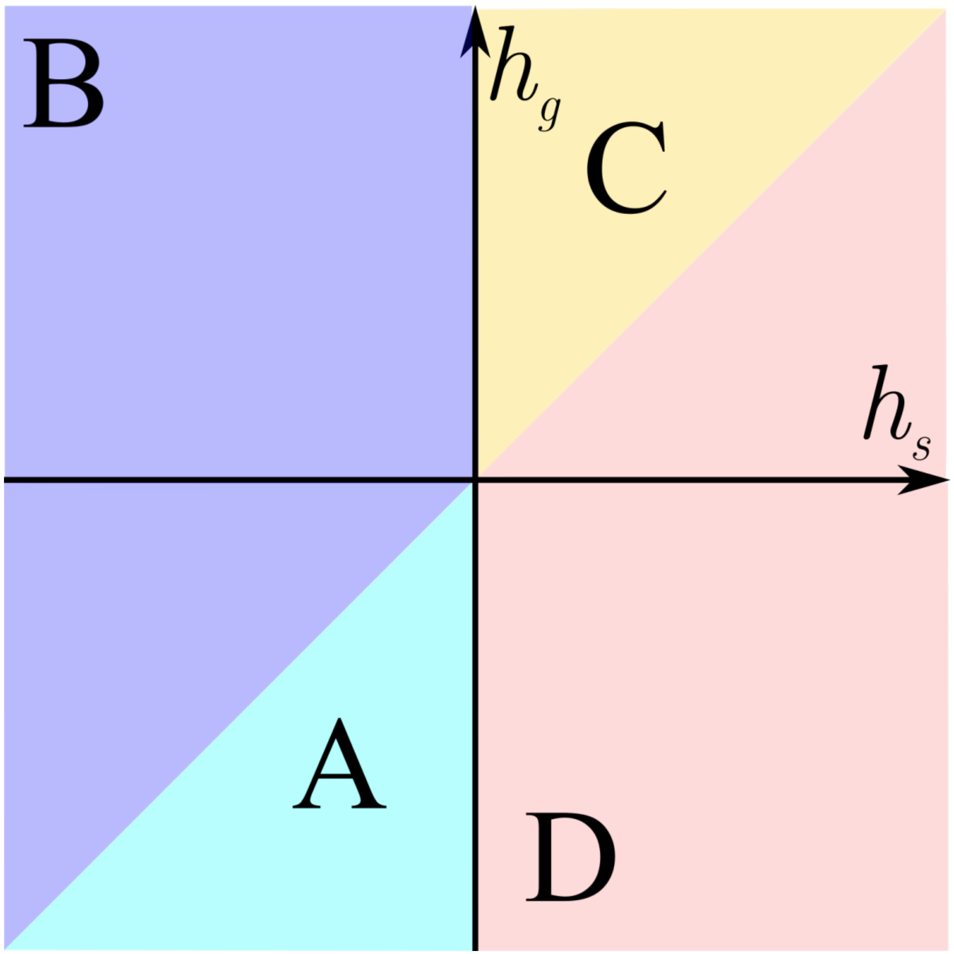
Discretization of diffusion coefficient of surface runoff model (denomenator)

$$K_s = \frac{h_s^{5/3}}{\nu\sqrt{|\nabla H_s|}}$$



$$H_{s,\alpha} = H_{s,j} + (x_\alpha - x_j)\frac{\partial H_{s,j}}{\partial x} + (y_\alpha - y_j)\frac{\partial H_{s,j}}{\partial y}$$

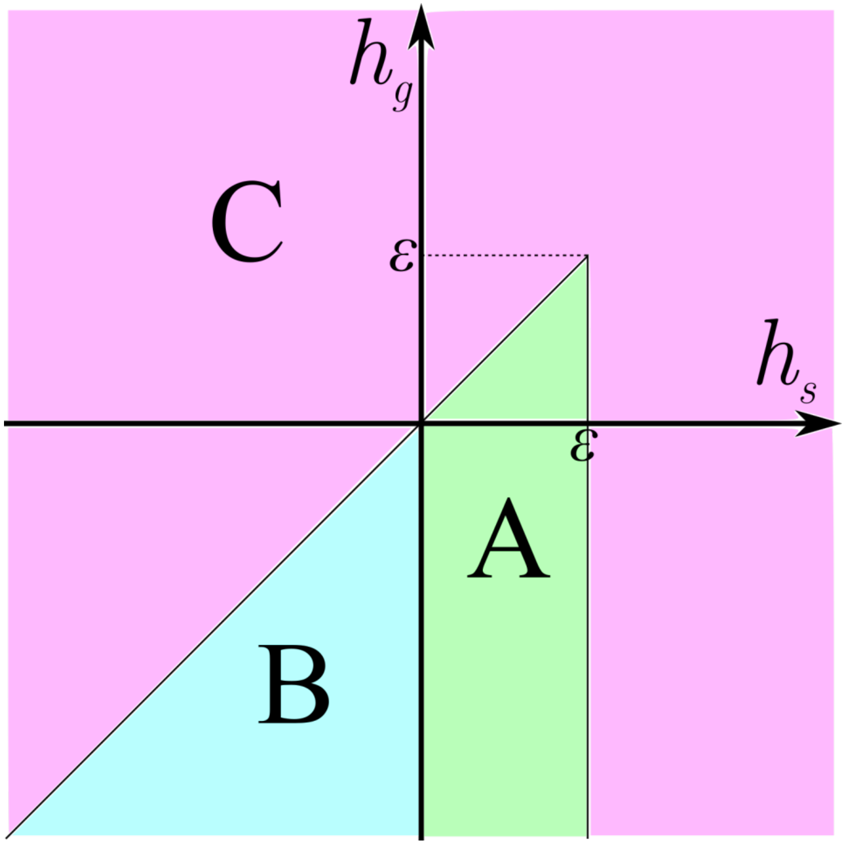
Coupling of the models



$$q_{ss} = \begin{cases} 0 \text{ in A} \\ \frac{K_{ss}}{d}(h_s - h_g) \text{ in B,C,D} \end{cases}$$

