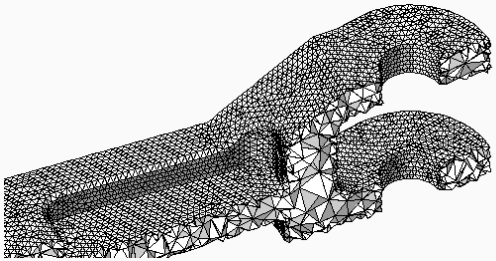
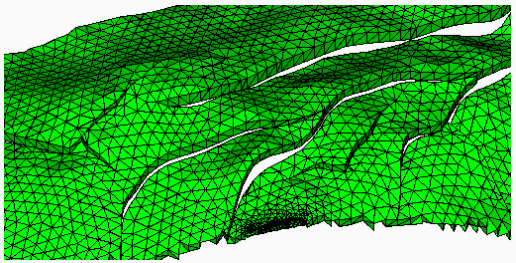
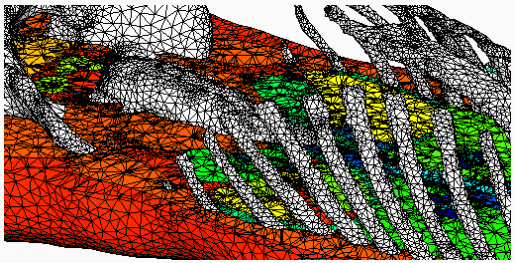
Unstructured mesh generation for complex domains

Alexander Danilov

INM RAS, MIPT, Sechenov University, Moscow, Russia





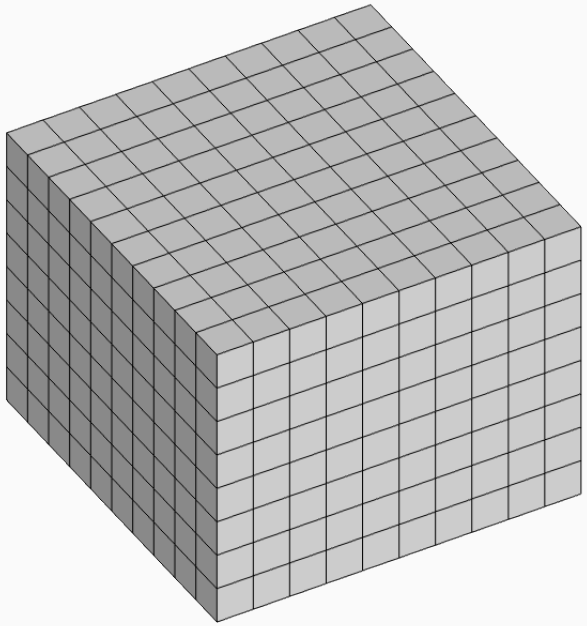


Introduction

Mesh conformity

Conforming meshes:

- Mesh elements fill the whole domain
- Elements do not intersect each other
- Neighbor elements share common vertices, edges, or faces



Cubic grid

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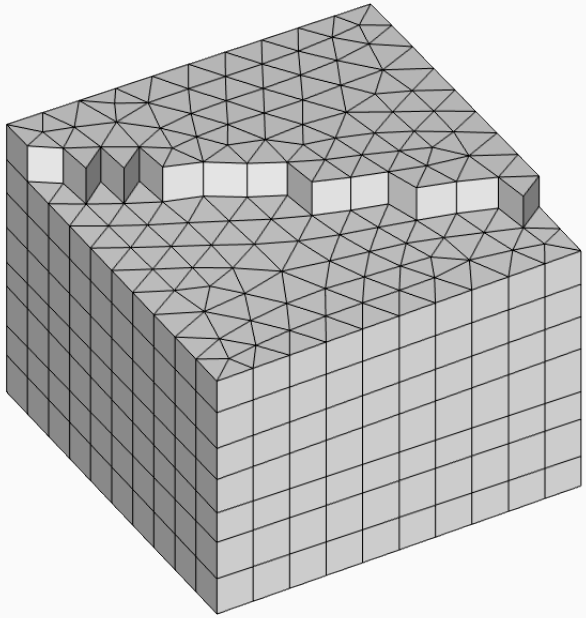

Prismatic grid

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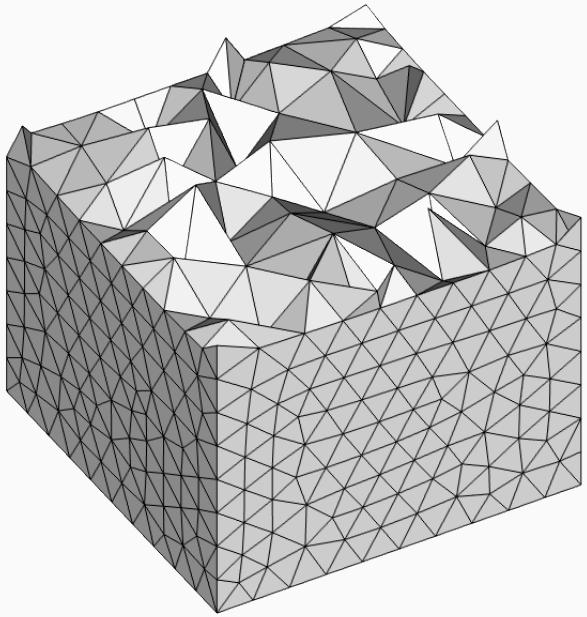

Tetrahedral grid

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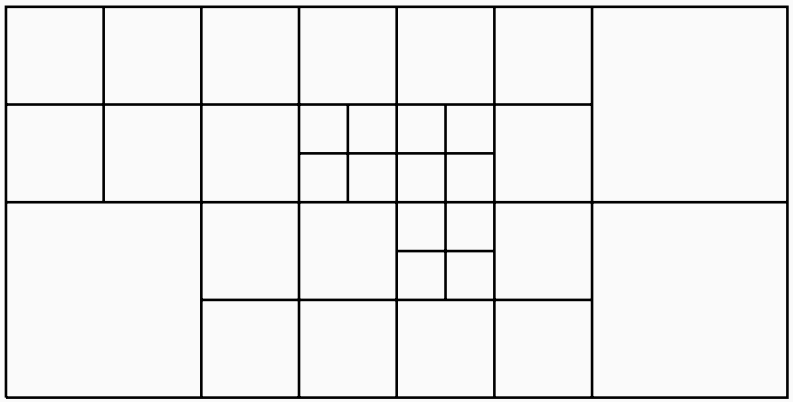
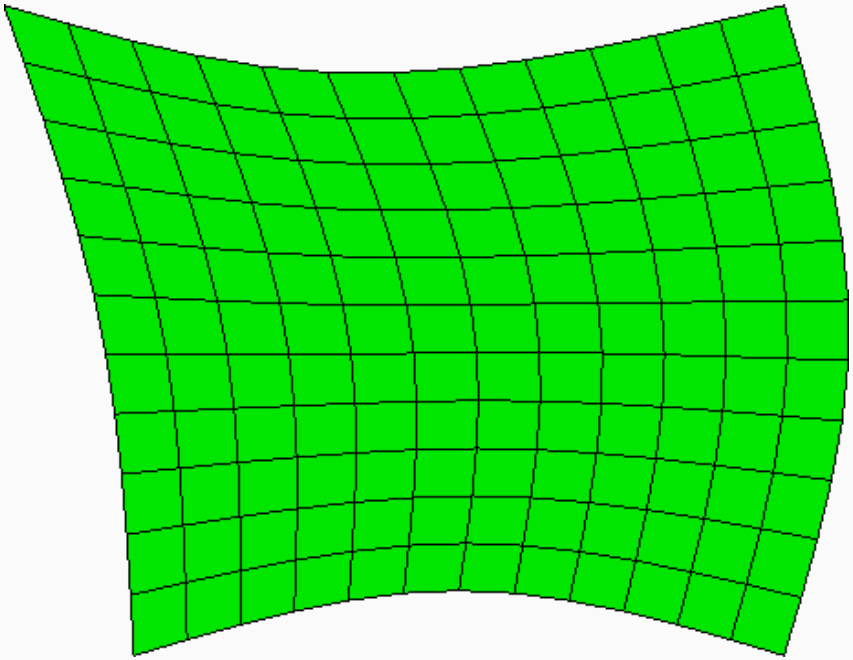
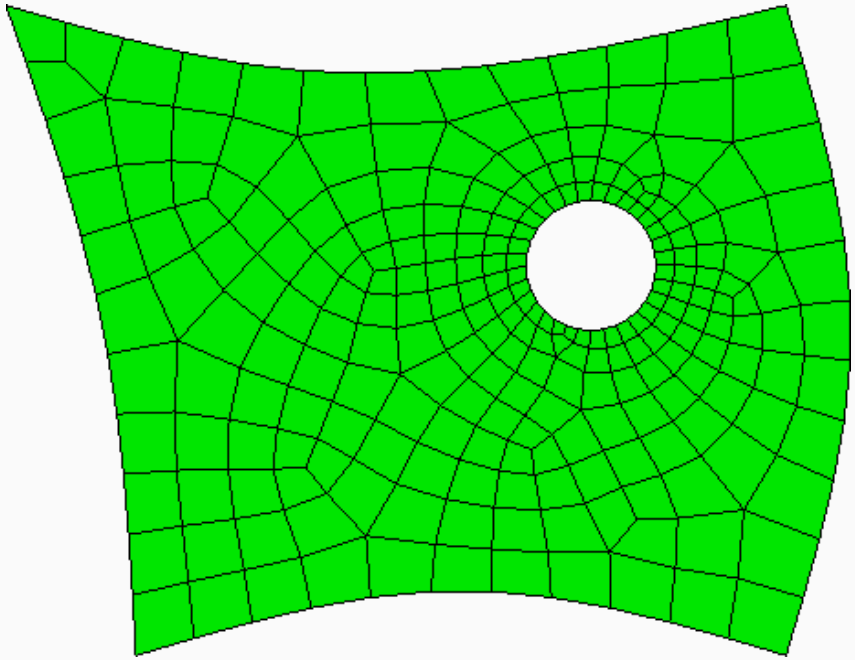


Quad-tree grid

Structured vs. Unstructured

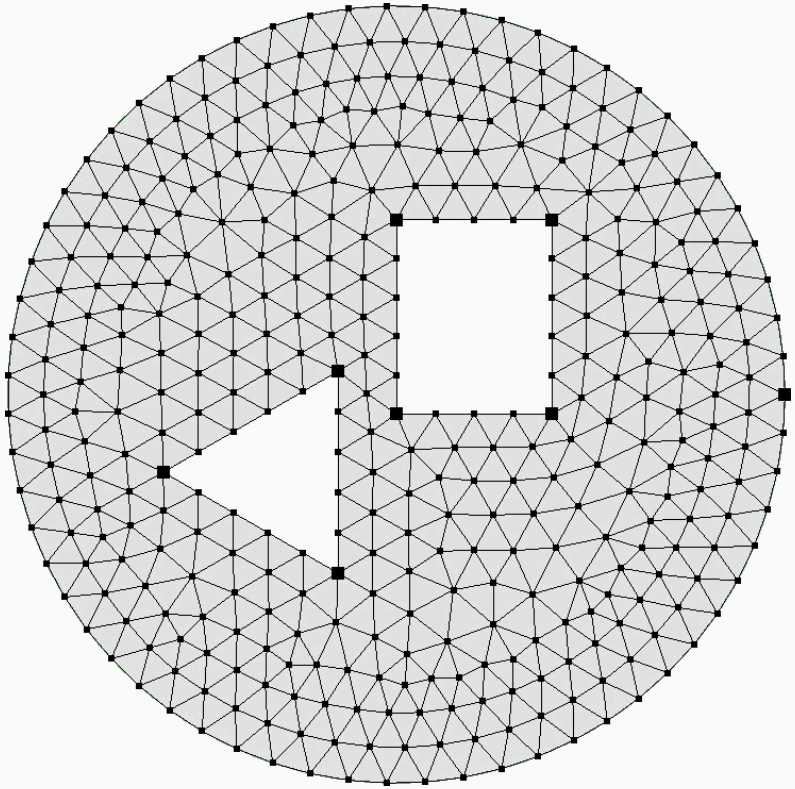


Structured

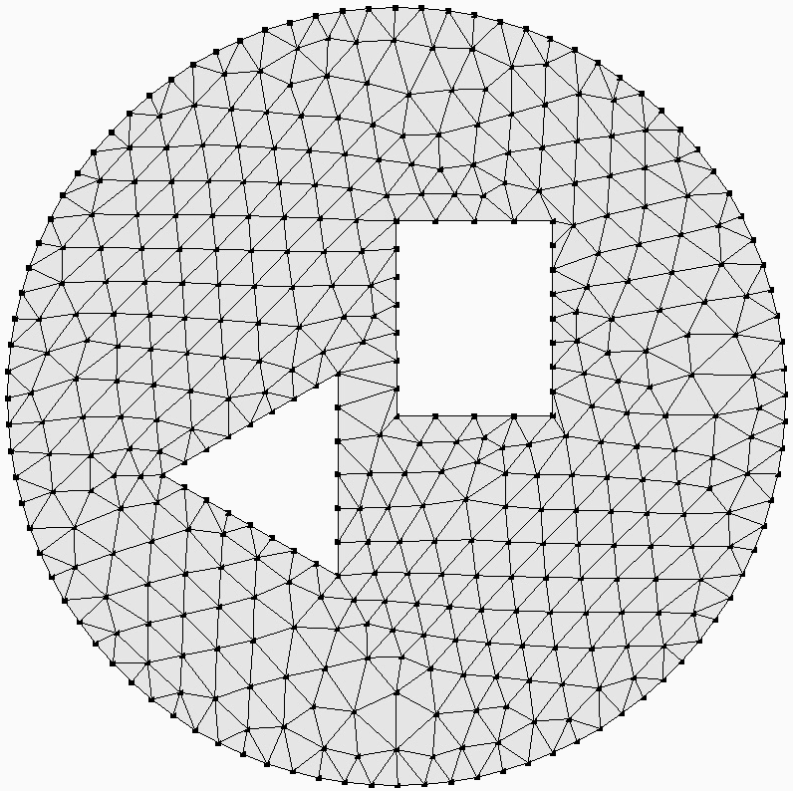


Unstructured

Two meshing approaches



Bottom-up approach
Start from geometry



Overlay grid
Start from mesh

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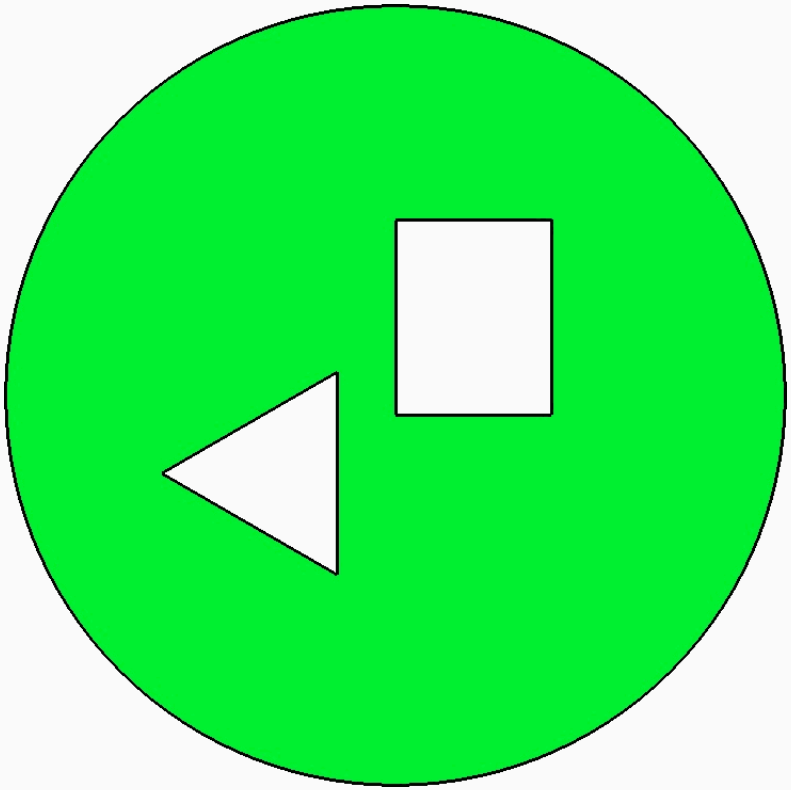

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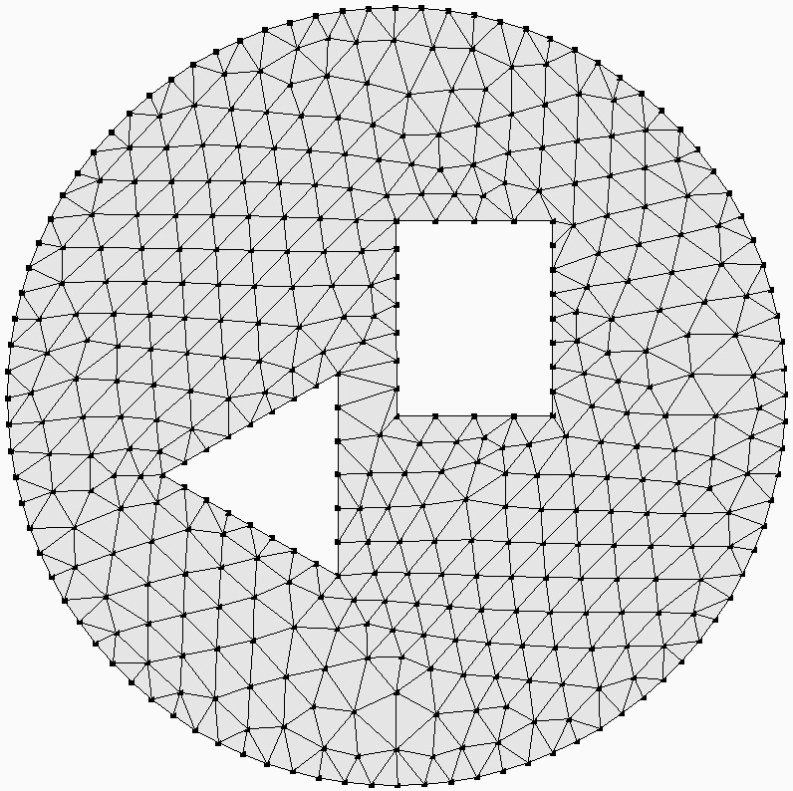

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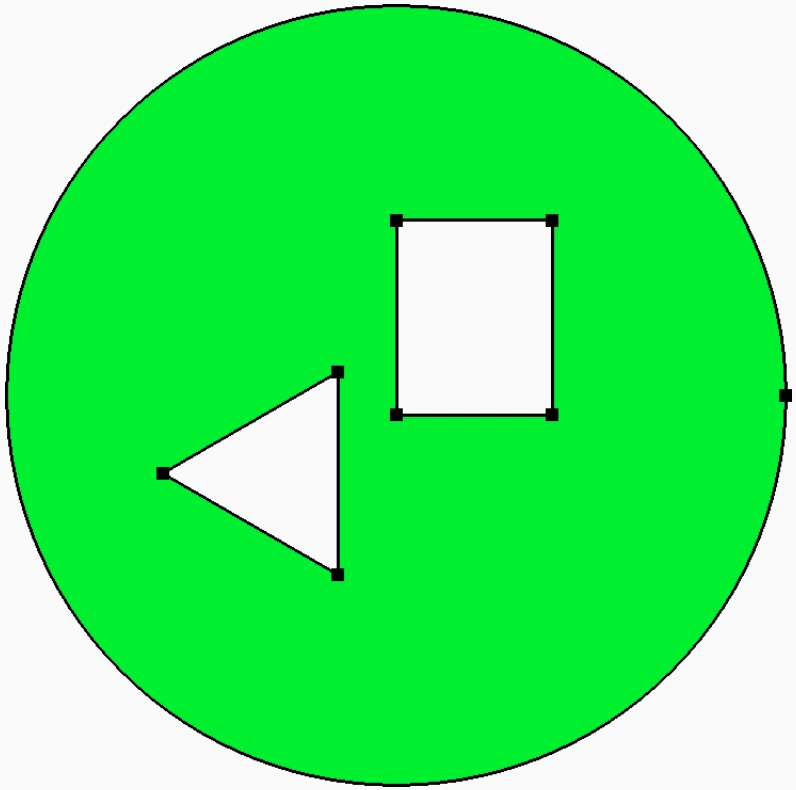