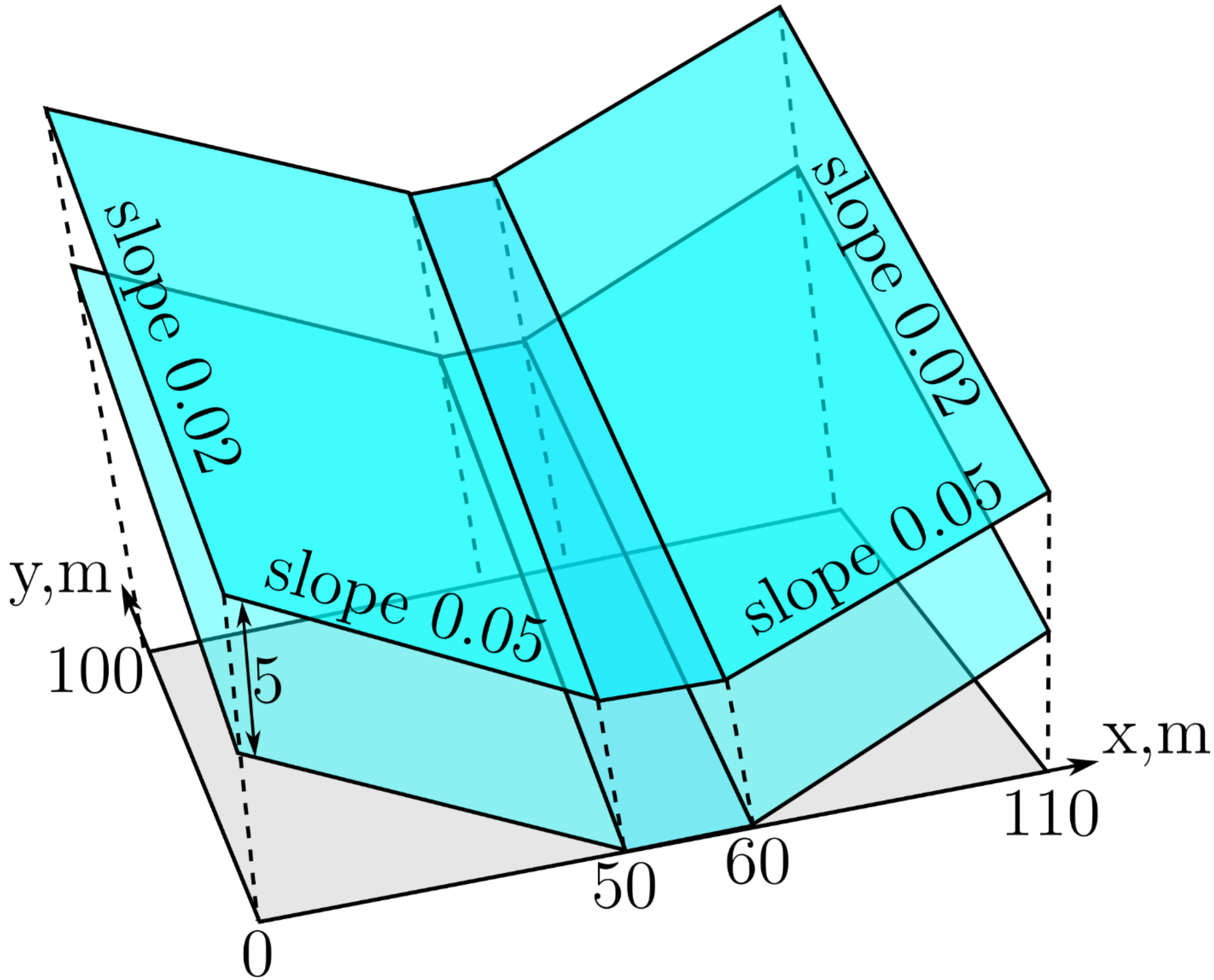
Smoothing surface-subsurface flux

$$q_{ss} = \begin{cases} \frac{K_{ss}}{d\varepsilon^2} h_s^3 - \frac{K_{ss}}{d\varepsilon} h_s^2 - \frac{K_{ss}}{d}(h_g - \varepsilon)\frac{1 - \cos\frac{\pi h_s}{\varepsilon}}{2}, (h_s, h_g) \in A \\ 0, (h_s, h_g) \in B \\ \frac{K_{ss}}{d}(h_s - h_g), (h_s, h_g) \in C \end{cases}$$



Numerical experiments

Tilted v-catchment with subsurface

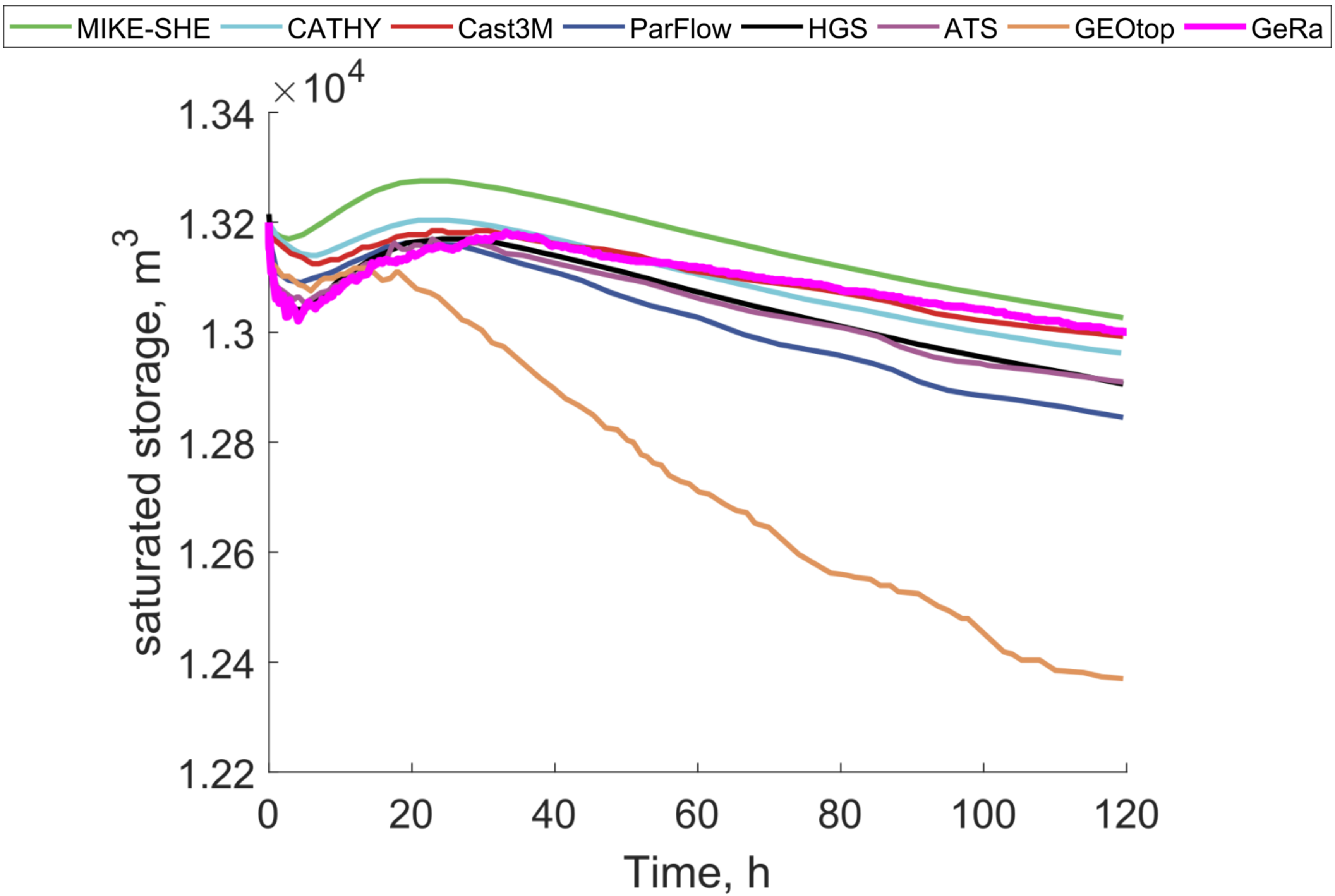


[S. Kollet et al. The integrated hydrologic model intercomparison project, ih-mip2: A second set of benchmark results to diagnose integrated hydrology and feedbacks // Water Resources Research 53 (1) (2017) 867-890]

Tilted v-catchment with subsurface

Parameter	Value
Manning's roughness channel, $(h/m^{1/3})$	1.74×10^{-3}
Manning's roughness banks, $(h/m^{1/3})$	1.74×10^{-4}
Saturated hydraulic conductivity, (m/h)	10
Residual volumetric water content, (-)	0.08
Saturated volumetric water content, (-)	0.4
Precipitation rate, (m/h)	Scenario I: 0; Scenario II: 0.1 for 20 h, 0 afterwards
Van Genuchten parameter n, (-)	2
Van Genuchten parameter α, (m)	6
Experiment duration, (h)	120
Bottom sediment width, (m)	0.2
Bottom sediment conductivity, (m/day)	20
Initial conditions	Water 2m below surface

Tilted v-catchment with subsurface first scenario



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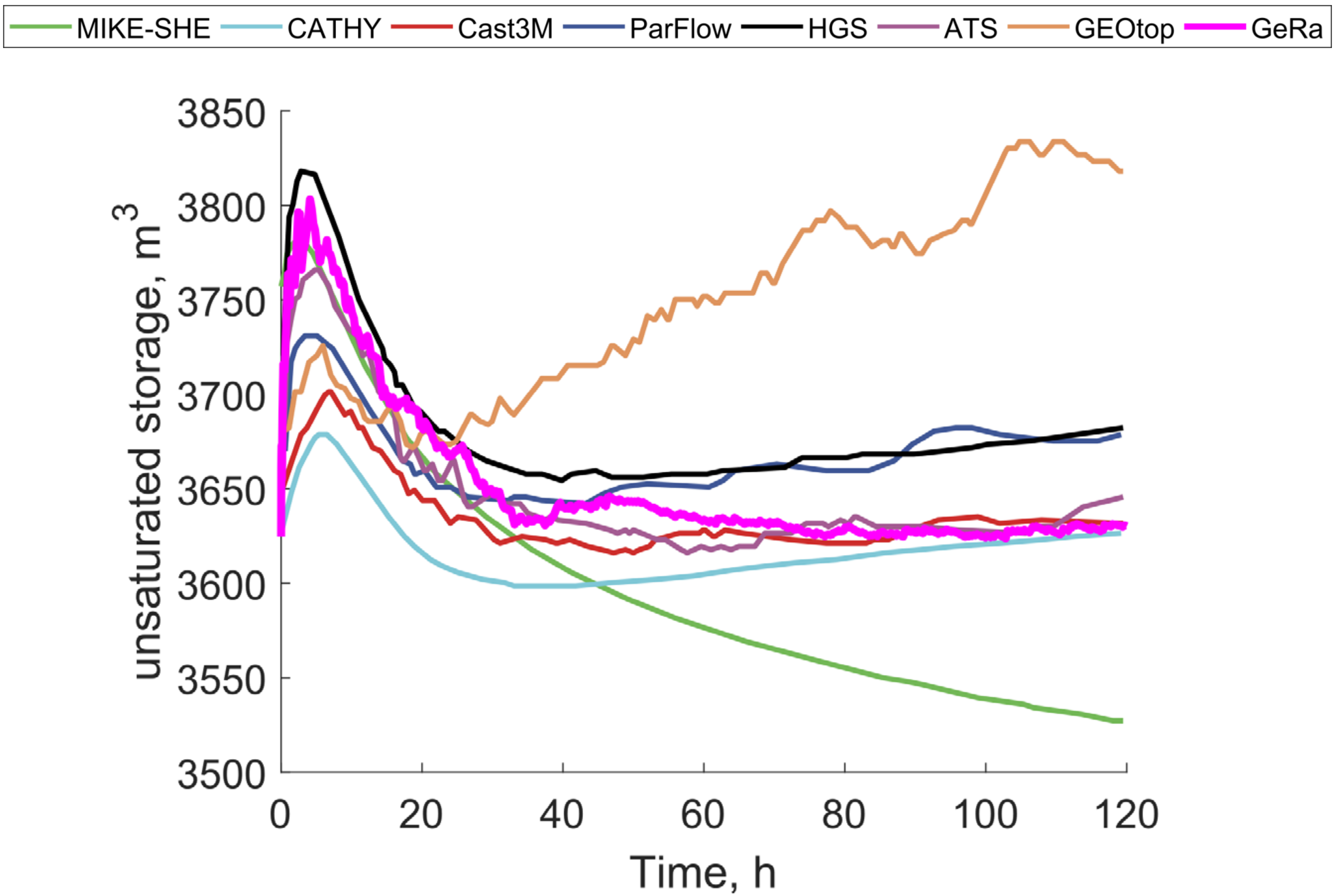