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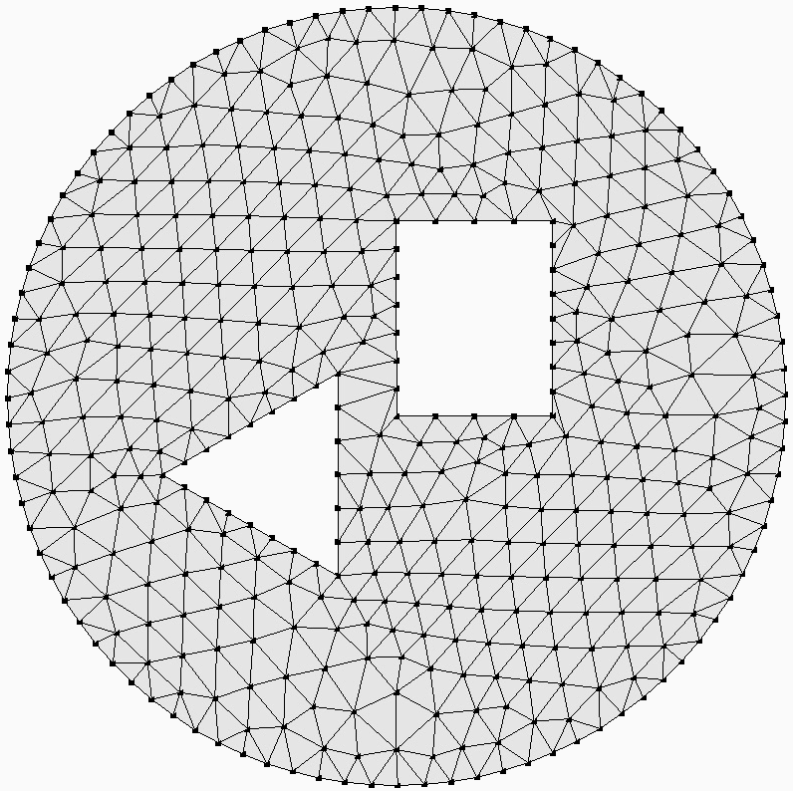

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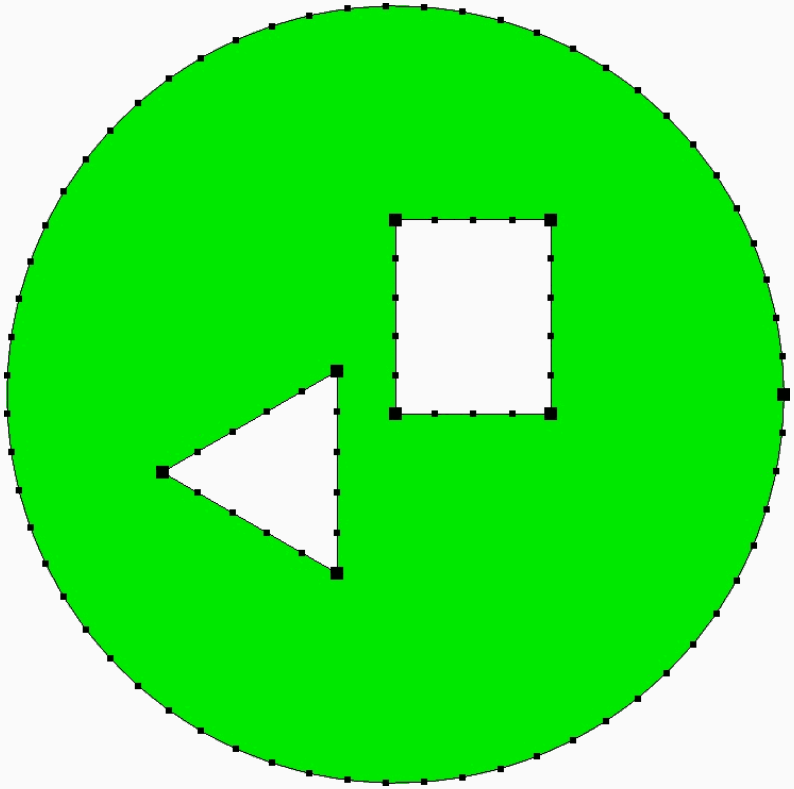

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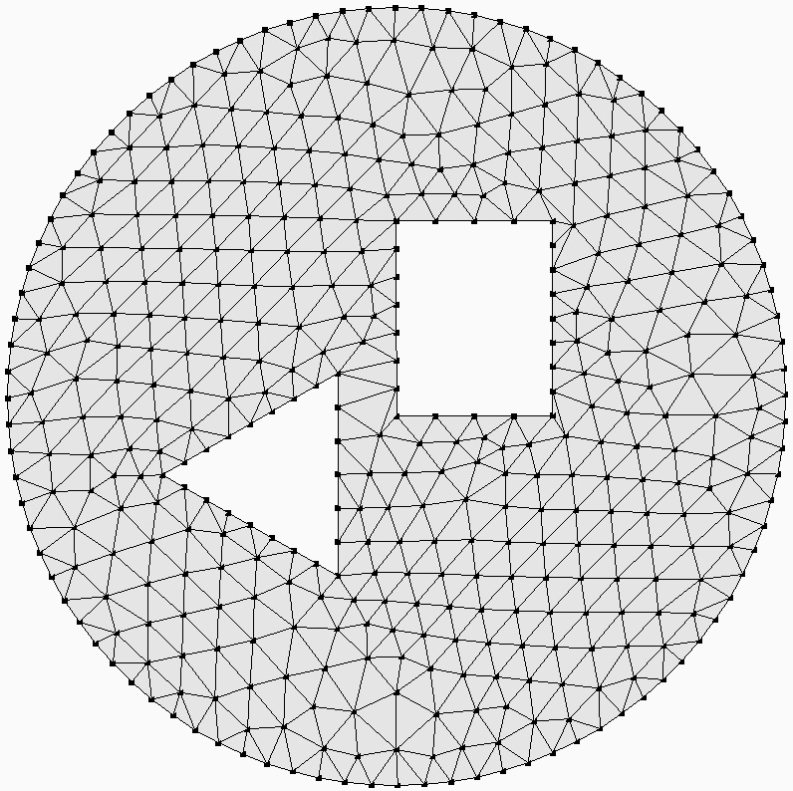

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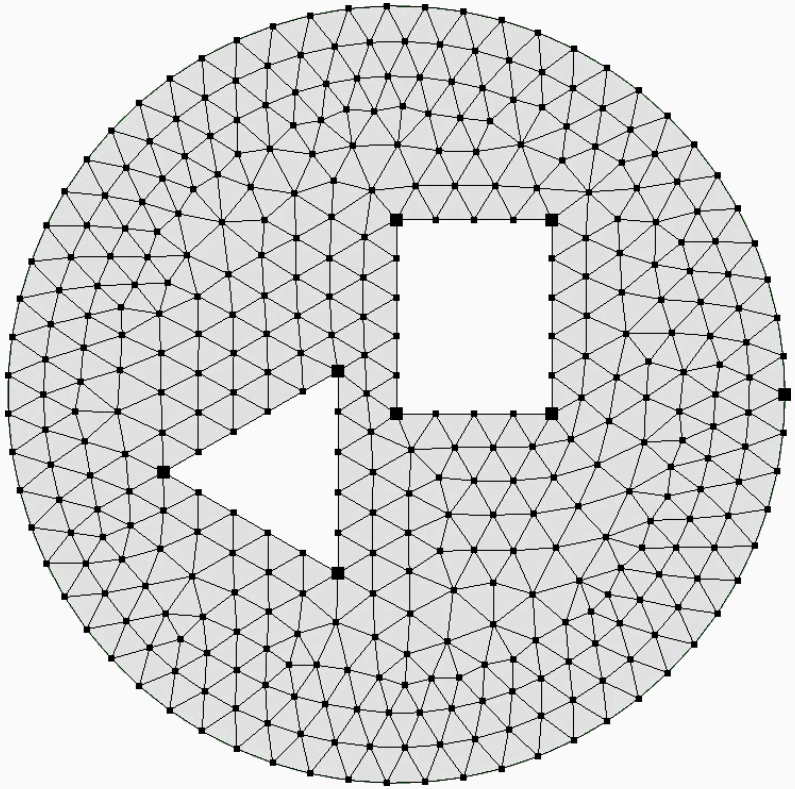

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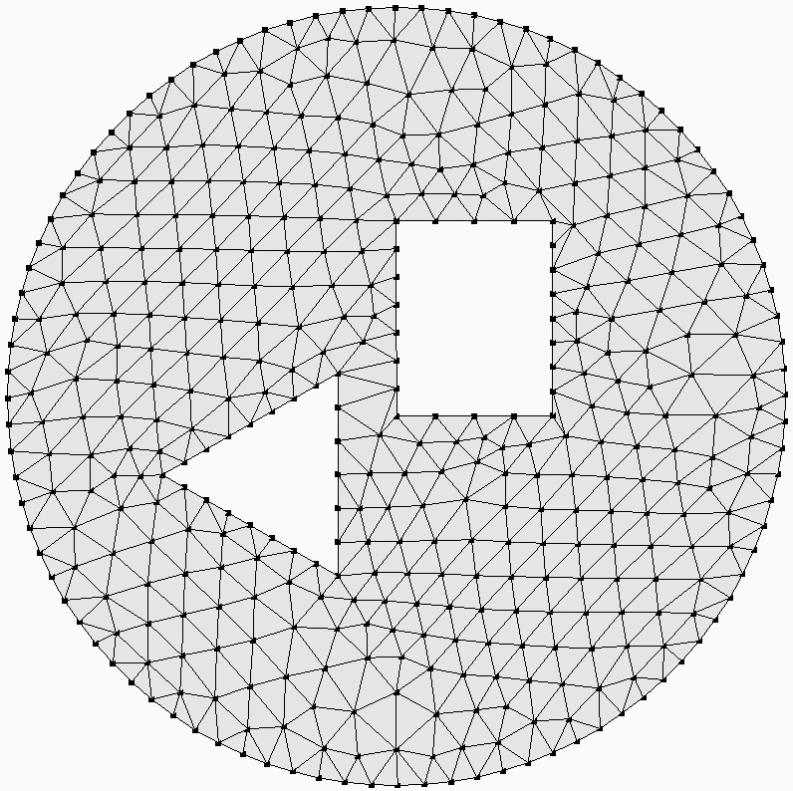

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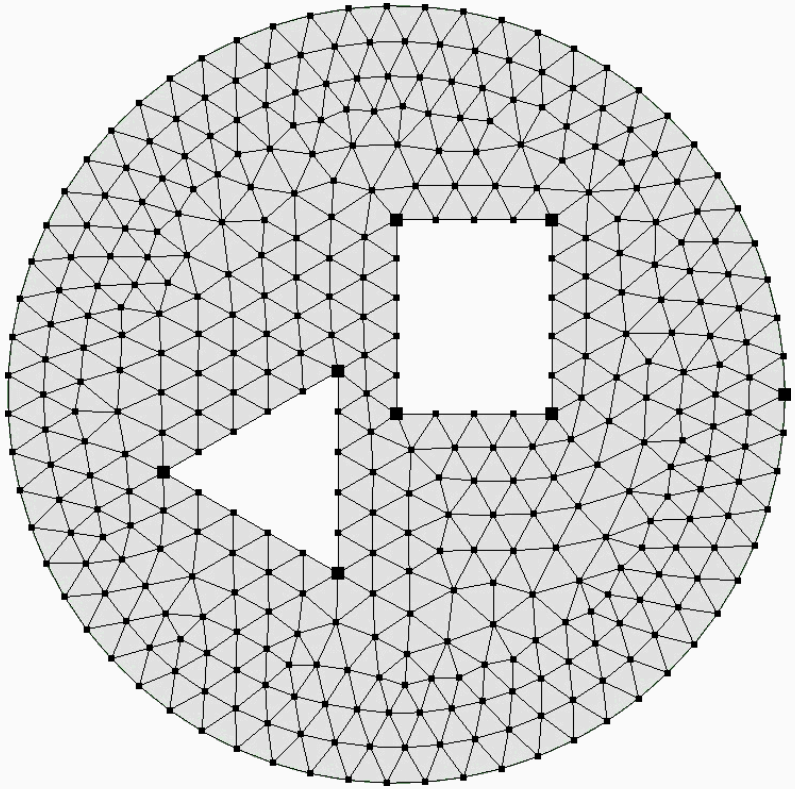

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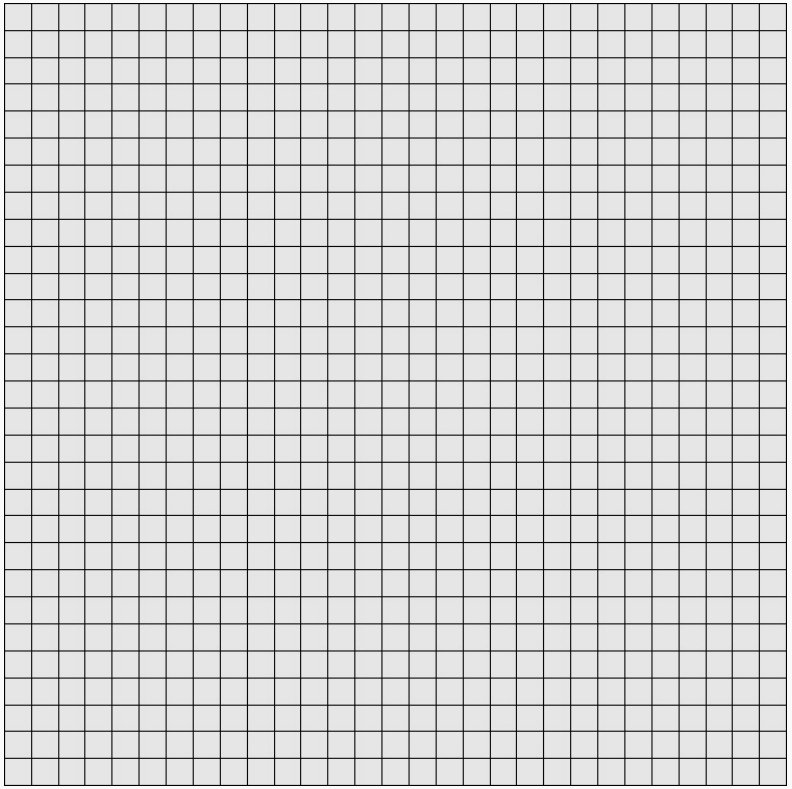

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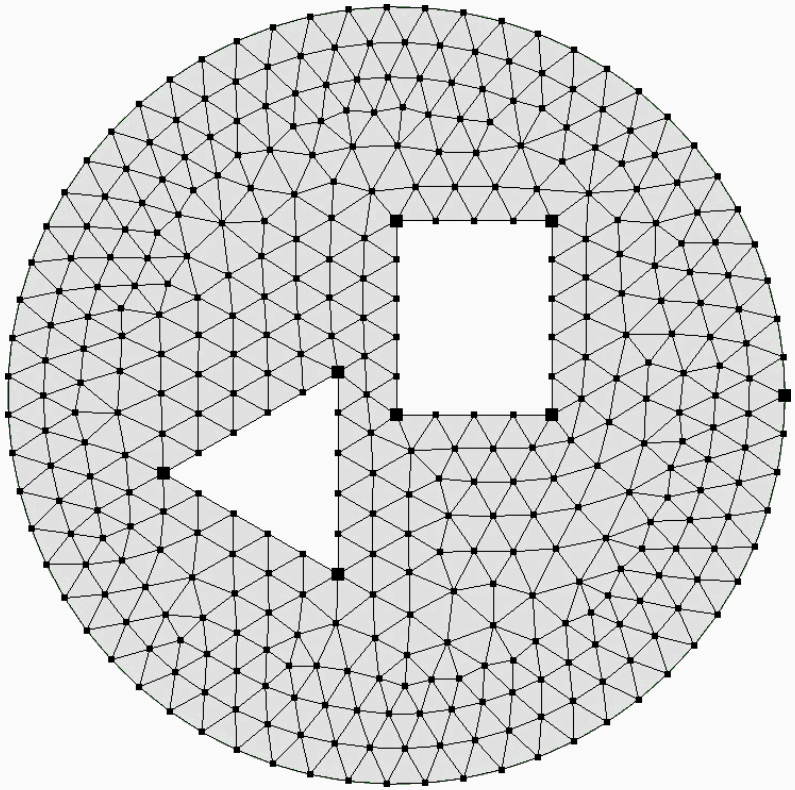

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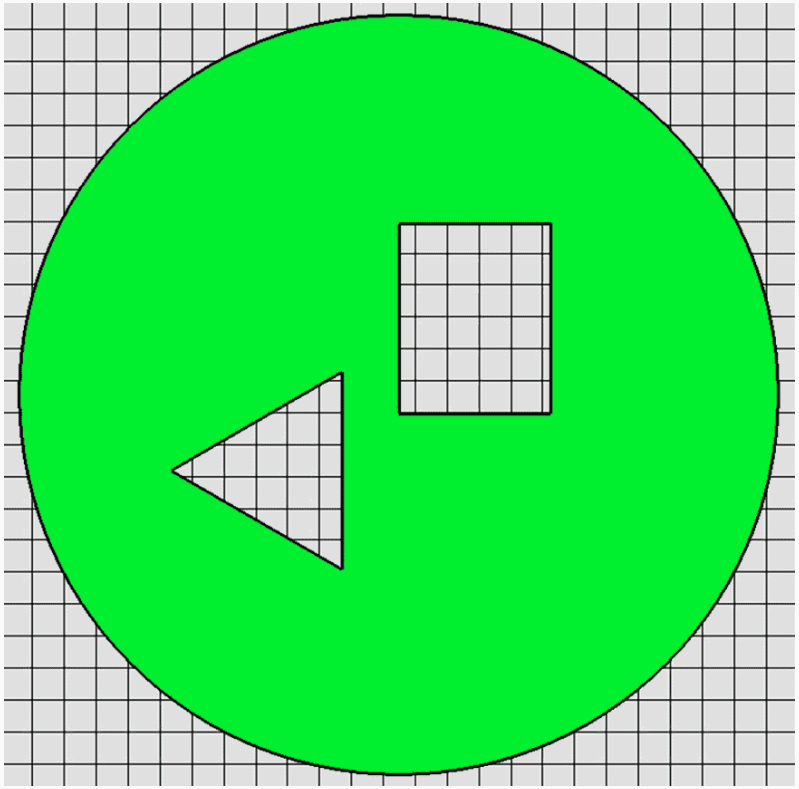