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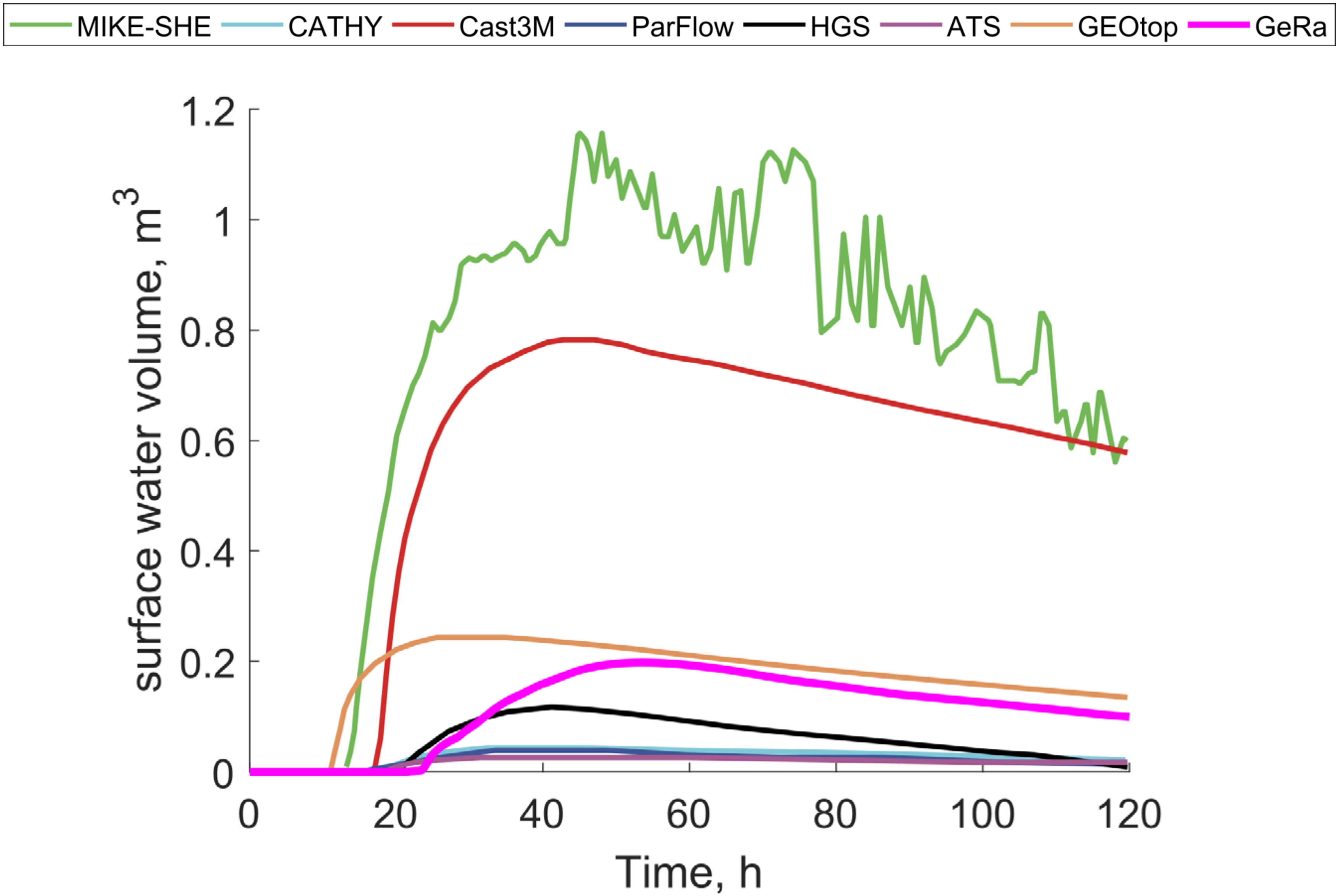