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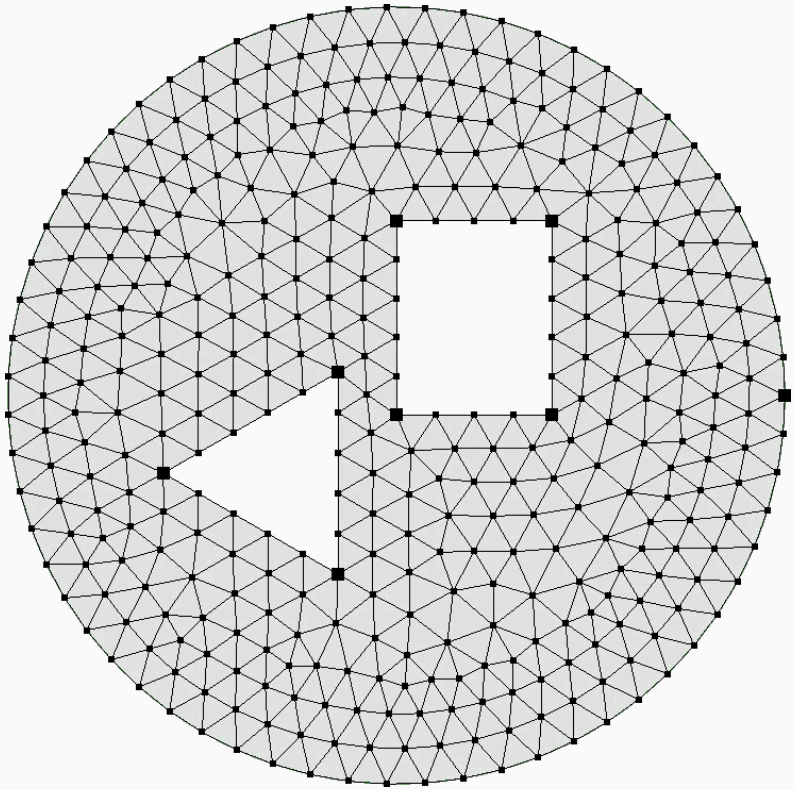

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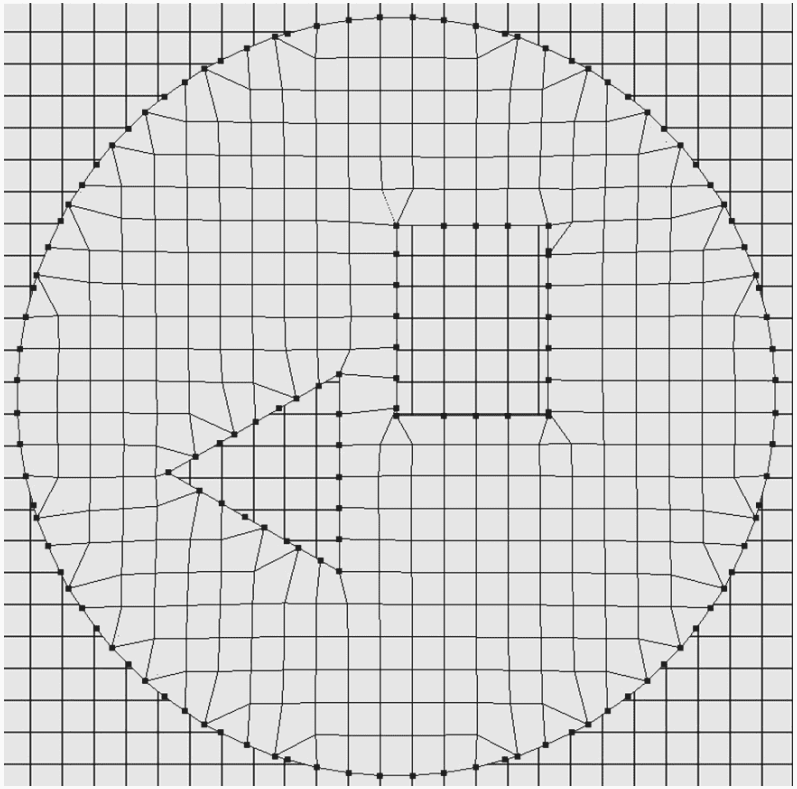

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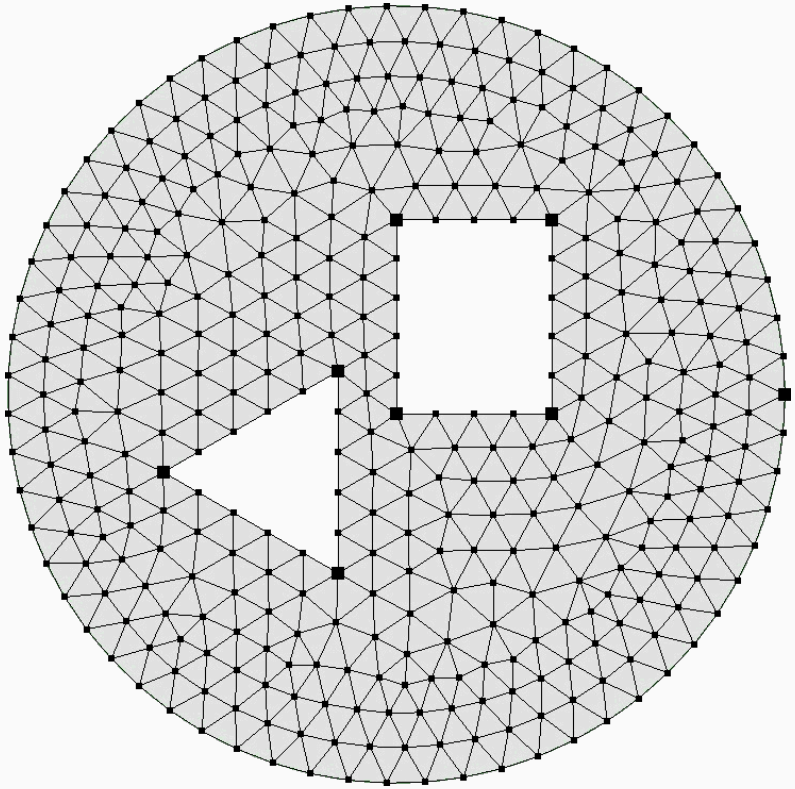

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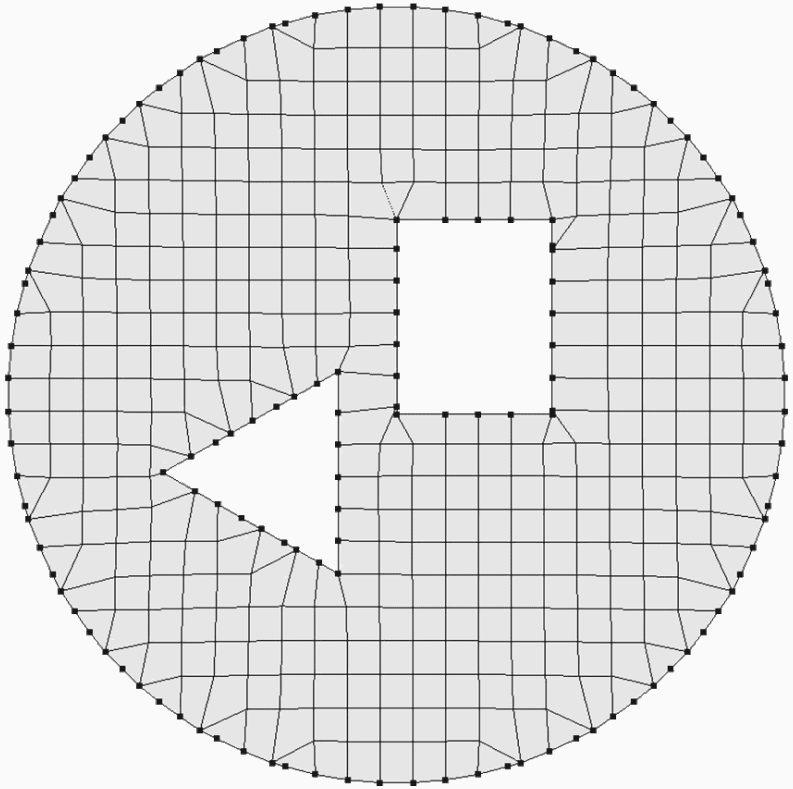

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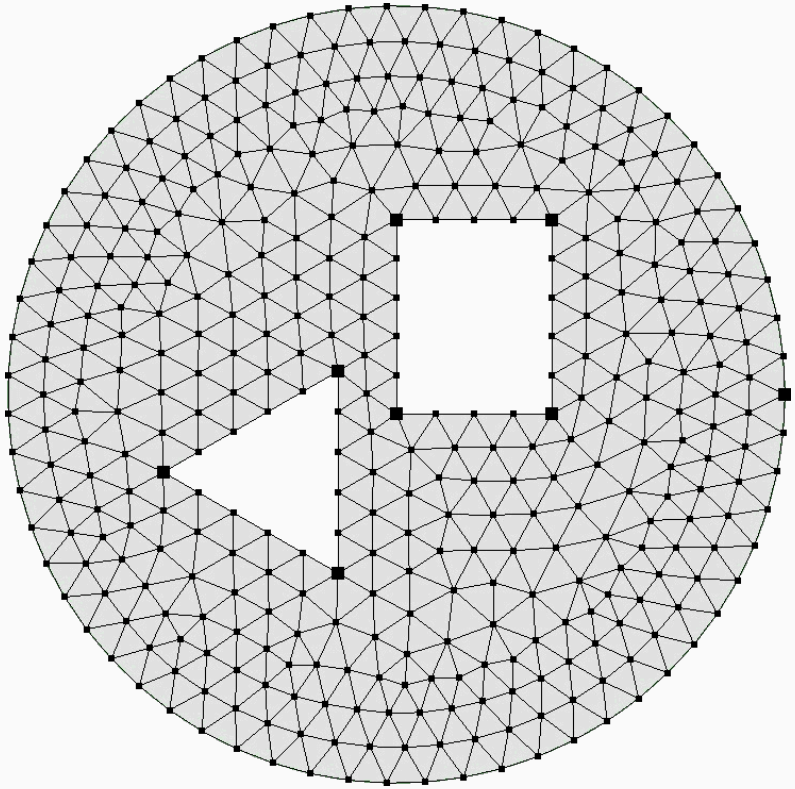

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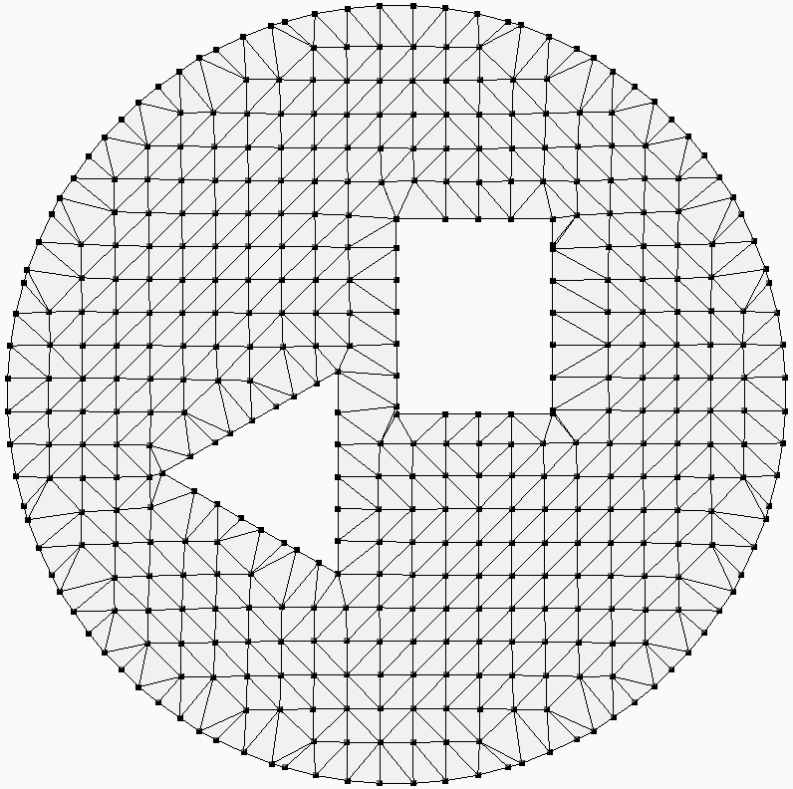

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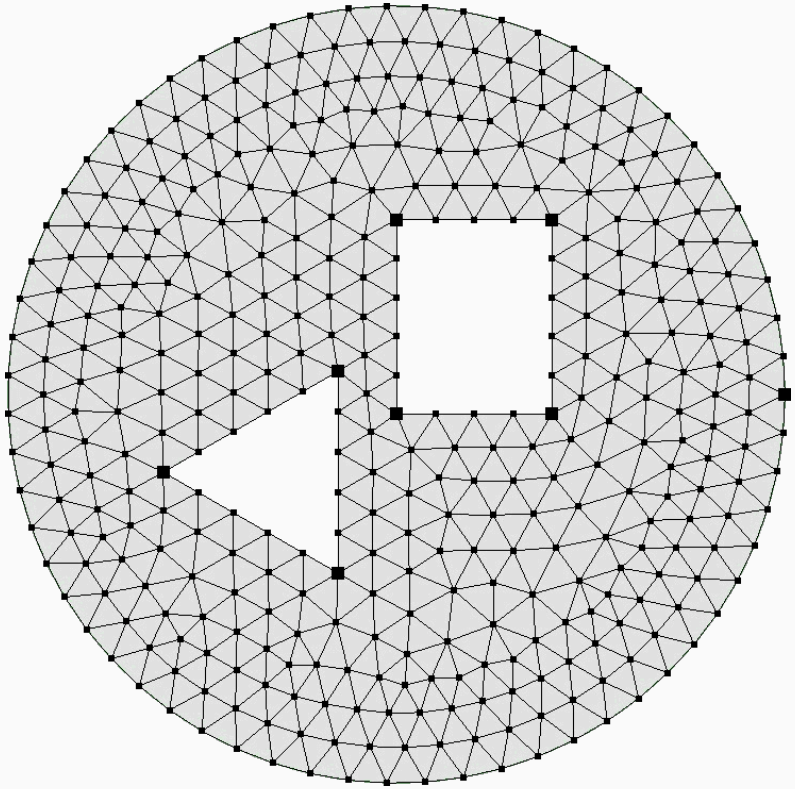

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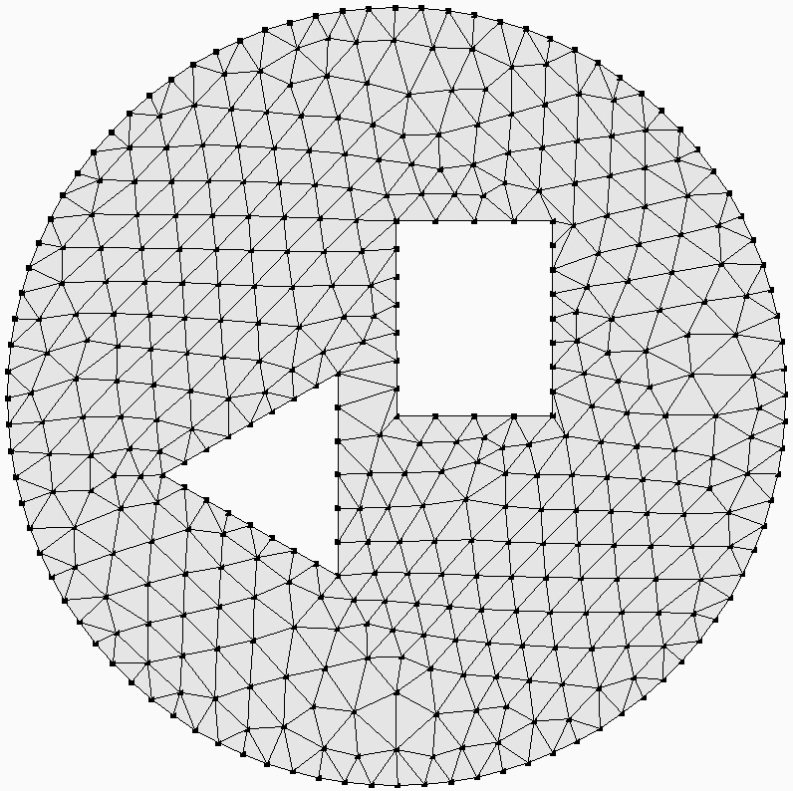

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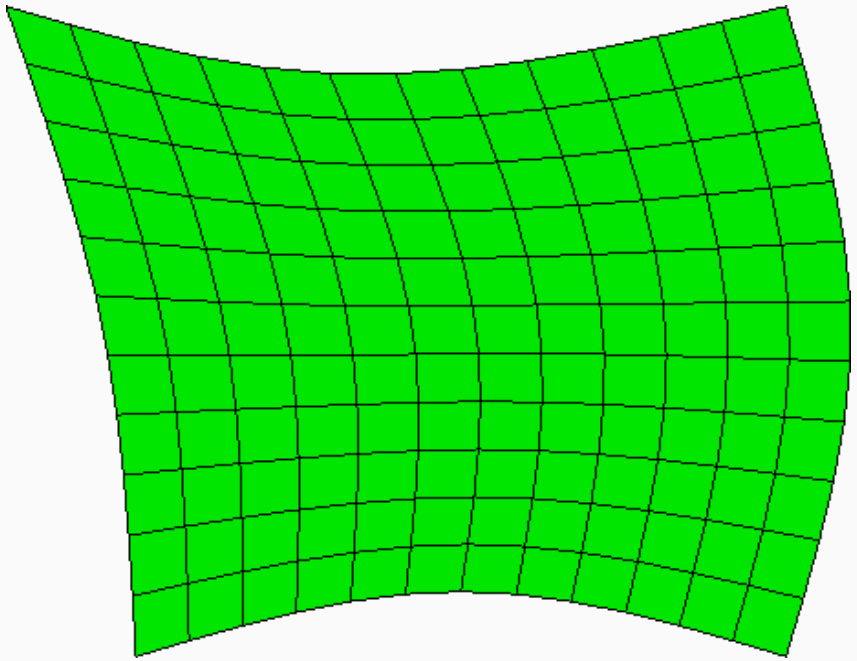
Overview of meshing algorithms

- Structured meshes – various mappings
- Geometry decomposition: sweeping, sub-mapping, multi-block
- Unstructured tetrahedra – AFT, Delaunay
- Unstructured hexahedral – T-Hex, grid-based, and many others
- Unstructured polyhedral – cutcells, Voronoi grids



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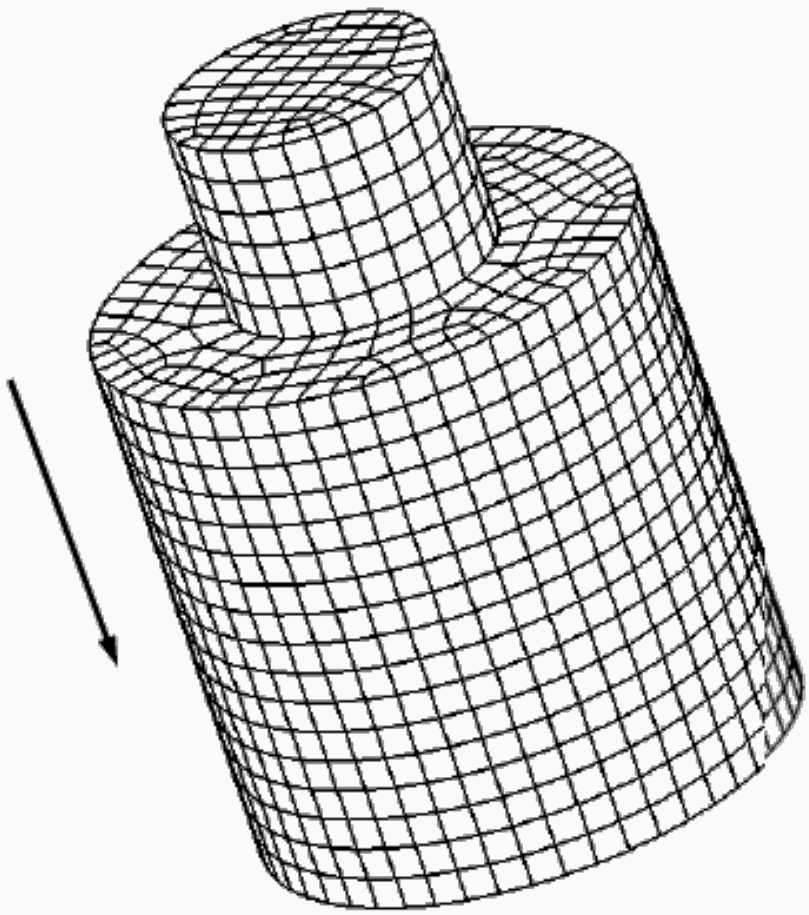

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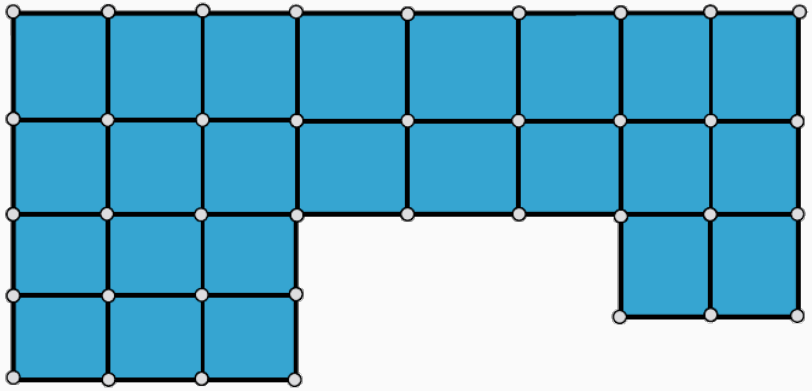

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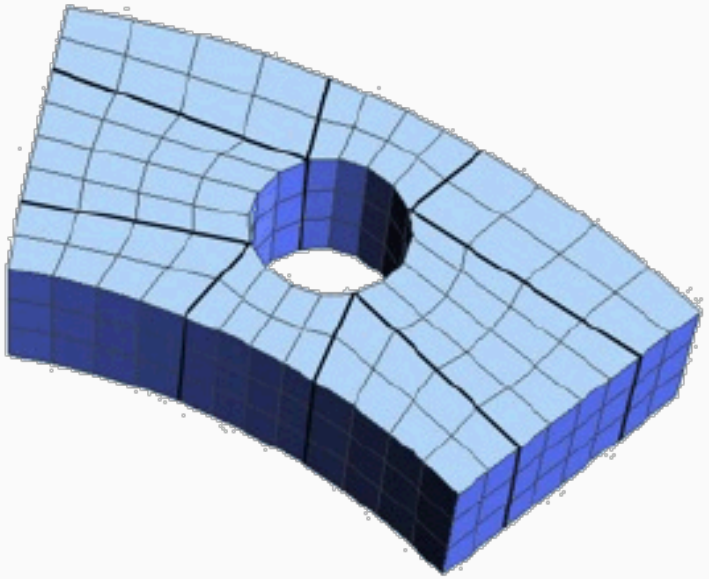

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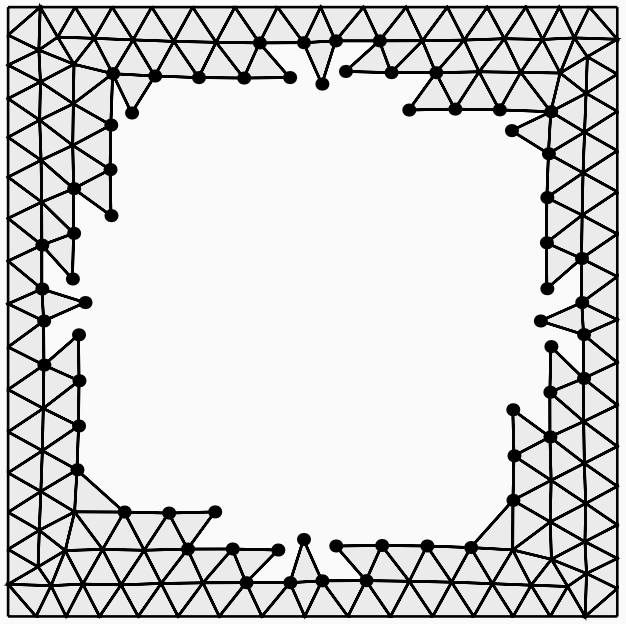

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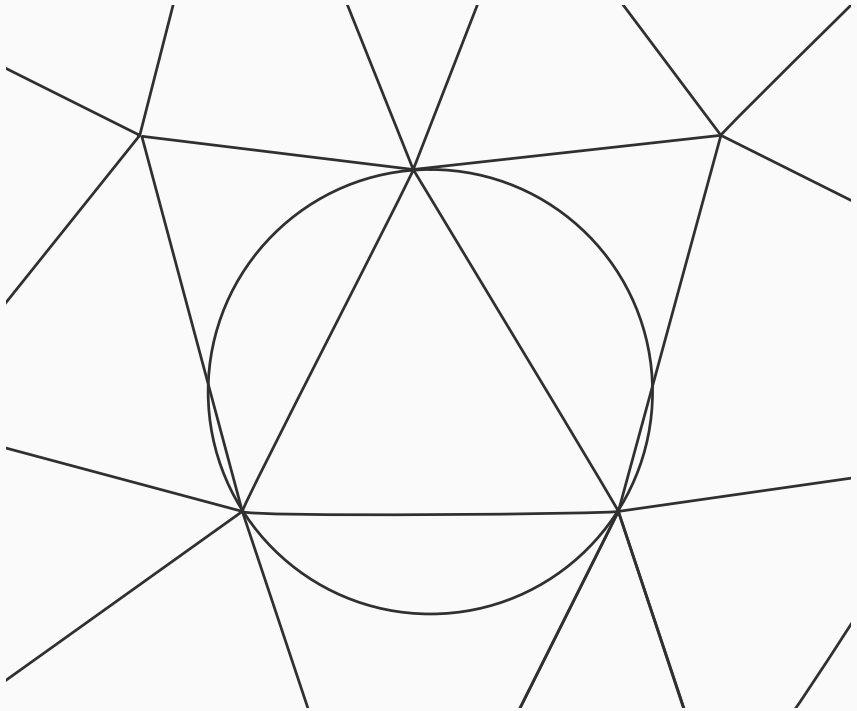

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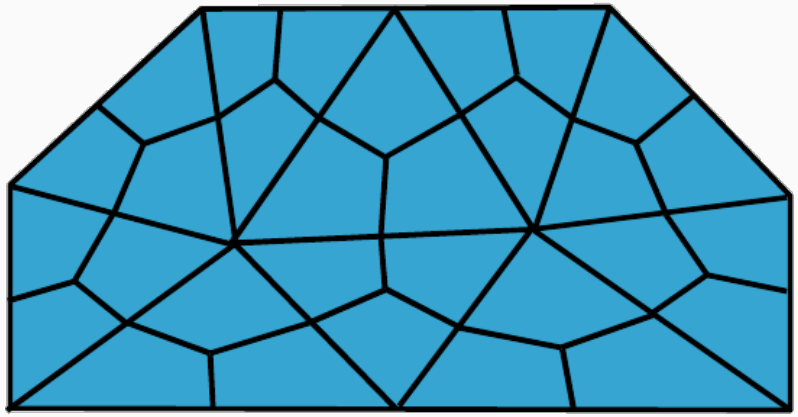

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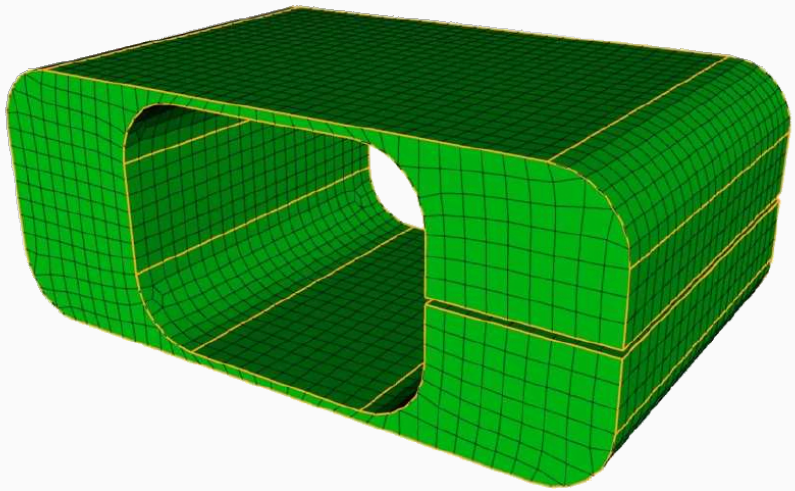

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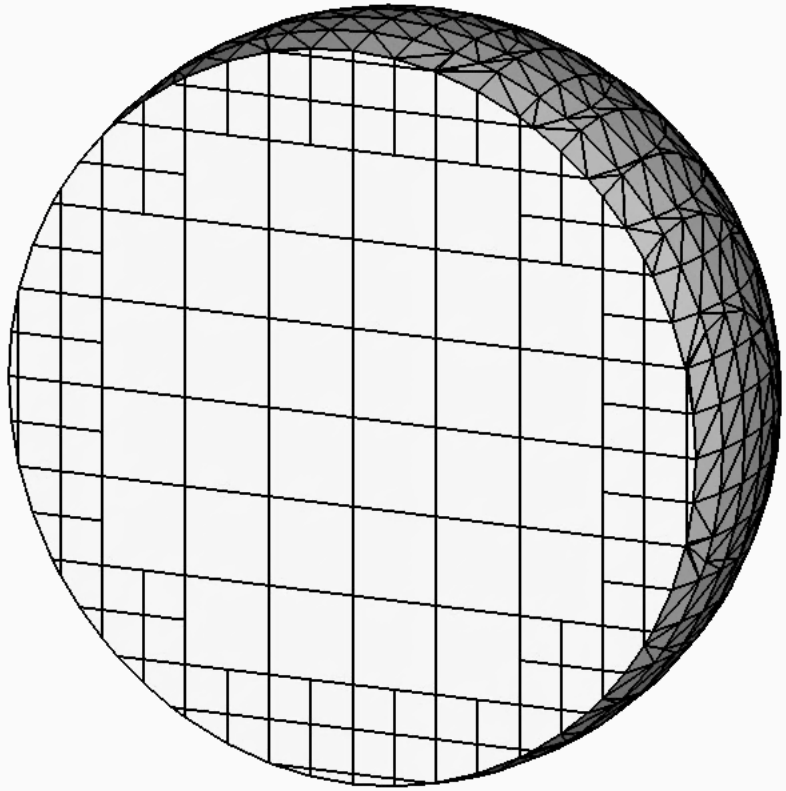

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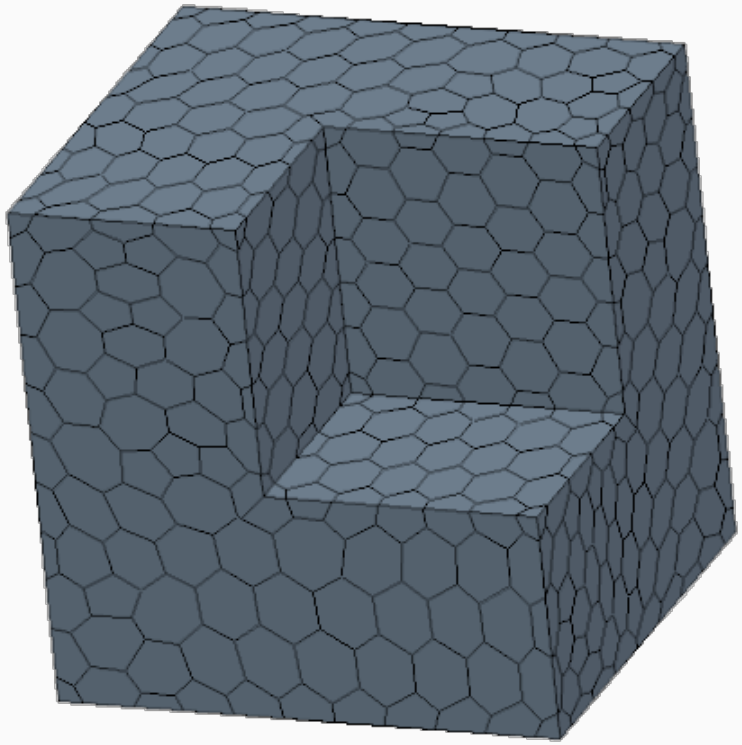


Meshing software

Tetrahedral meshes:

- GHS3D – `team.inria.fr/gamma3/gamma-software`
- TetGen – `wias-berlin.de/software/tetgen`
- Netgen – `sf.net/p/netgen-mesher`
- Gmsh – `gmsh.info`
- CGALmesh – `cgal.org`
- Ani3D – `sf.net/p/ani3d`

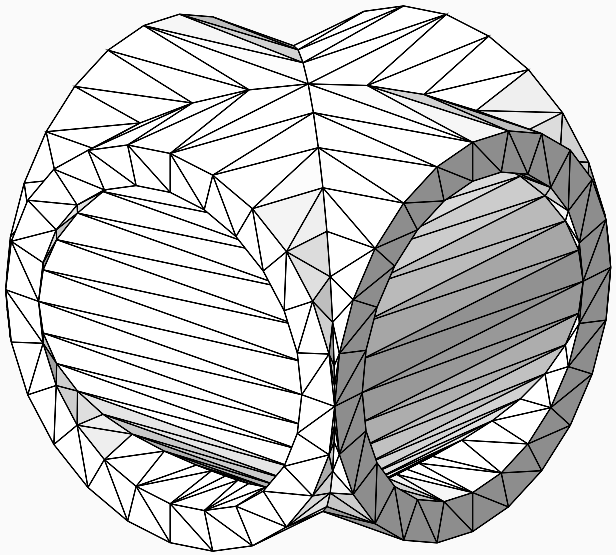
Hexahedral meshes:

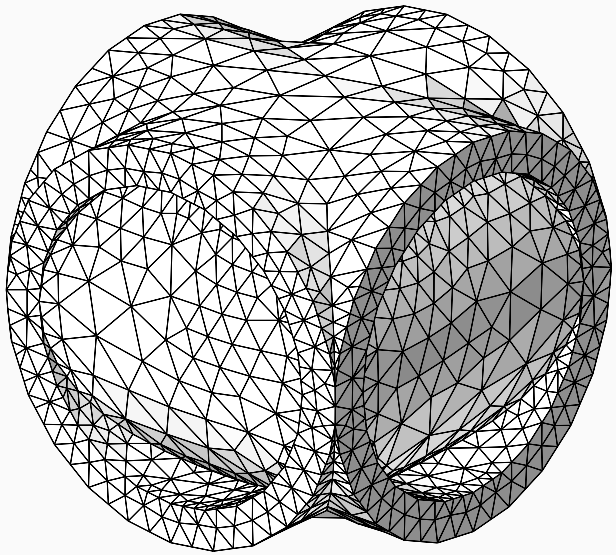
- Cubit – `cubit.sandia.gov`
- Hexotic – `team.inria.fr/gamma3/gamma-software`

CAD models

Surface mesh

- Triangulated surface – typical export format in CAD systems
- Minimal number of triangles – good for visualization
- May require additional pre-processing for mesh generation





Boundary representation

Vertices $\longrightarrow$ Curves $\longrightarrow$ Surfaces $\longrightarrow$ Volumes $\longrightarrow$ Body
Loops Shells

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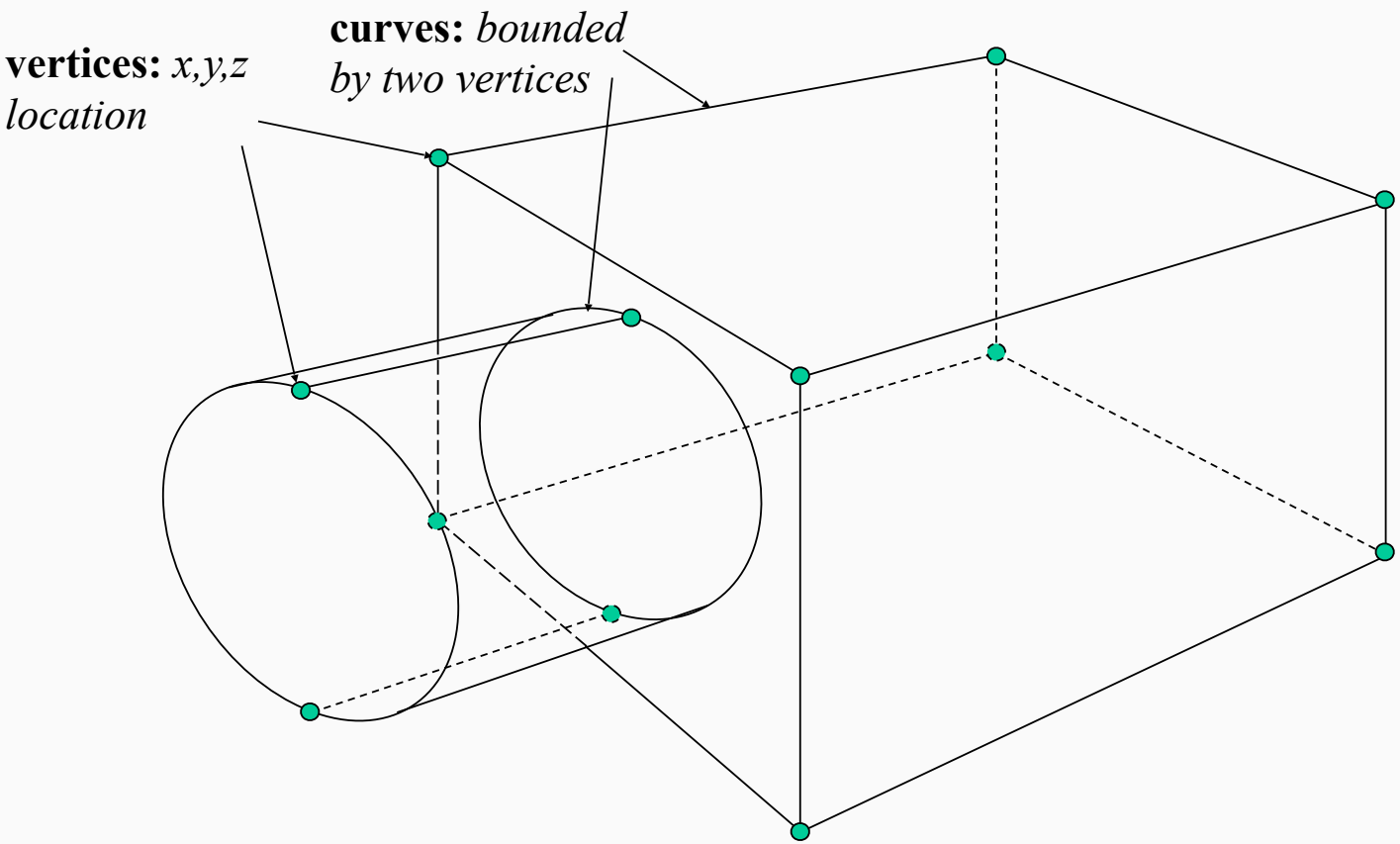