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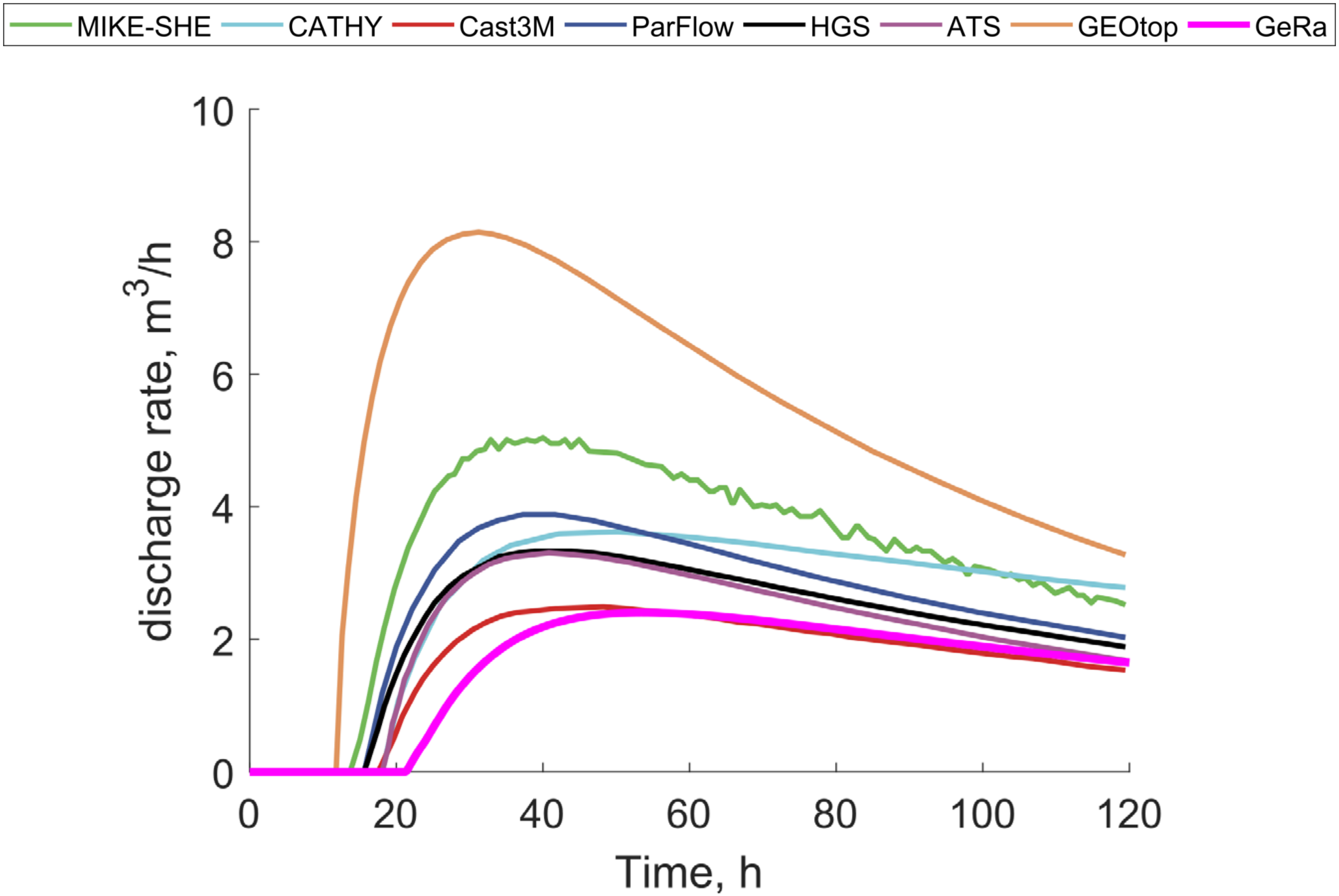
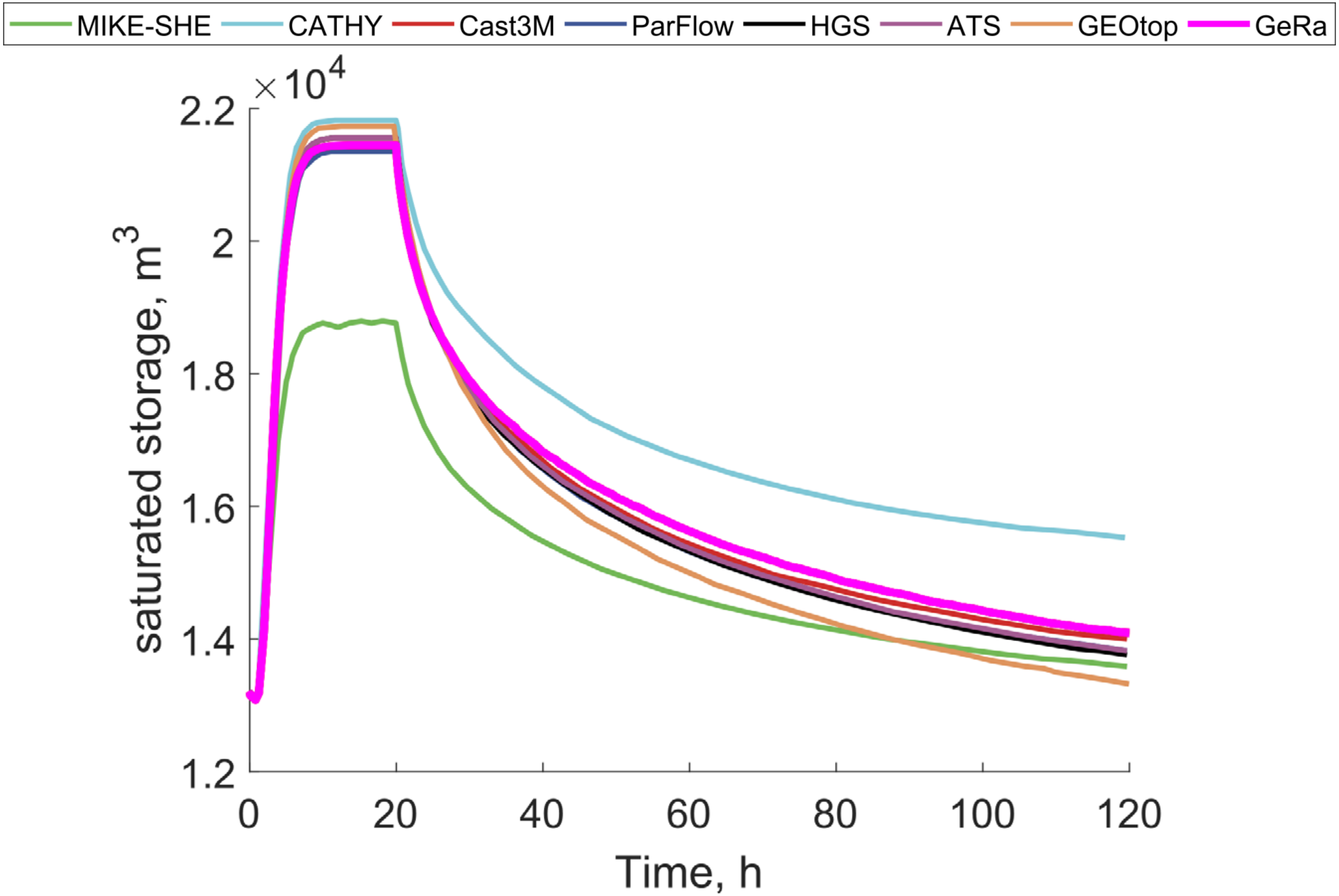
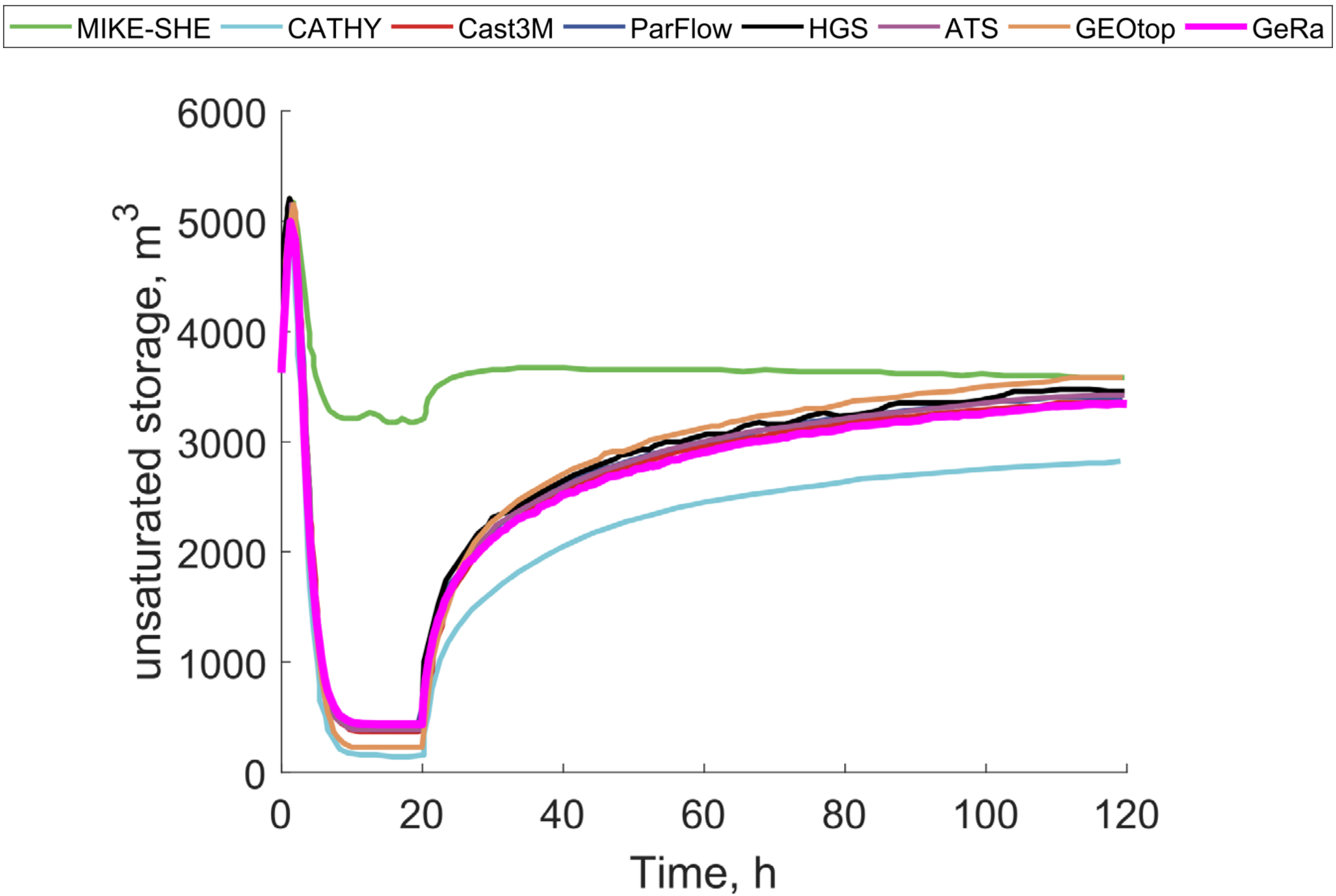