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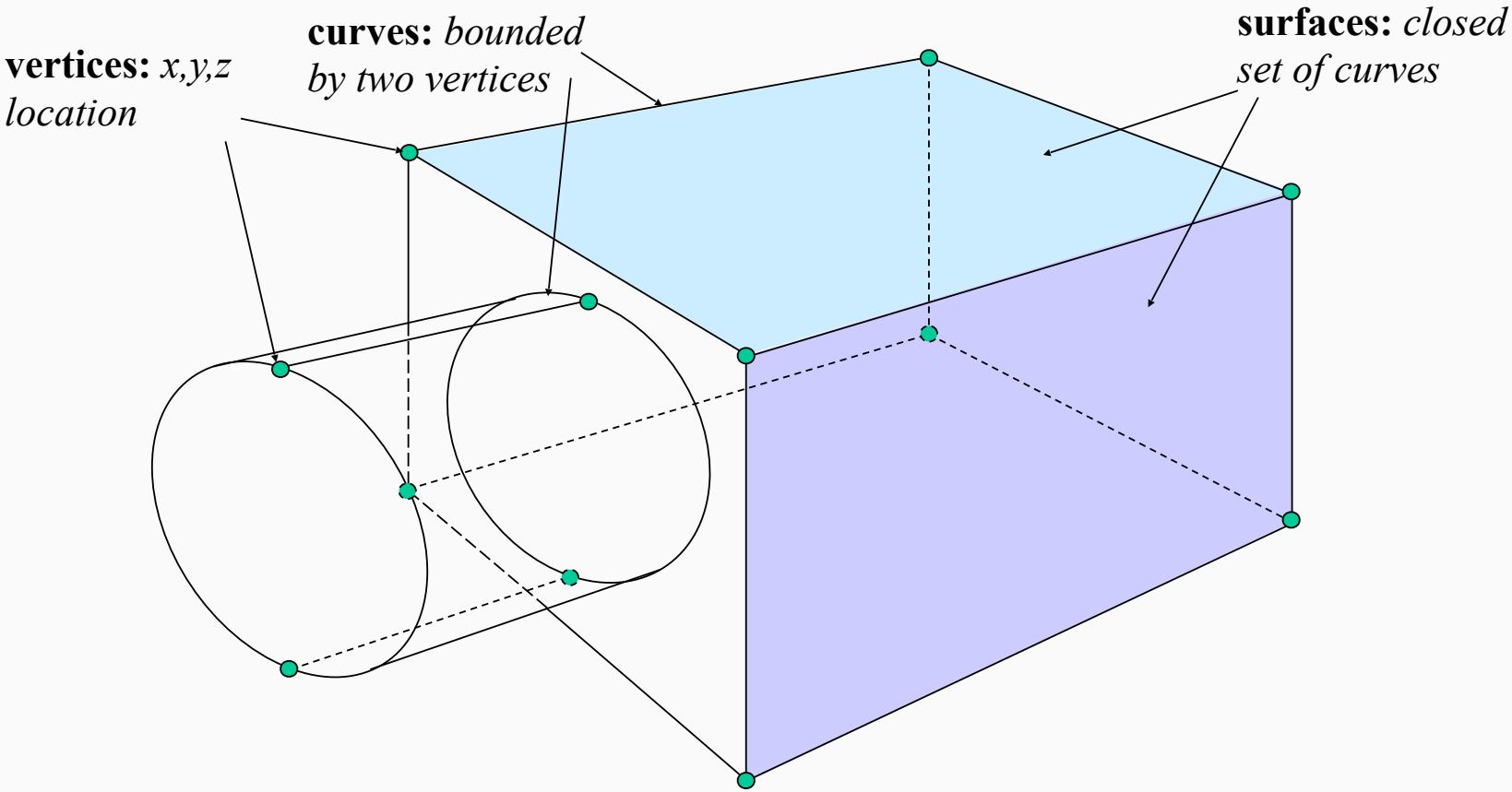

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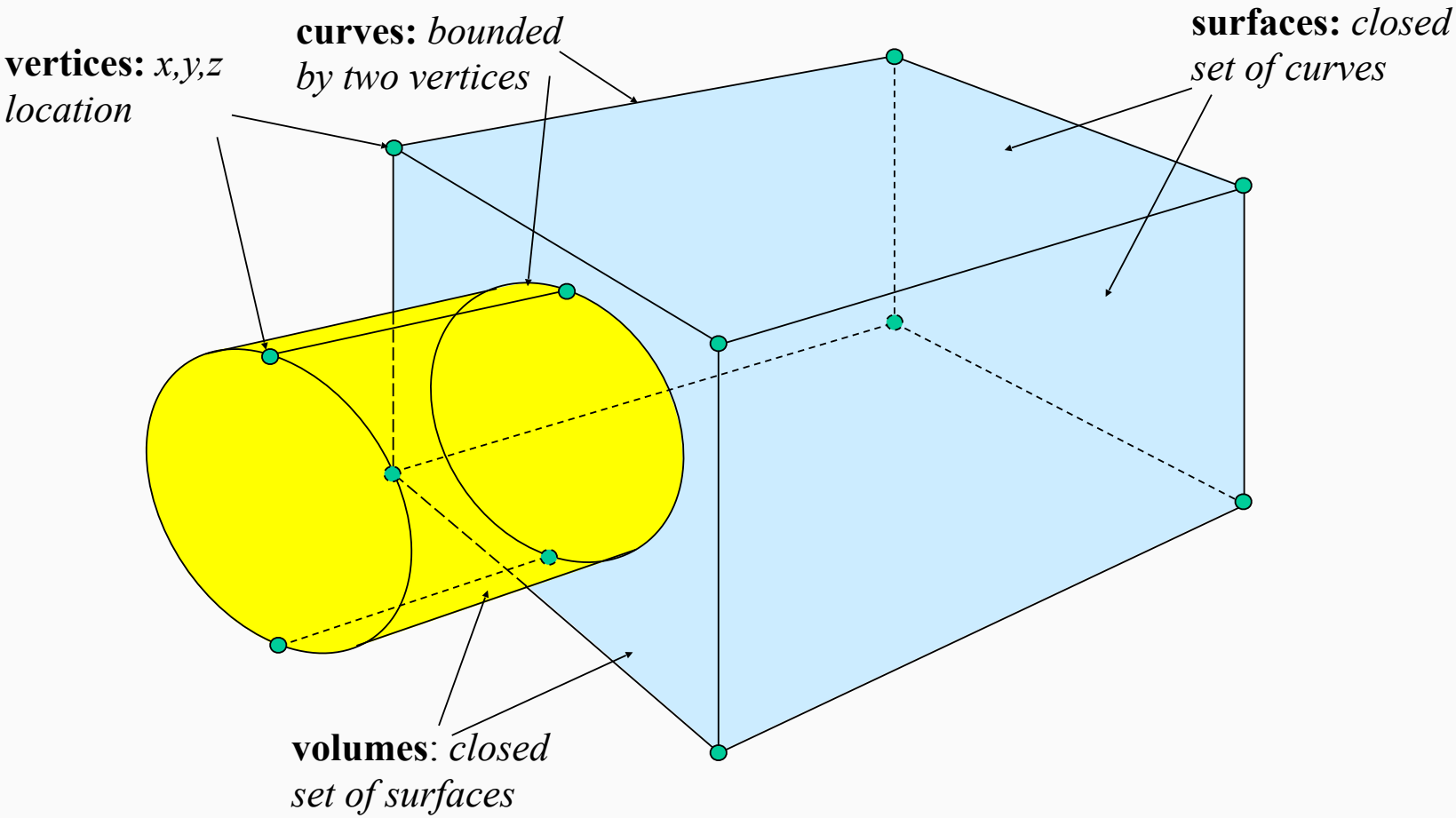

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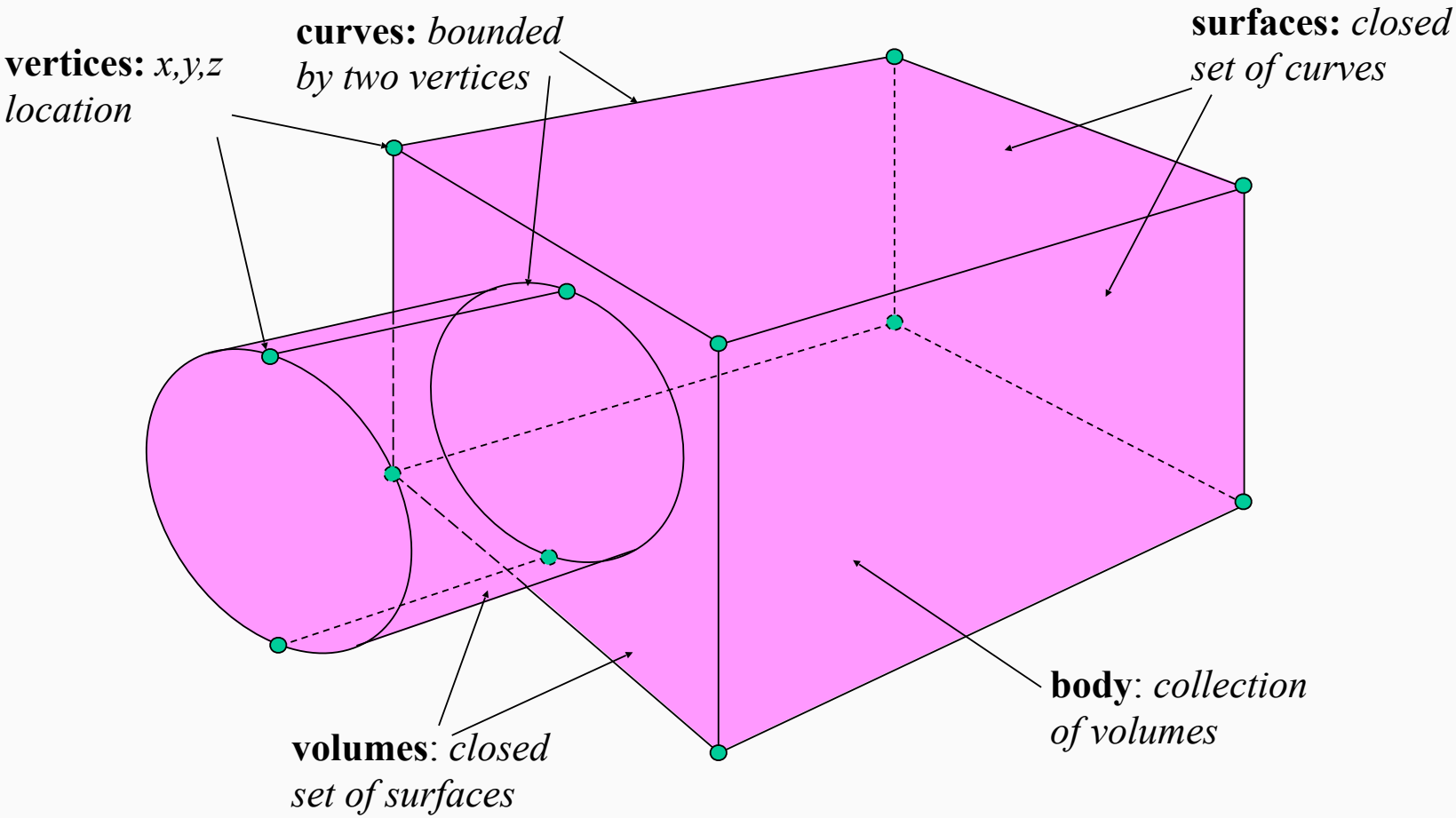

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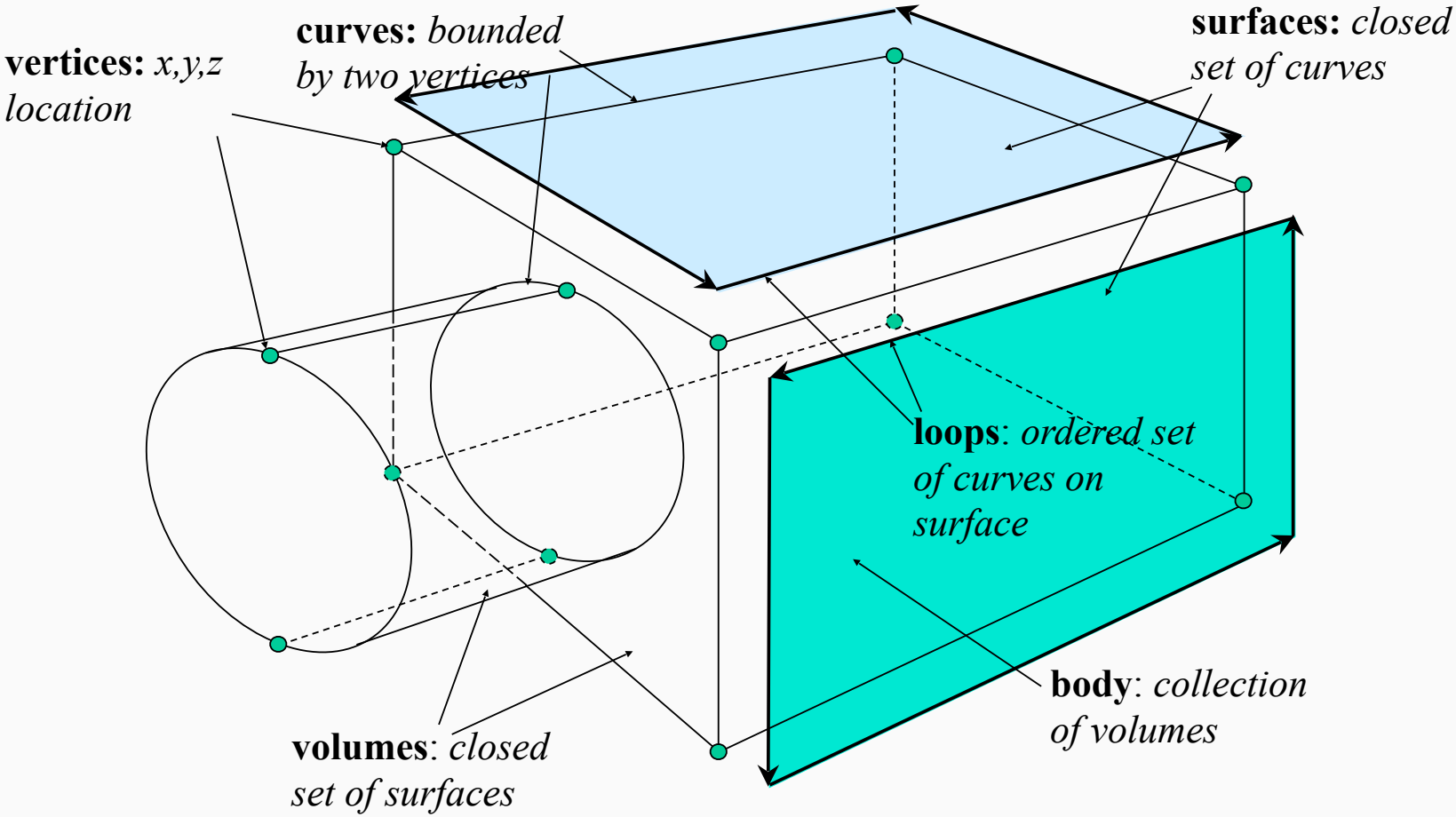

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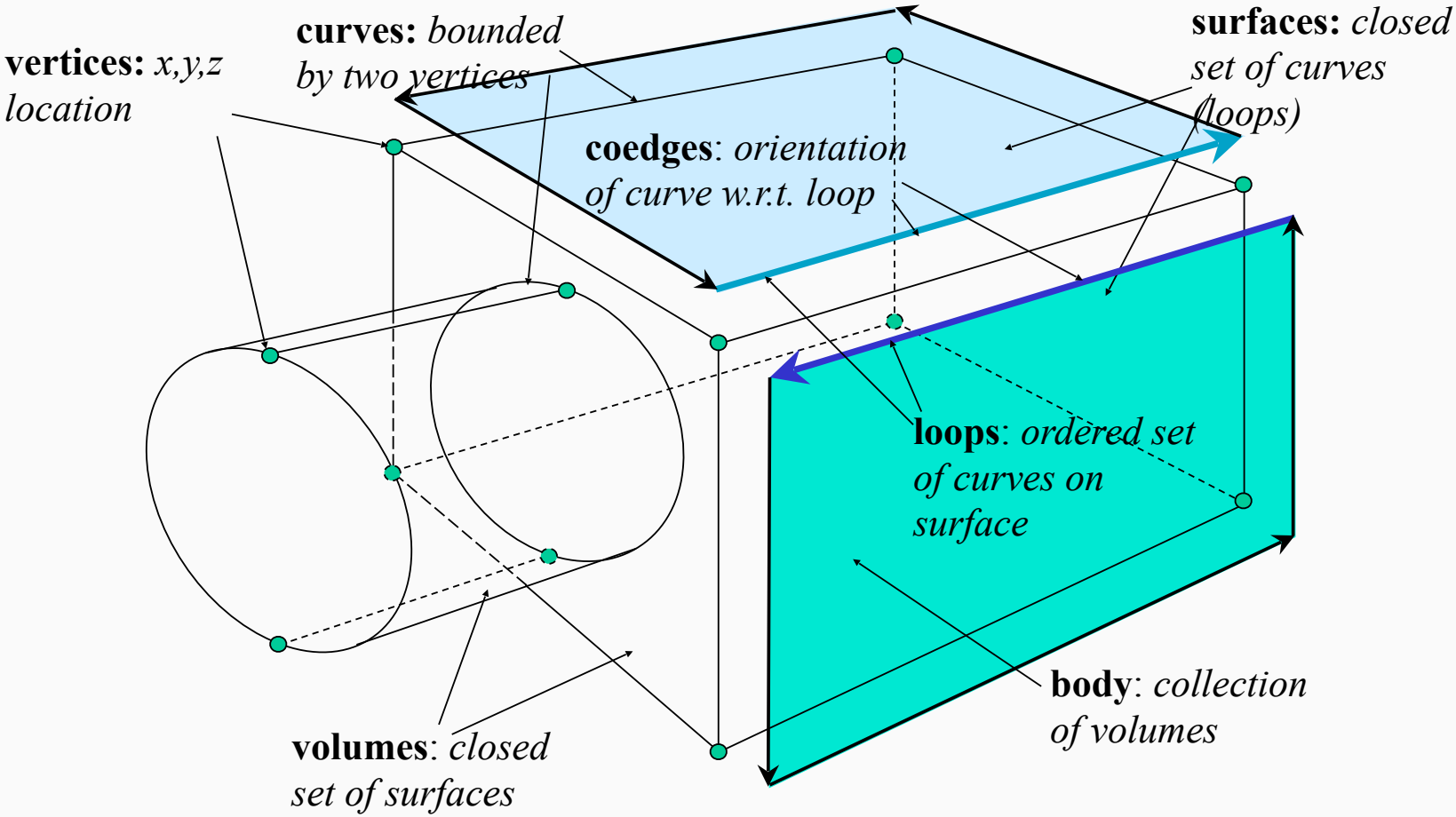

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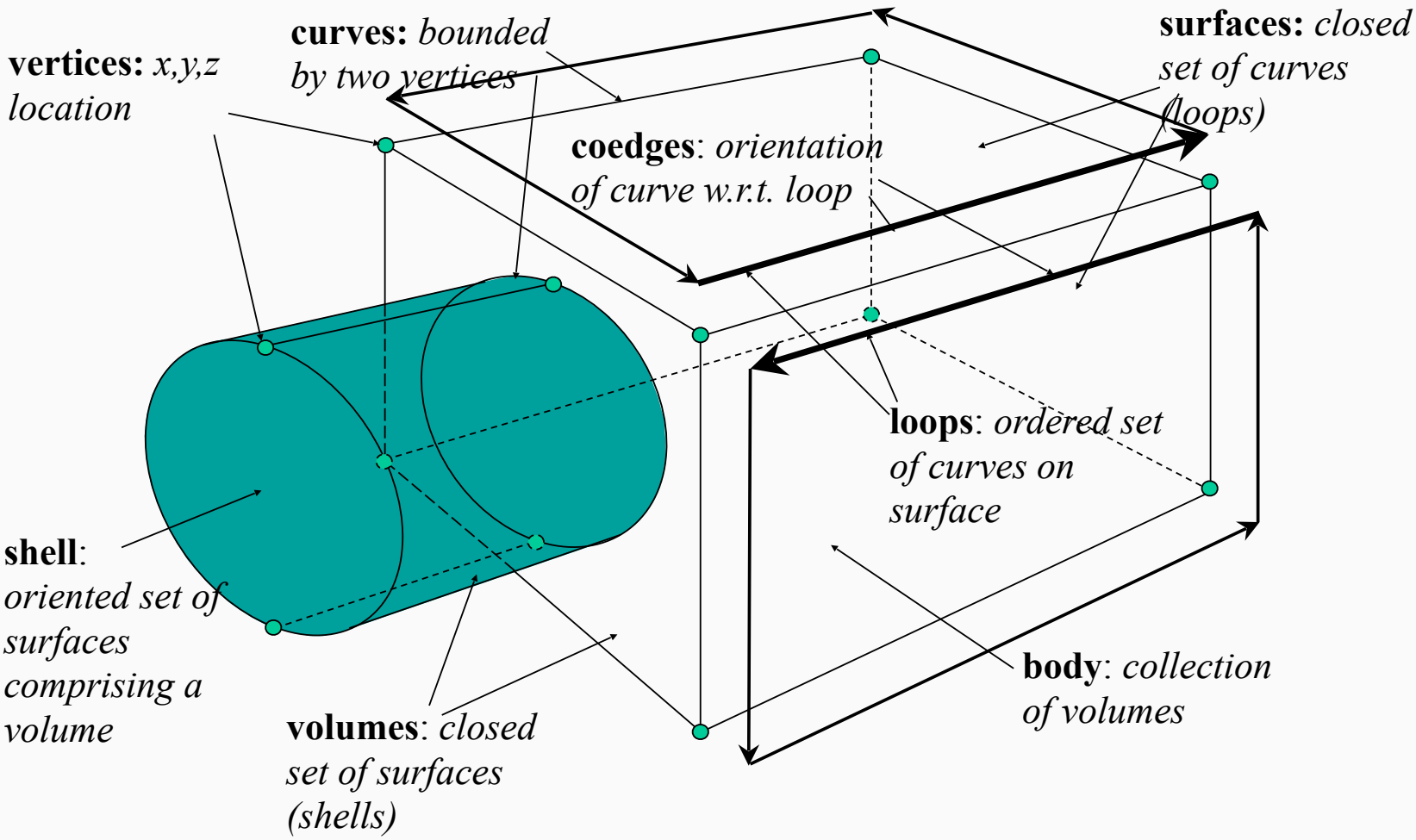
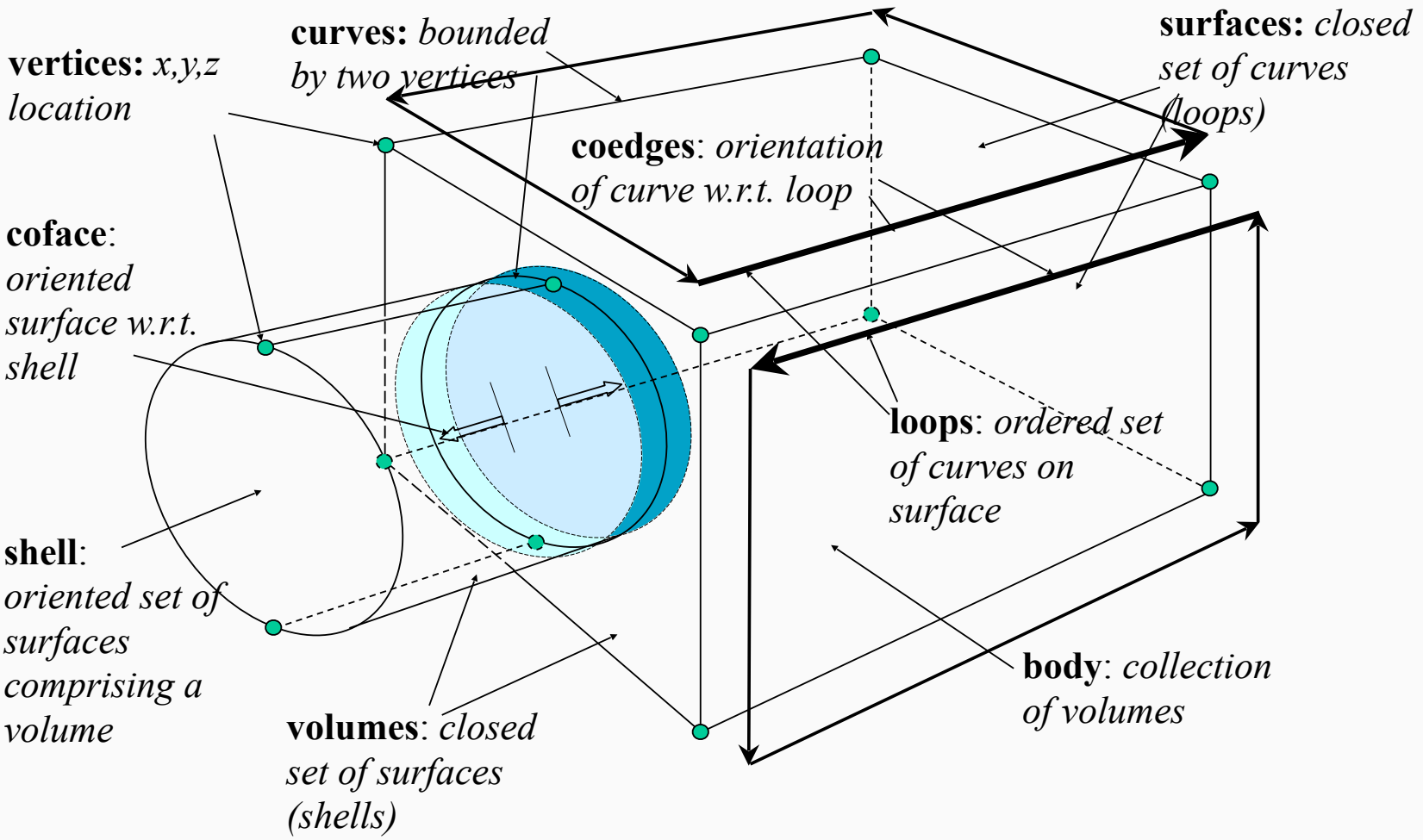
Boundary representation



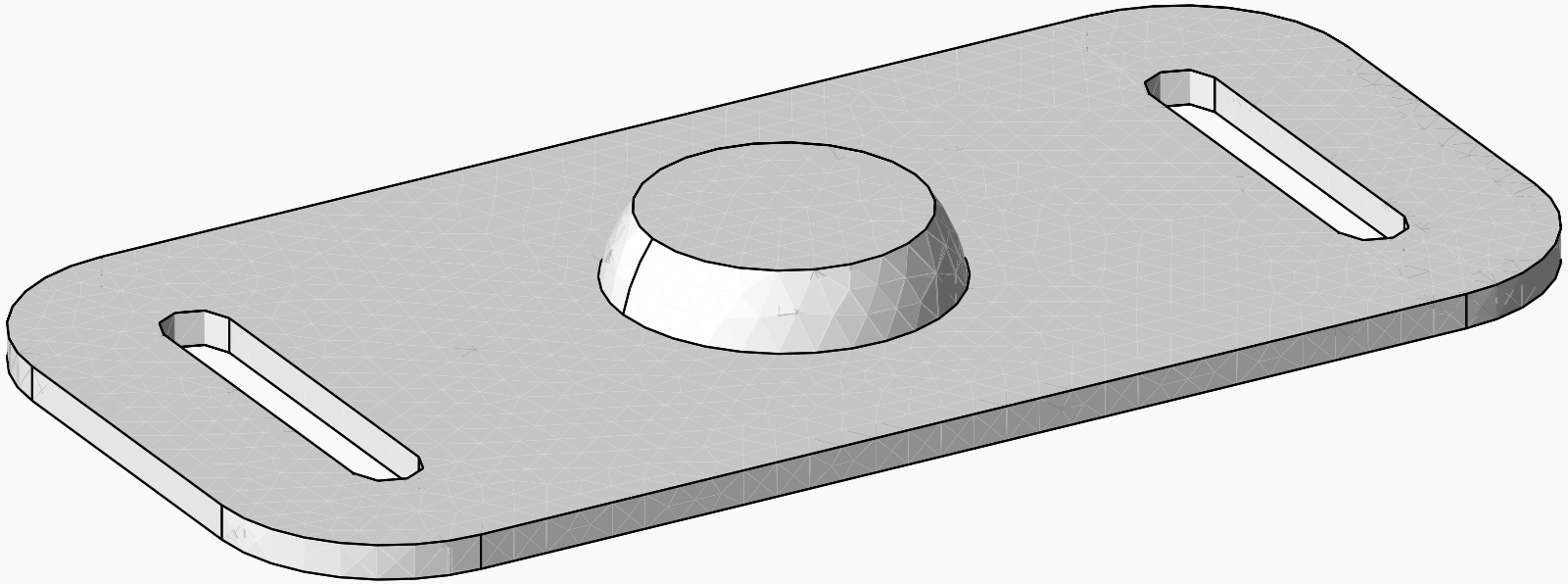
Vertices $\longrightarrow$ Curves $\longrightarrow$ Surfaces $\longrightarrow$ Volumes $\longrightarrow$ Body
Loops Shells

Boundary representation



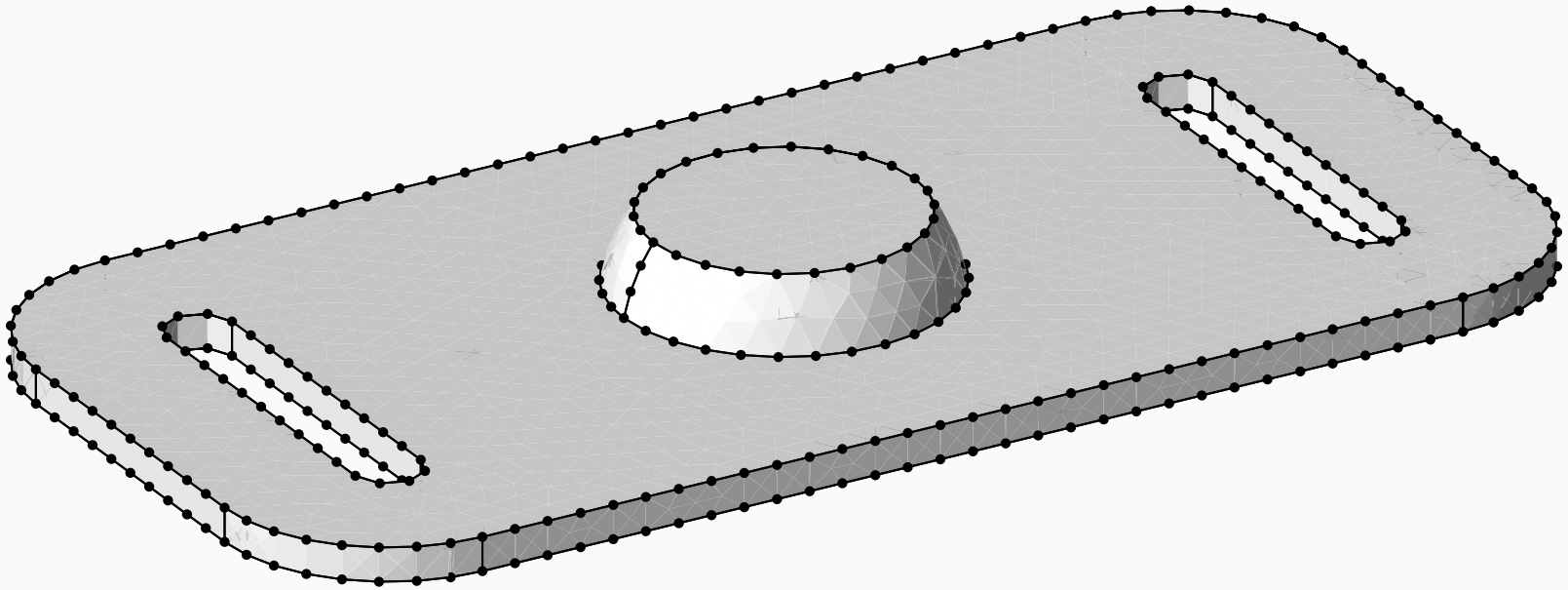
Vertices $\longrightarrow$ Curves $\longrightarrow$ Surfaces $\longrightarrow$ Volumes $\longrightarrow$ Body
Loops Shells