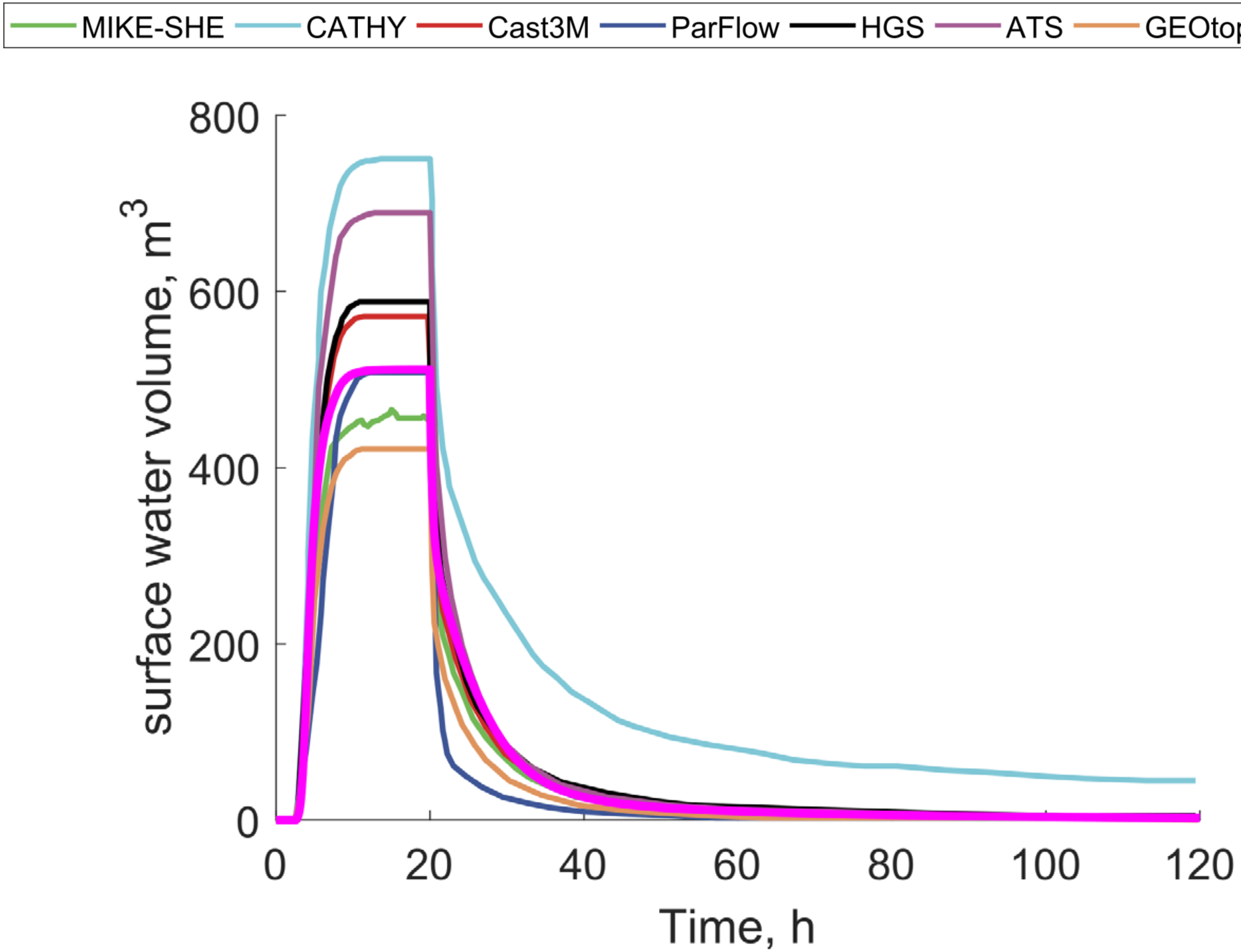
Tilted v-catchment with subsurface second scenario



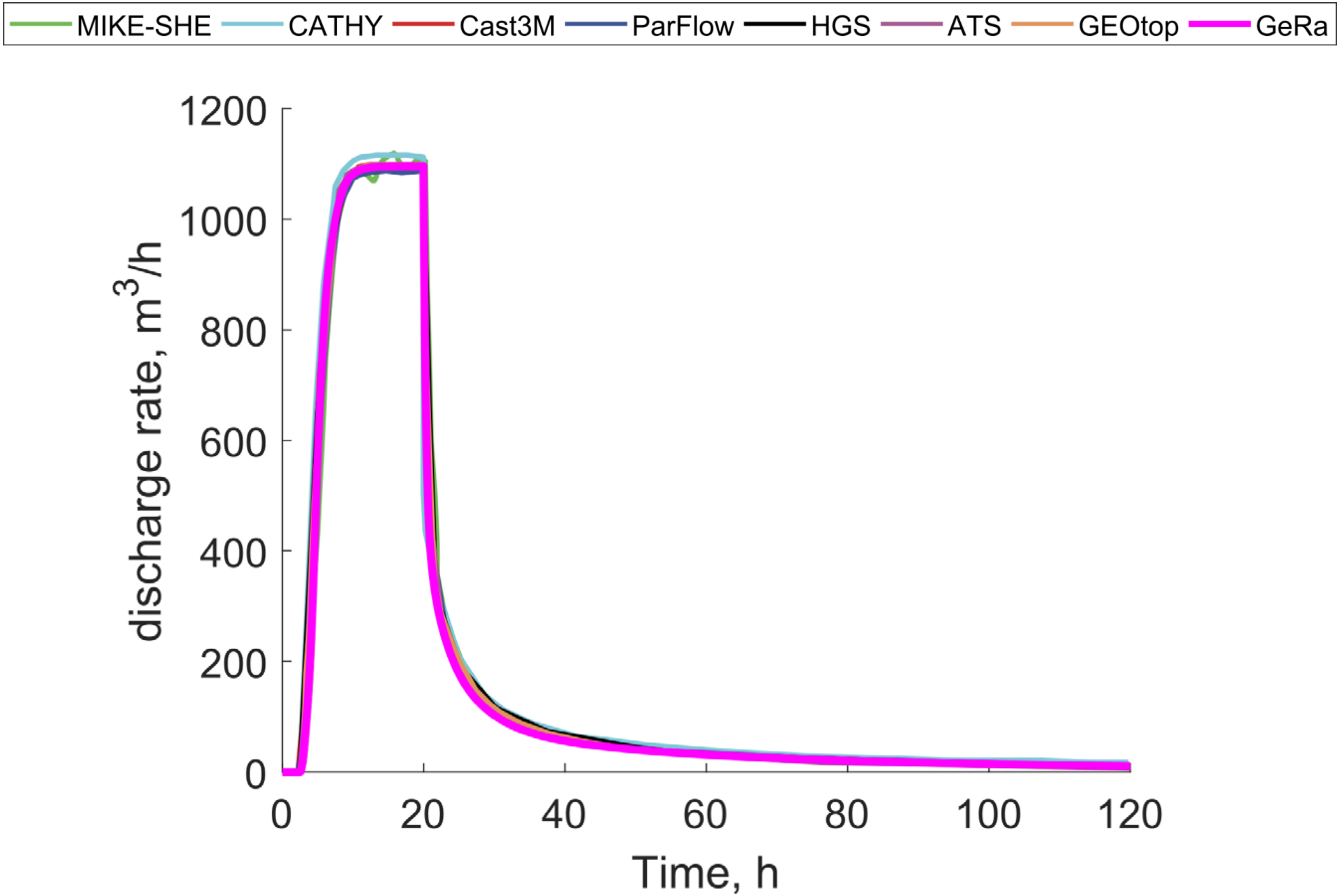
Tilted v-catchment with subsurface second scenario



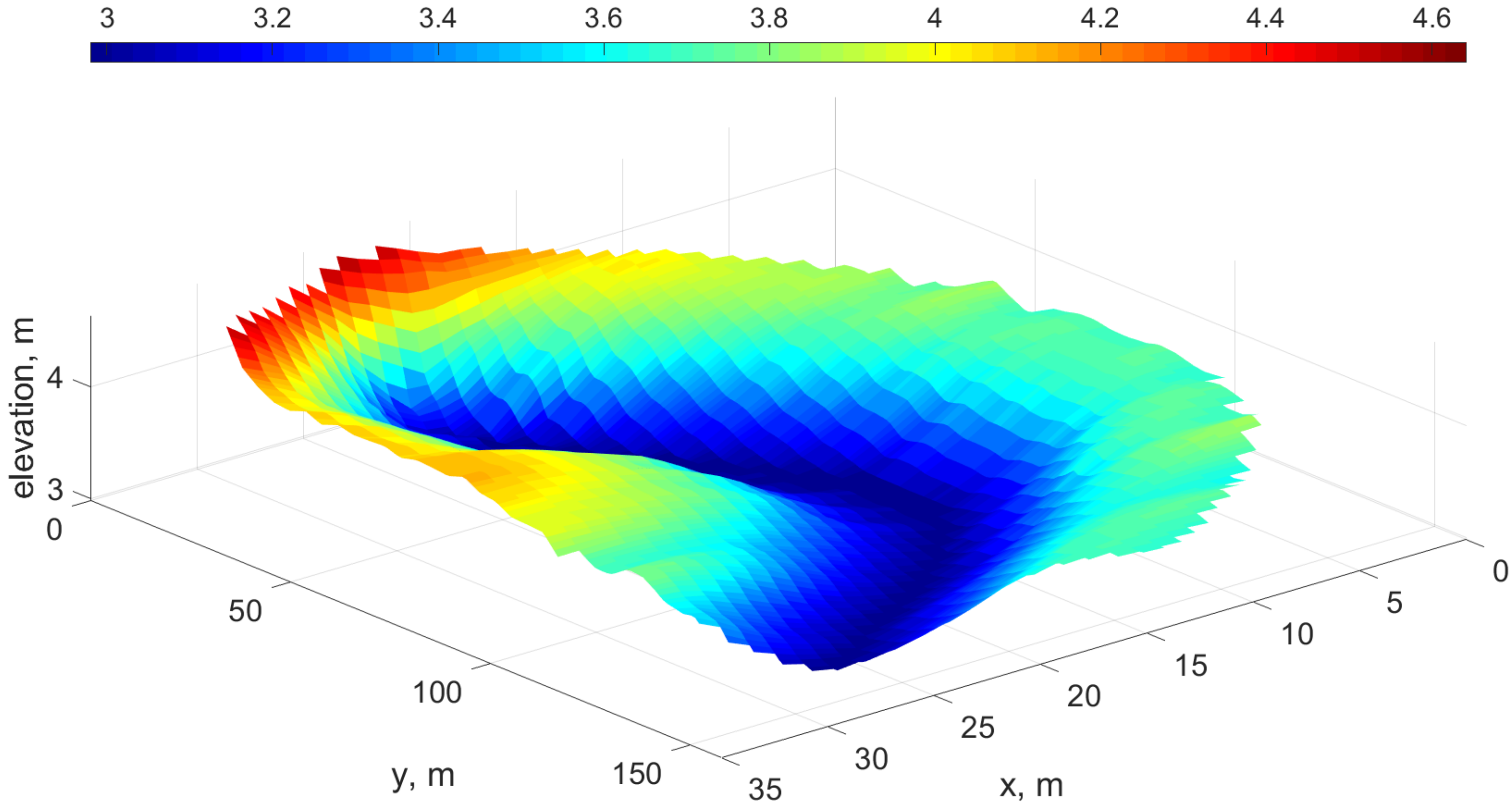
Tilted v-catchment with subsurface second scenario