Bottom-up approach



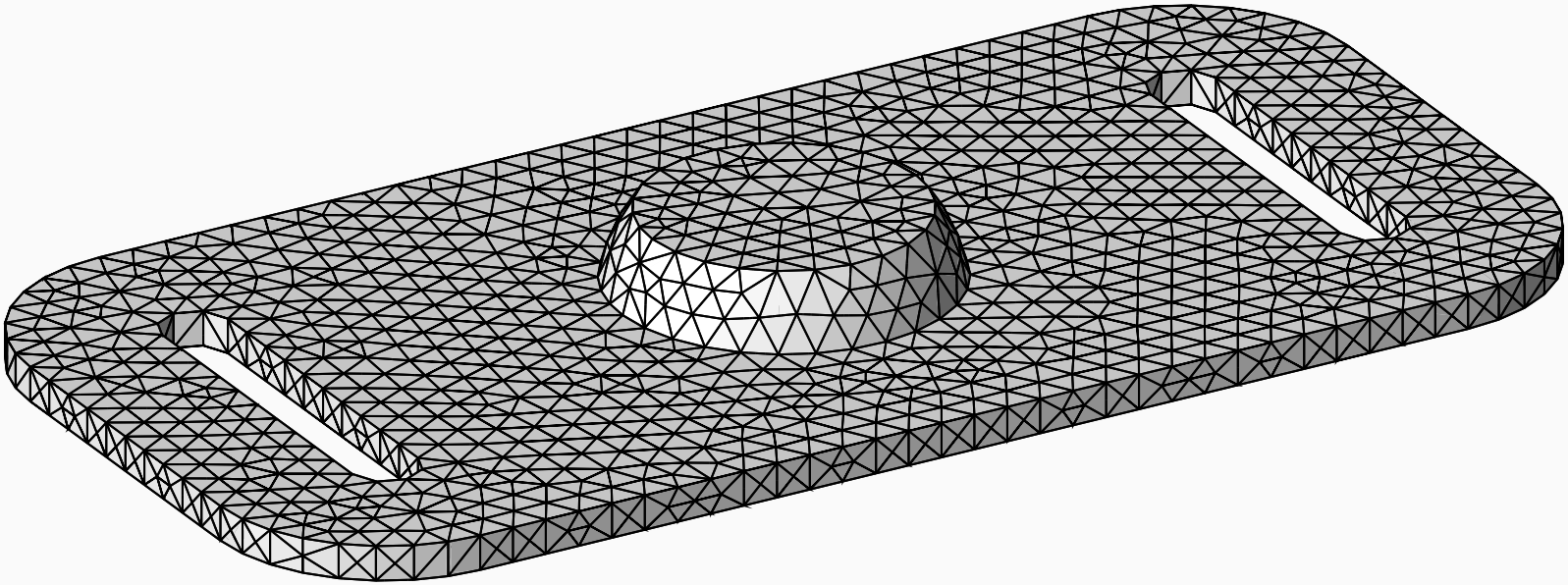
Initial CAD model

Bottom-up approach



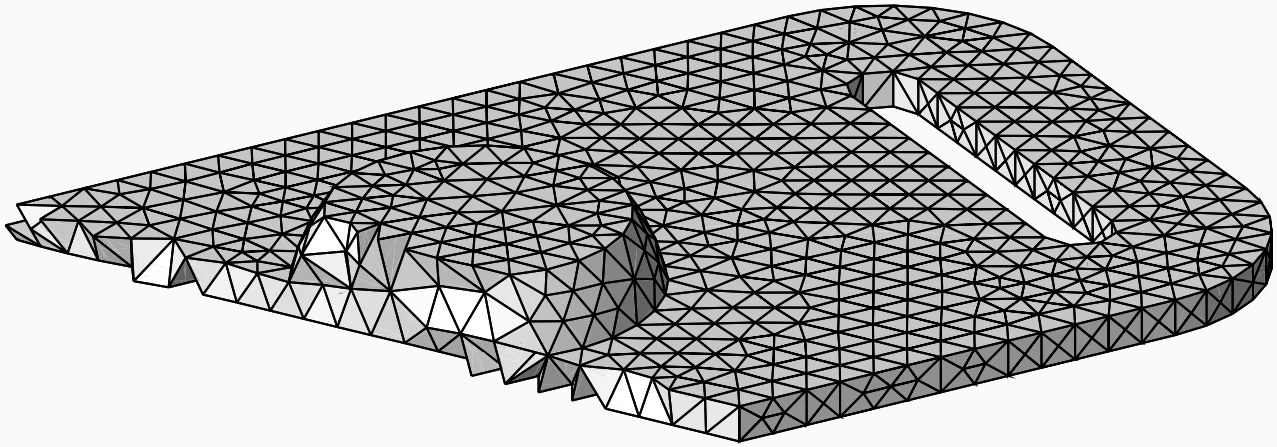
Discretization of the edges

Bottom-up approach



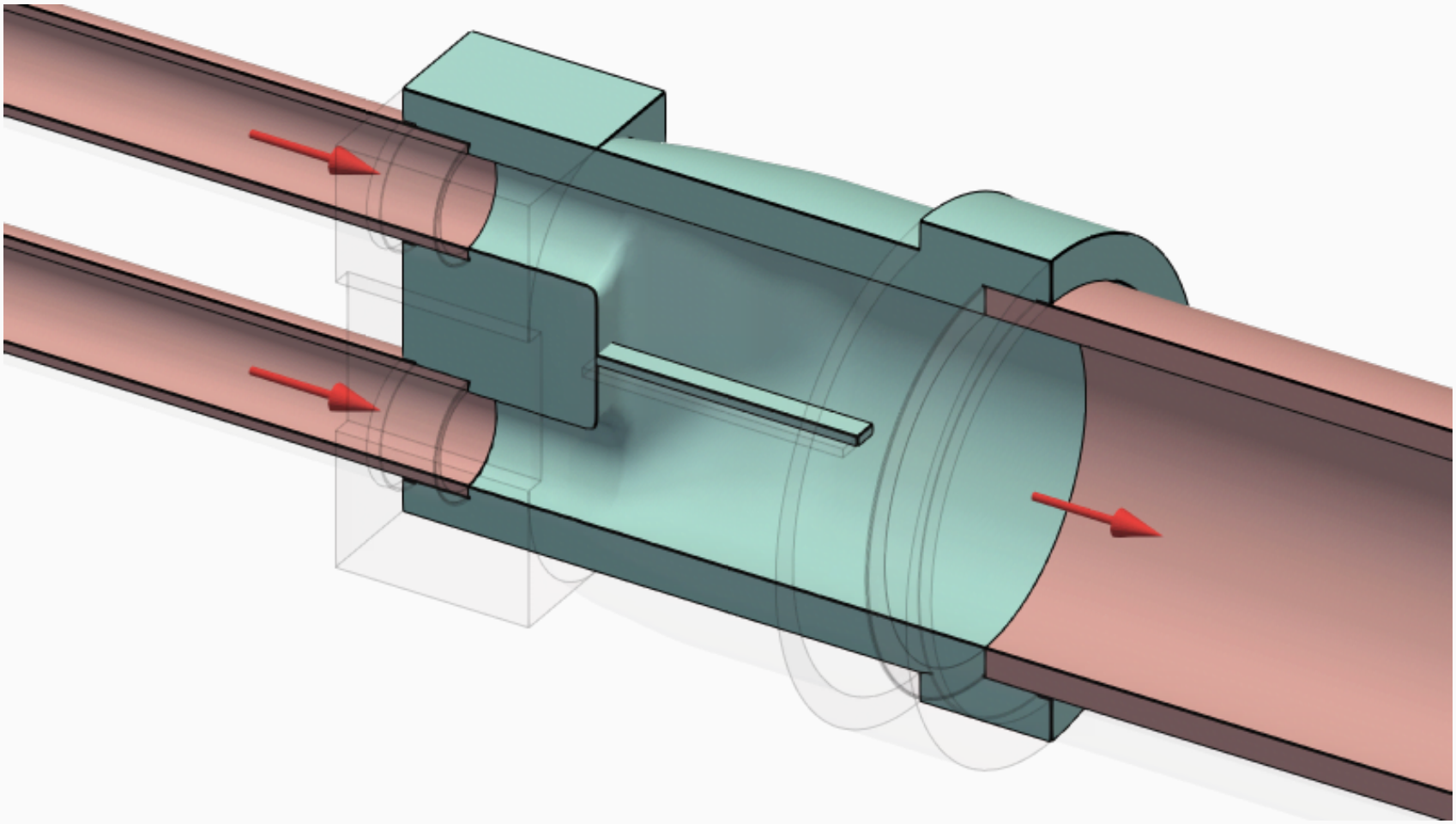
Triangulation of the faces

Bottom-up approach

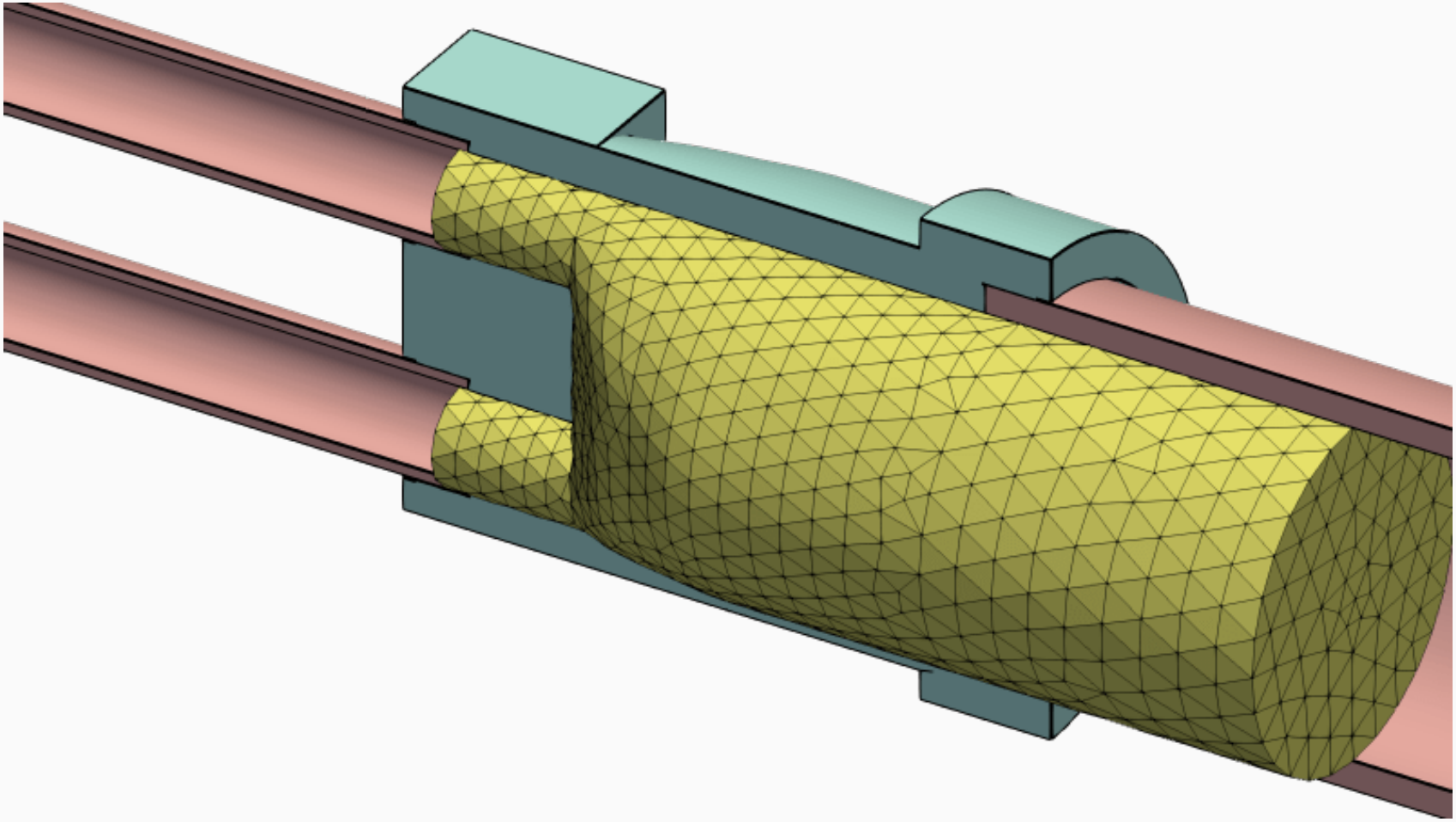


Tetrahedral volume mesh

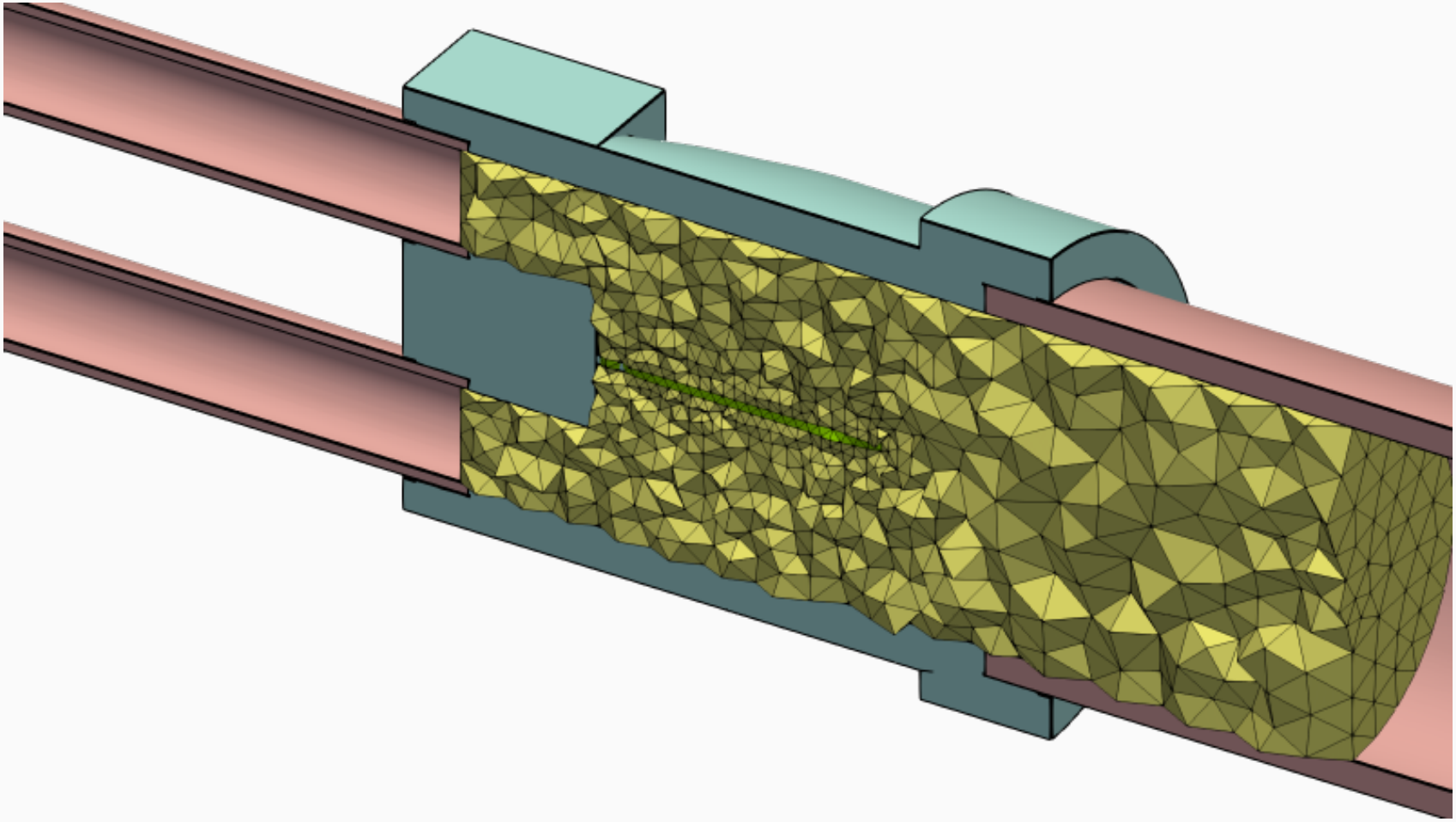
Example – tetrahedral mesh



Example – tetrahedral mesh



Example – tetrahedral mesh

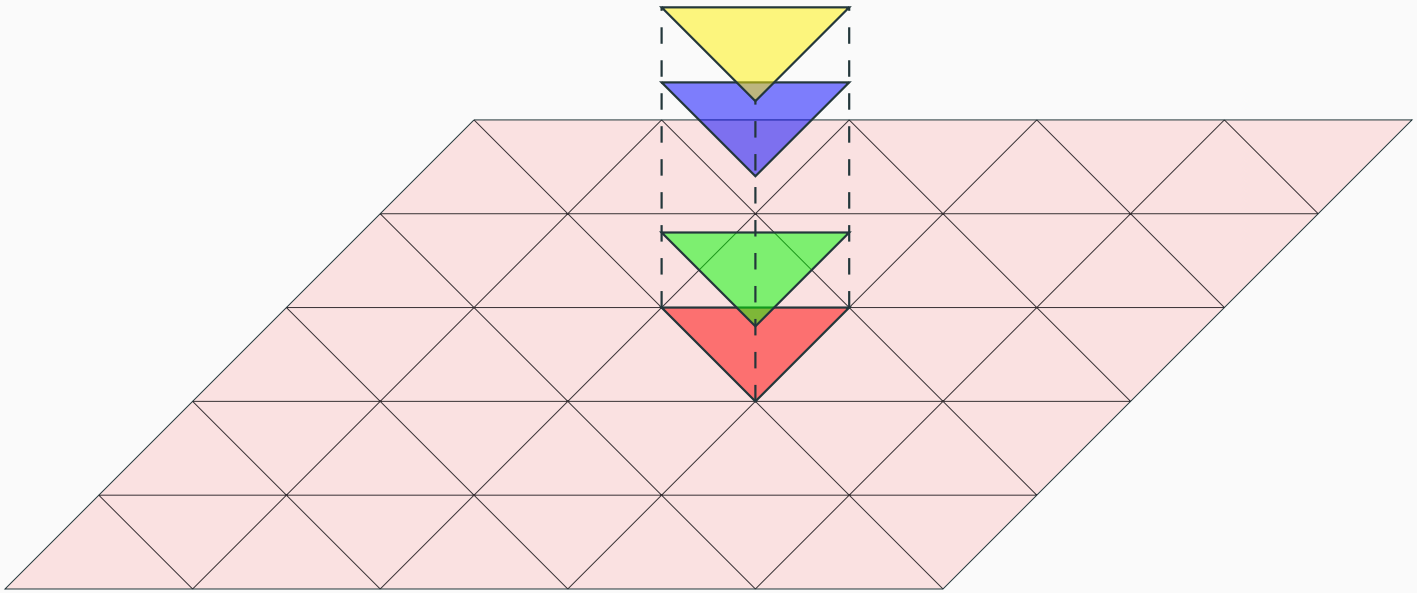


Important notes

- CAD model surface should be watertight
- Surface intersections and nonconformity should be fixed
- Initial surface mesh may be remeshed to improve quality
- Bottom-up approach is usually preferred

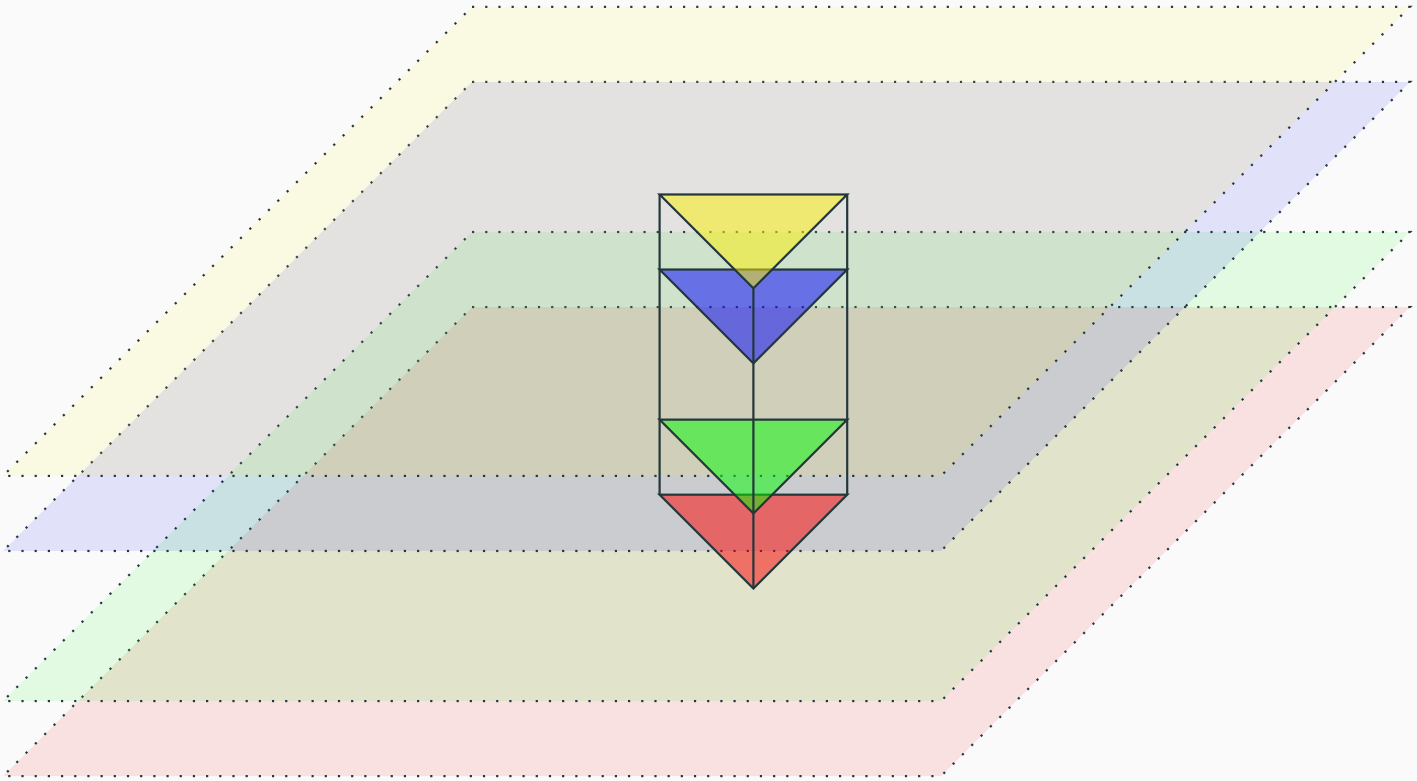
Geological layers

Layered domains



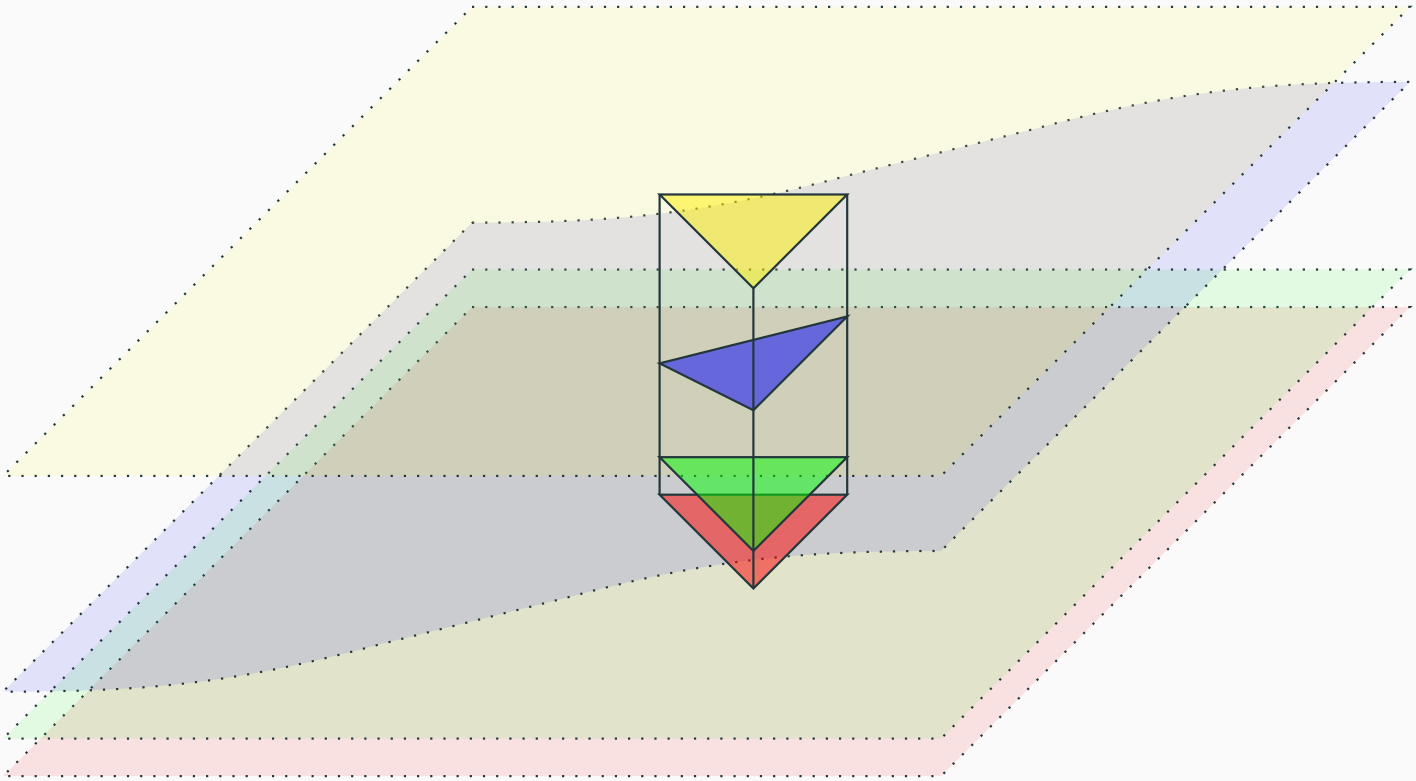
Triangular grid is extruded in vertical direction