Tilted v-catchment with subsurface second scenario



Borden experiment



[S. Kollet et al. The integrated hydrologic model intercomparison project, ih-mip2: A second set of benchmark results to diagnose integrated hydrology and feedbacks // Water Resources Research 53 (1) (2017) 867-890]

Borden experiment

Parameter	Value
Manning's roughness channel, $(s/m^{1/3})$	0.03
Manning's roughness banks, $(s/m^{1/3})$	0.3
Saturated hydraulic conductivity, (m/h)	0.036
Residual volumetric water content, (-)	0.067
Saturated volumetric water content, (-)	0.37
Precipitation rate, (m/h)	0.02 first 50 minutes, 0 next 50 minutes
Van Genuchten parameter n, (-)	6
Van Genuchten parameter α, (m)	1.9
Experiment duration, (min)	100
Bottom sediment width, (m)	0.2
Bottom sediment conductivity, (m/day)	0.47
Initial conditions	Water level 2.78m

Borden experiment

[S. Kollet et al. The integrated hydrologic model intercomparison project, ih-mip2: A second set of benchmark results to diagnose integrated hydrology and feedbacks // Water Resources Research 53 (1) (2017) 867-890]

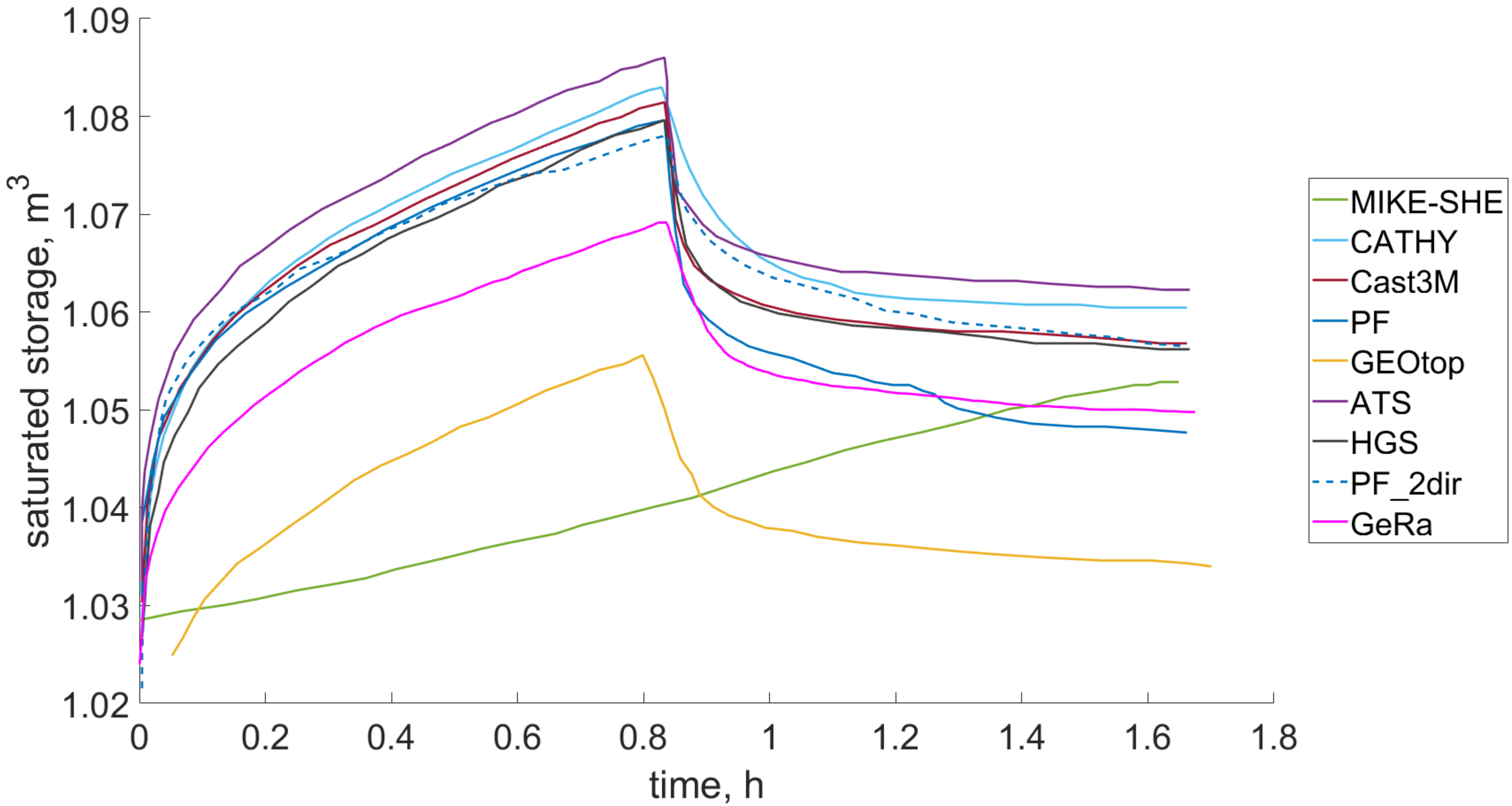
Manning's roughness banks, (s/$m^{1/3}$)	0.3

[Aquanty inc., HGS user manual. – Waterloo. – 2015.]

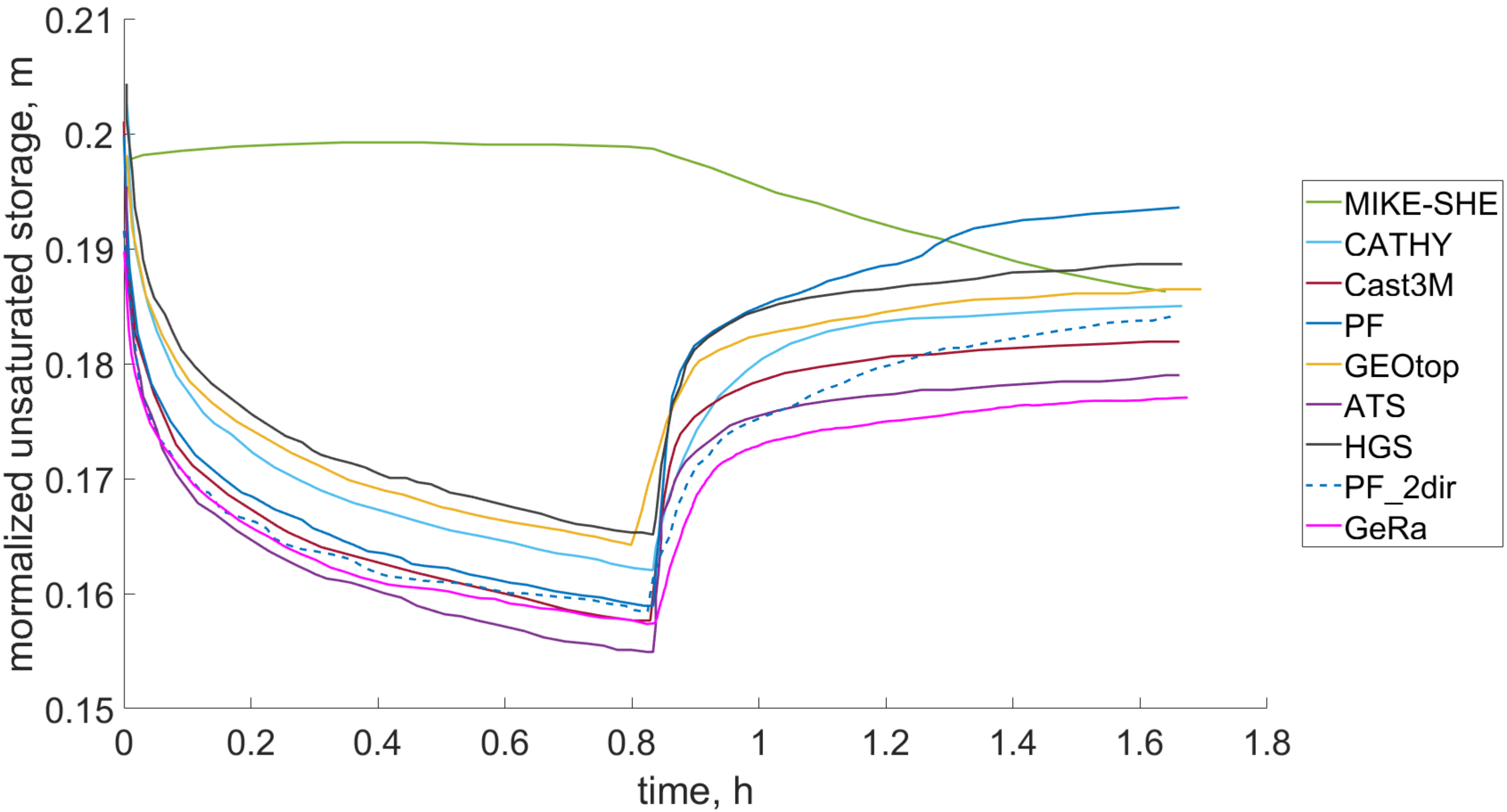
[VanderKwaak J., Numerical simulation o flow and chemical transport in integrated surface-subsurface hydrologic systems, Ph.D. thesis, University of Waterloo, Waterloo, Ontario, Canada (1999)]

Manning's roughness channel, (s/$m^{1/3}$)	0.03
Manning's roughness banks, (s/$m^{1/3}$)	0.3

Borden experiment



Borden experiment



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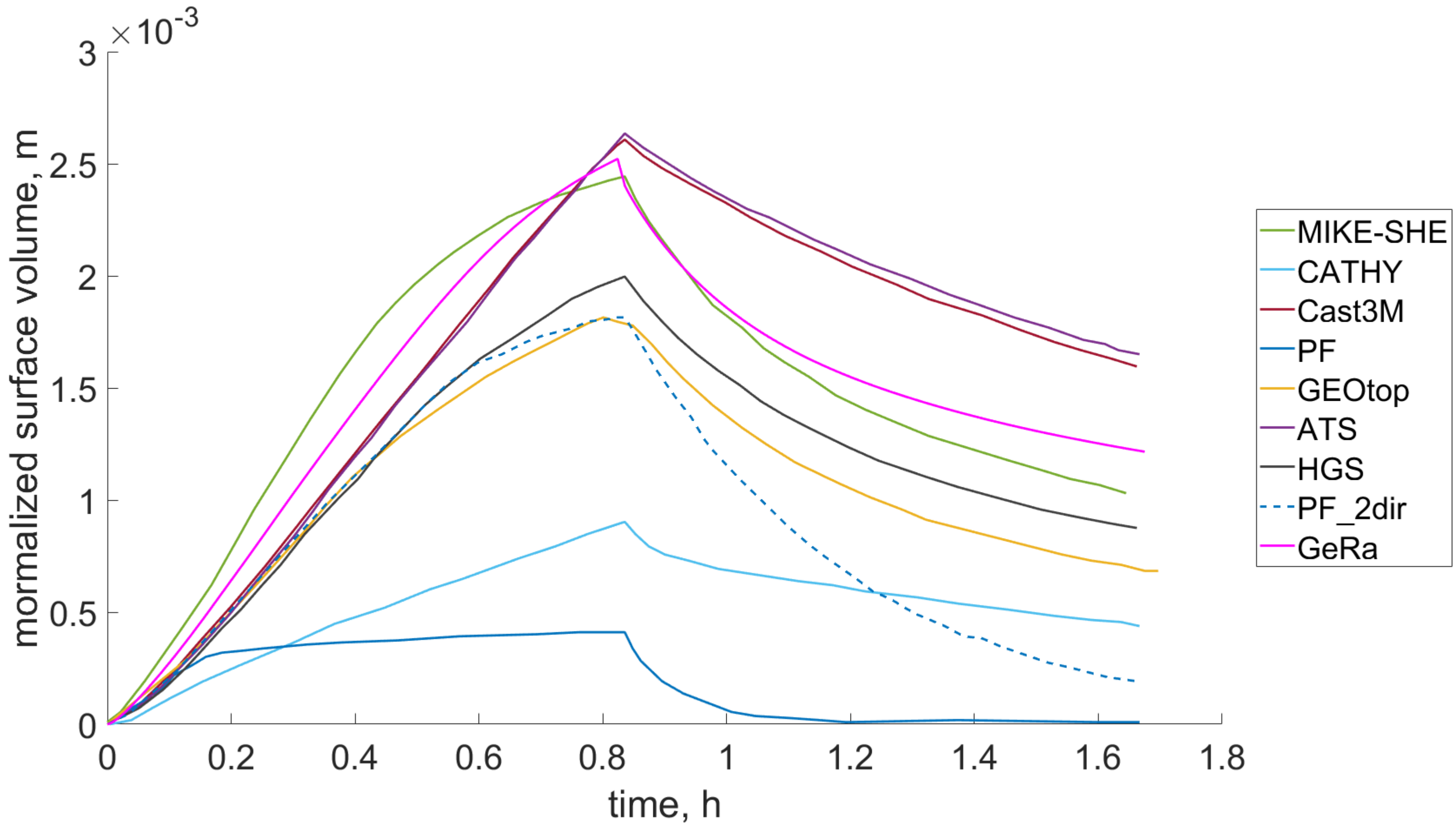
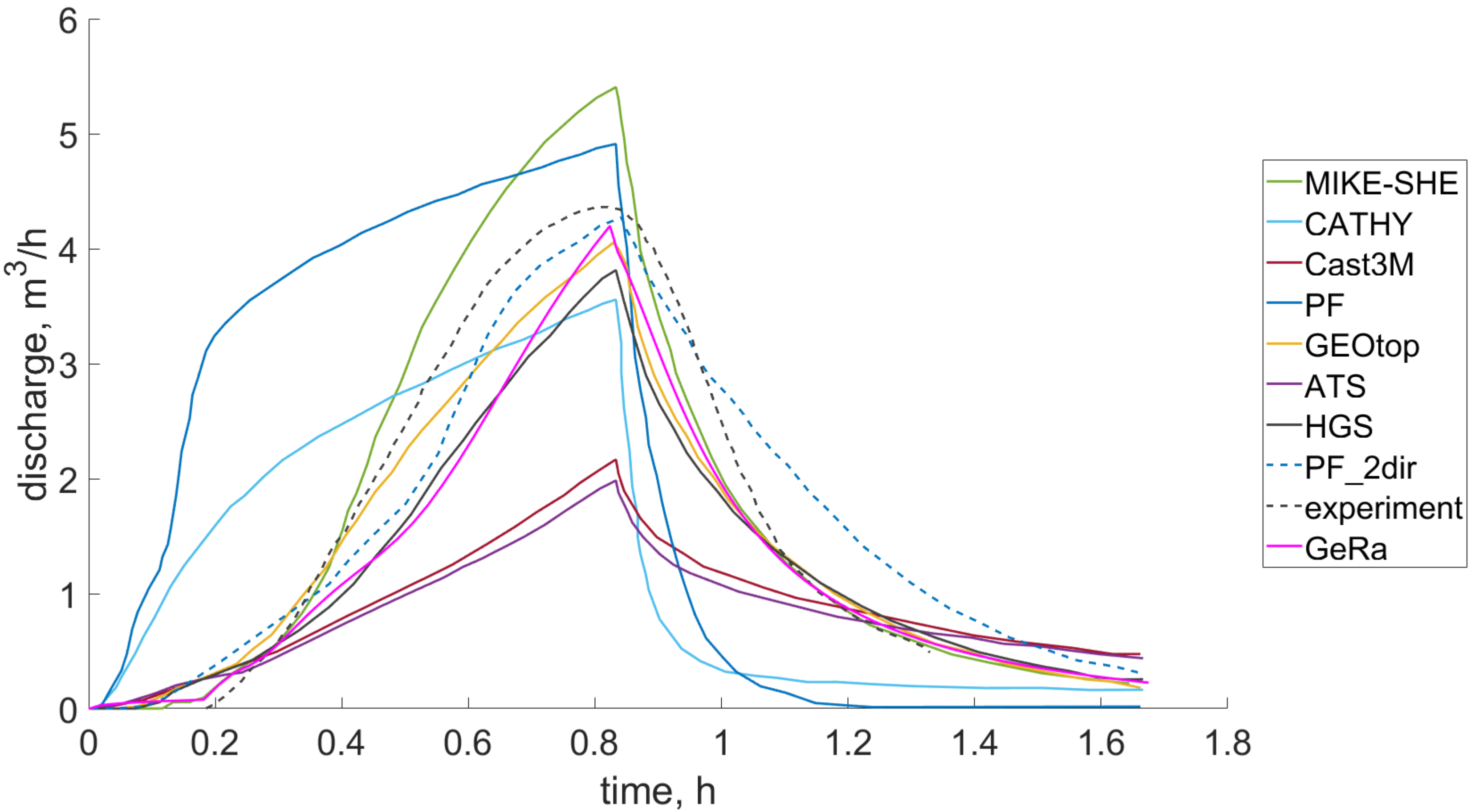
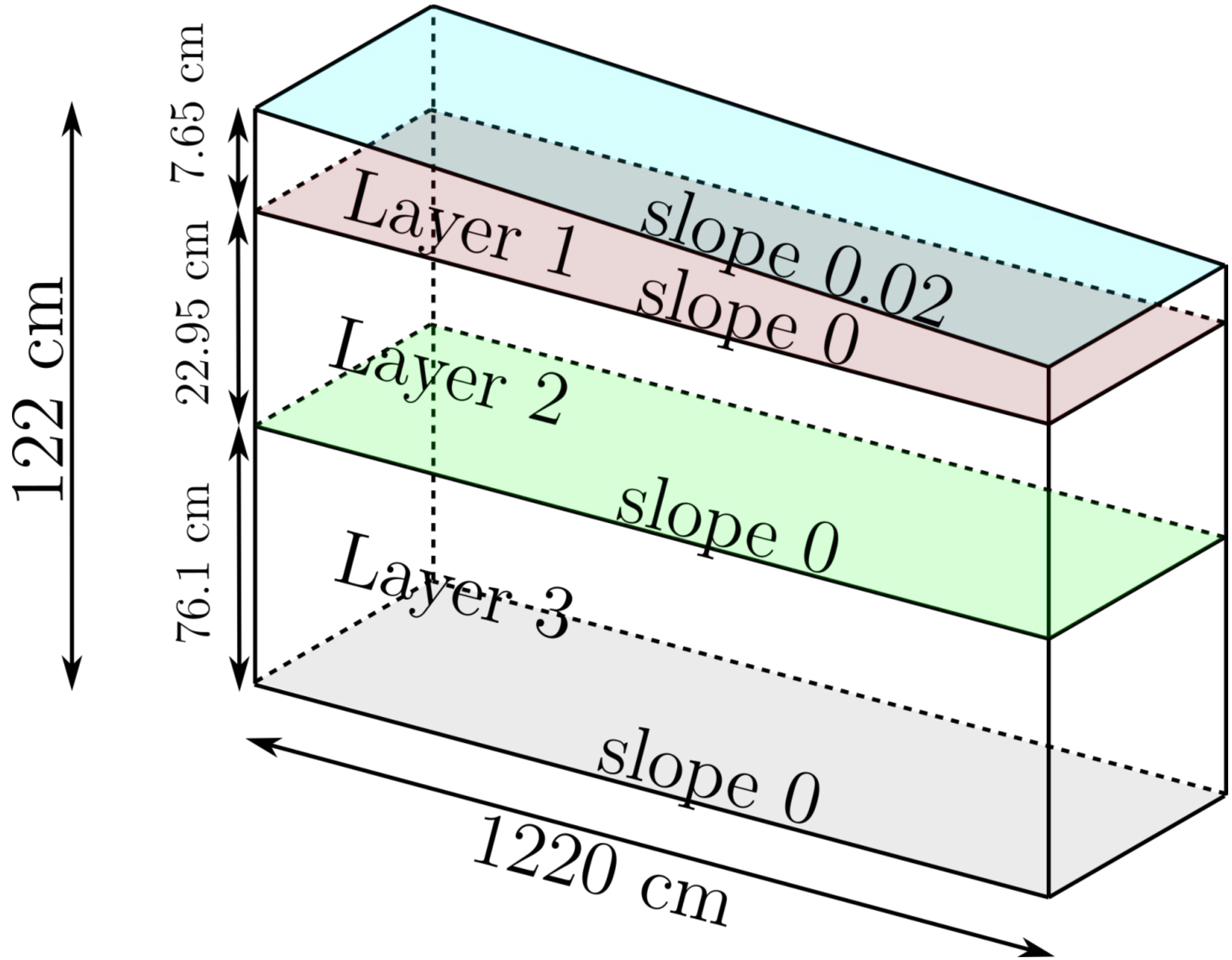


Borden experiment



Smith and Woolhiser

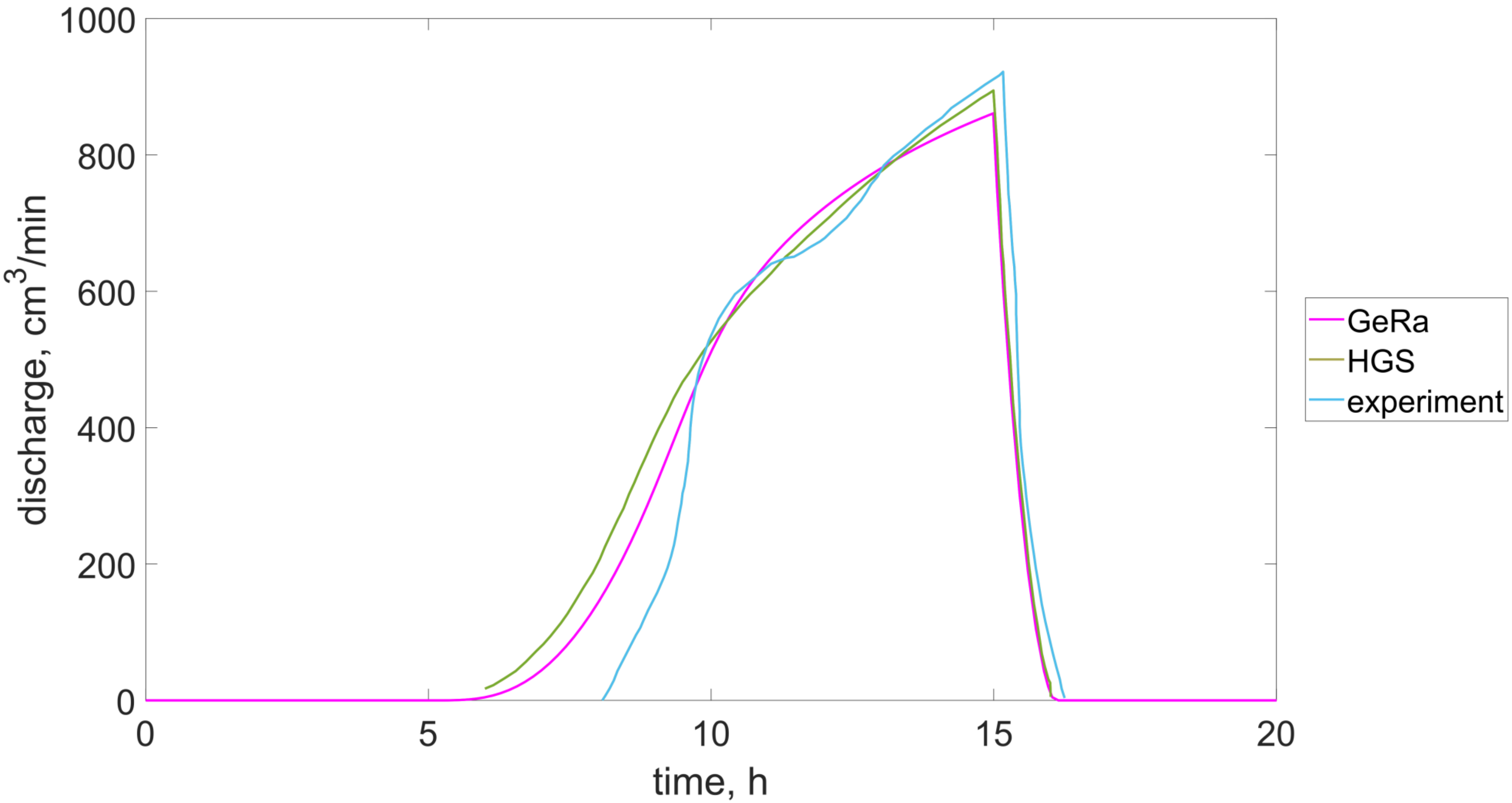


[R. Smith, D. Woolhiser, Overland Flow on an inltrating surface // Water Resources Research, 7 (4), (1971), 899-913.]

Smith and Woolhiser

Parameter	Value
Manning's roughness, $(min/cm^{1/3})$	0.000122
Saturated hydraulic conductivity, (cm/min)	Layer 1: 0.184 Layer 2: 0.1452 Layer 3: 0.1296
Residual volumetric water content, (-)	Layer 1: 0.05068 Layer 2: 0.05699 Layer 3: 0.05248
Saturated volumetric water content, (-)	Layer 1: 0.3946 Layer 2: 0.4387 Layer 3: 0.4764
Precipitation rate, (m/h)	0.416667 first 15 min, 0 next 5 min
Van Genuchten parameter n, (-)	Layer 1: 3.4265 Layer 2: 4.1371 Layer 3: 4.3565
Van Genuchten parameter α, (m)	Layer 1: 0.07 Layer 2: 0.056 Layer 3: 0.0443
Experiment duration, (min)	20
Bottom sediment width, (m)	0.01
Bottom sediment conductivity, (m/day)	100
Initial conditions	Saturation 0.2

Smith and Woolhiser



Summary

- Surface runoff modelling allows more precise description of precipitation
- Diffusive wave approximation of shallow water equations is used in GeRa package as a surface runoff model
- Numerical experiments demonstrate good agreement of GeRa with other packages