Layered domains



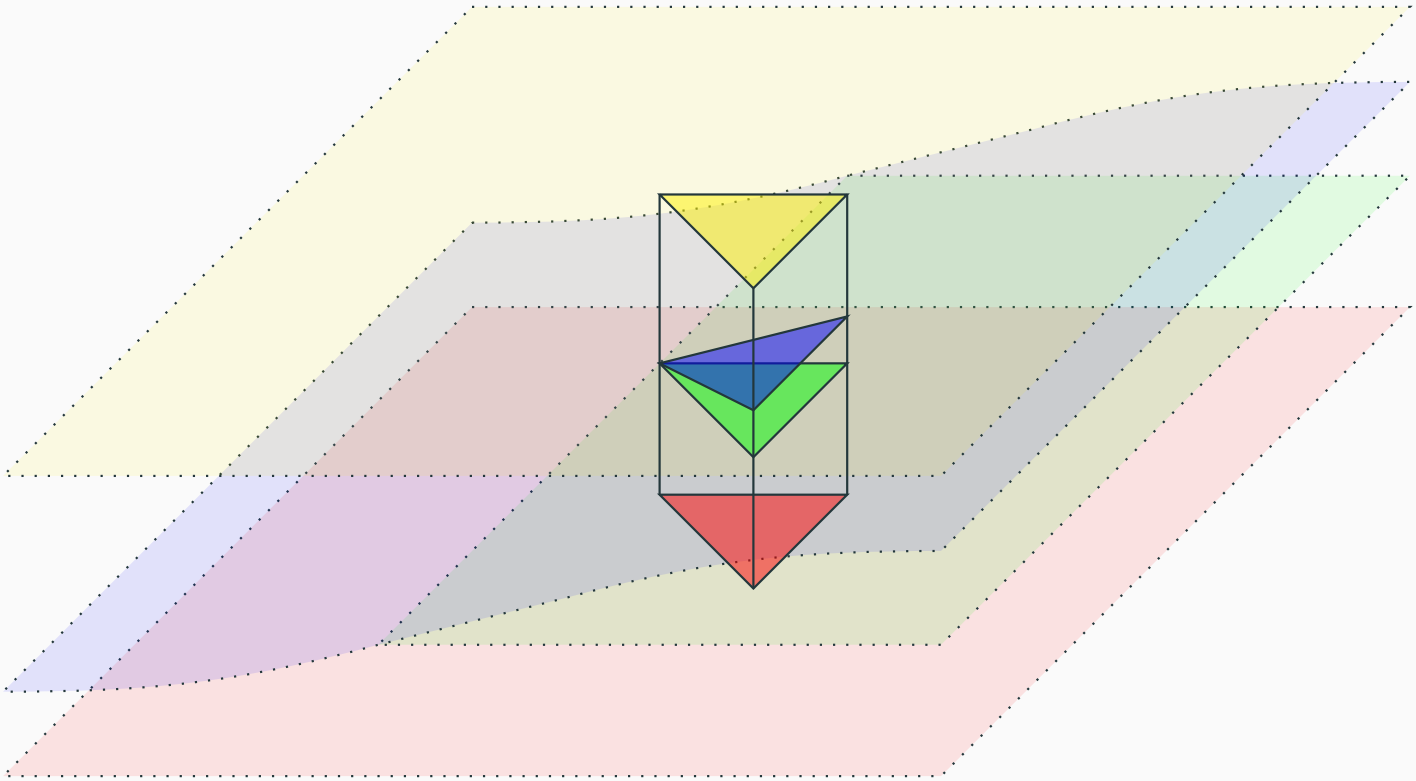
Each triangle produces one prism for each layer

Layered domains



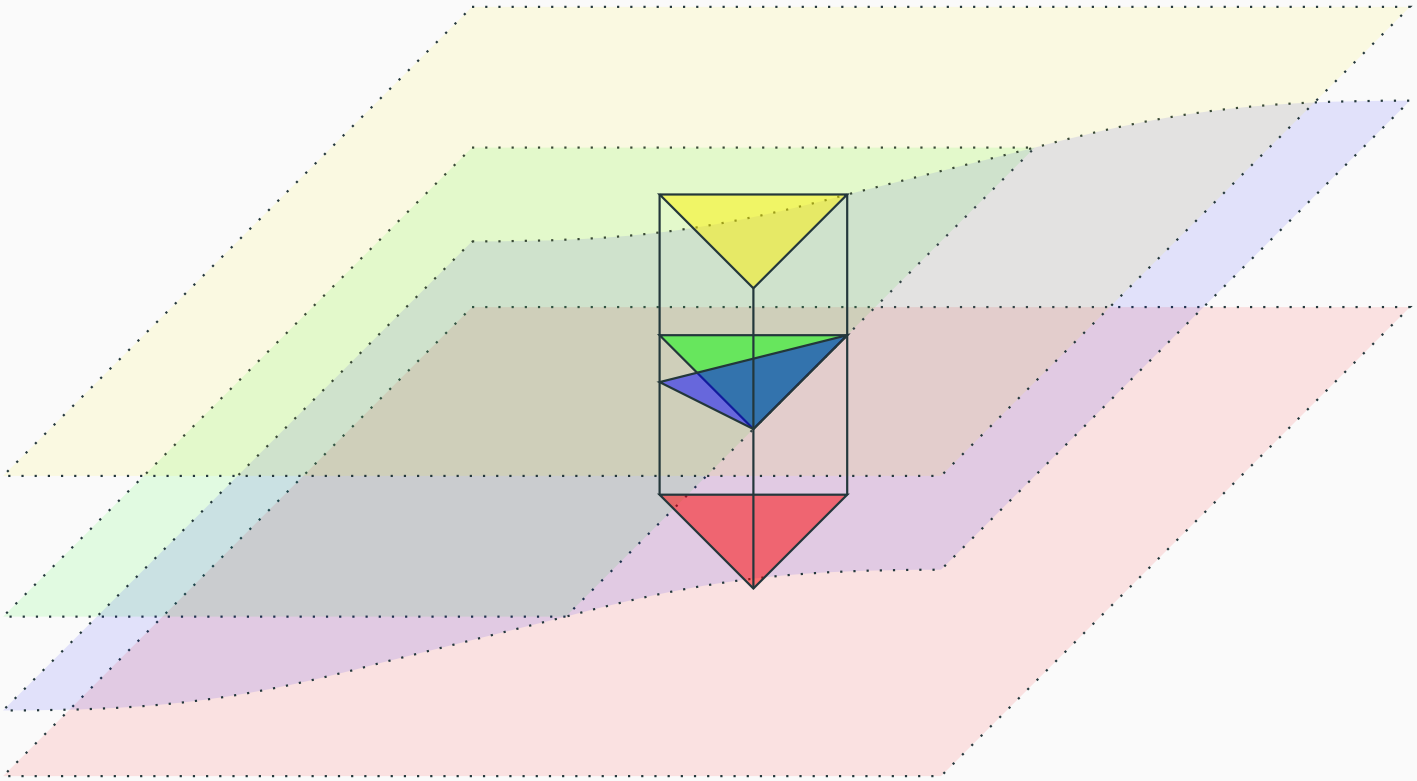
Curvilinear layers affect the shape of the prisms

Layered domains



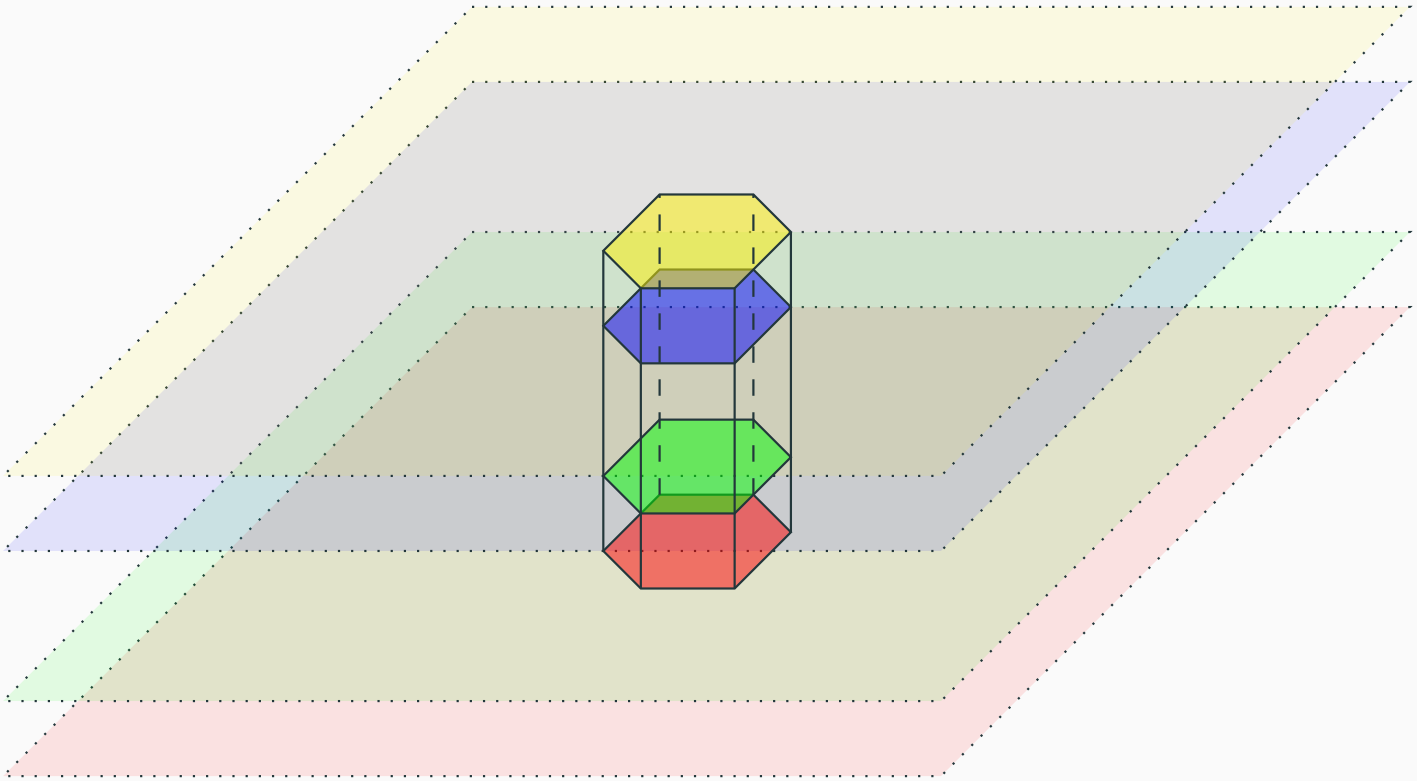
Pinch-outs may produce pyramids and tetrahedra

Layered domains



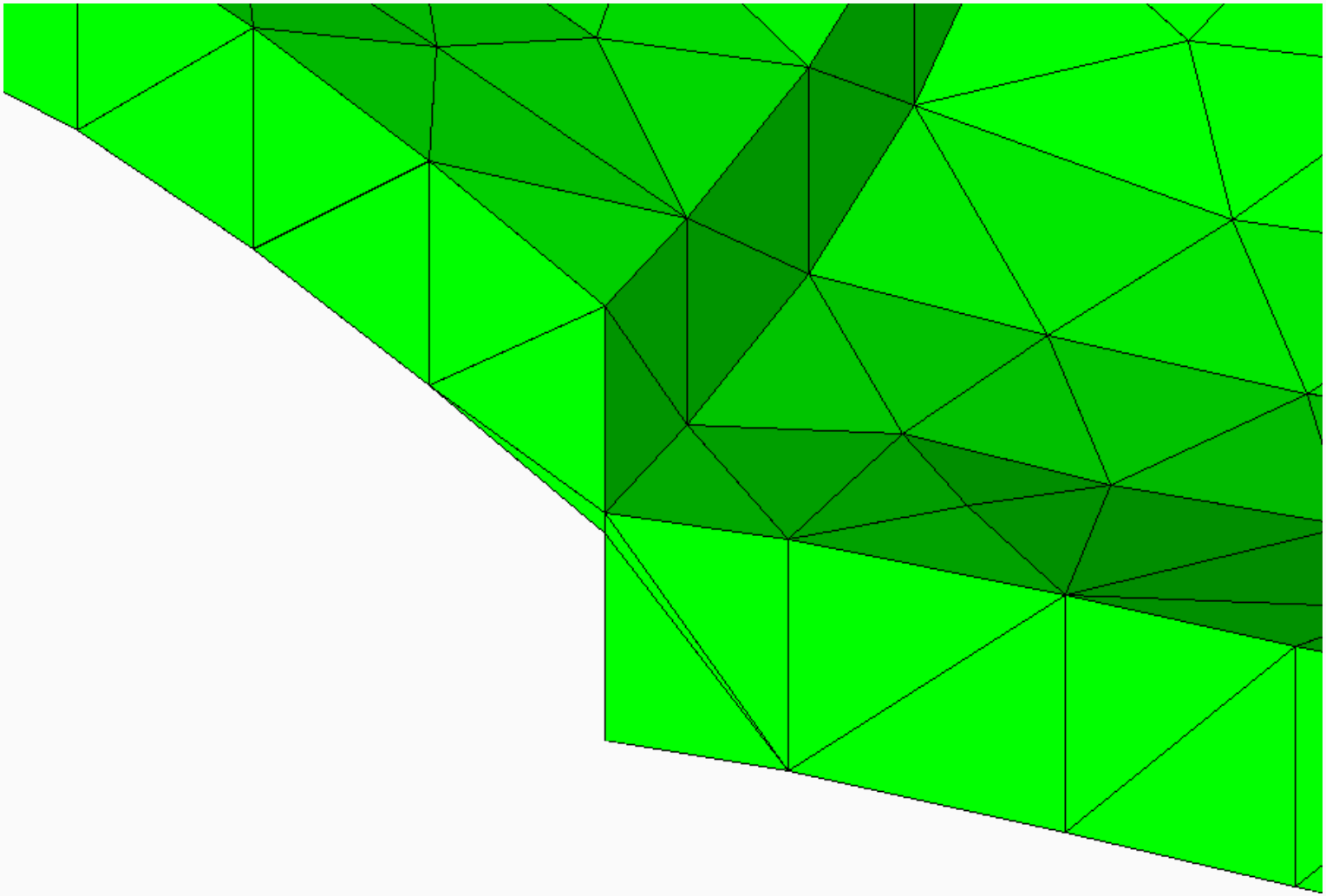
Pinch-outs may produce pyramids and tetrahedra

Layered domains



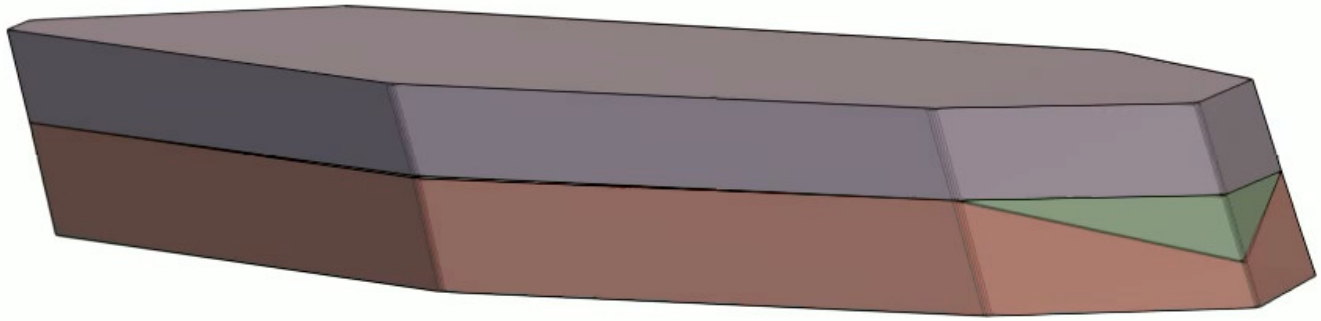
Polygonal base mesh will produce polyhedral volume mesh

Faults

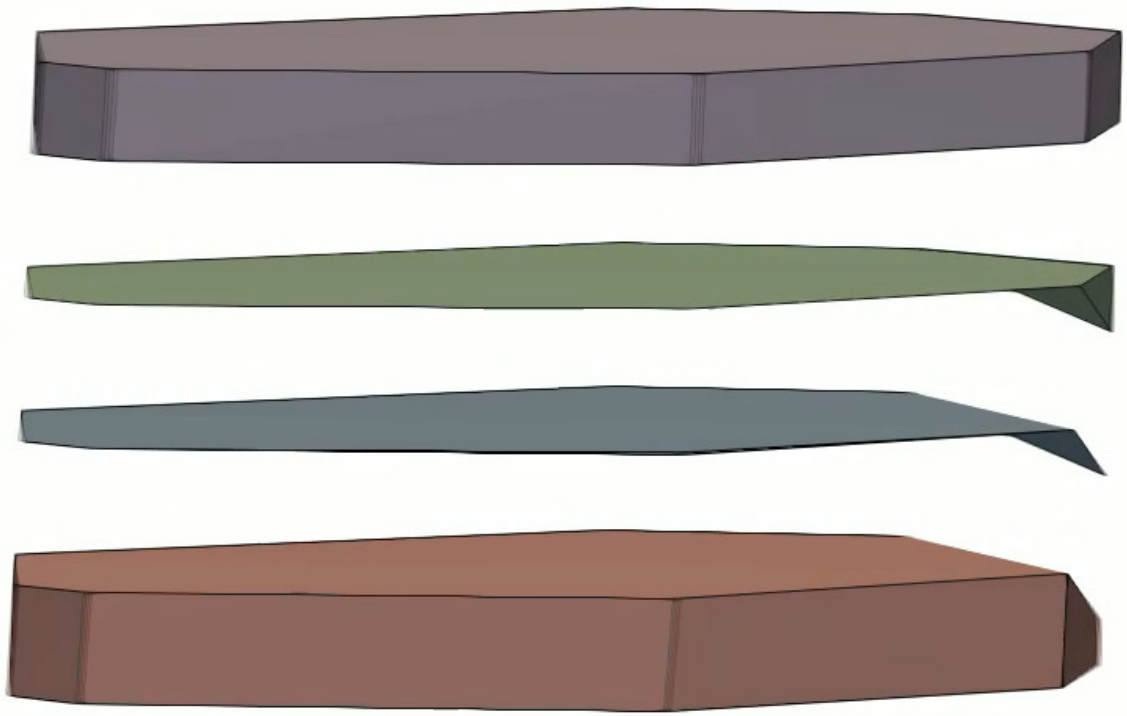


Mesh conformity should be restored

Complex cases

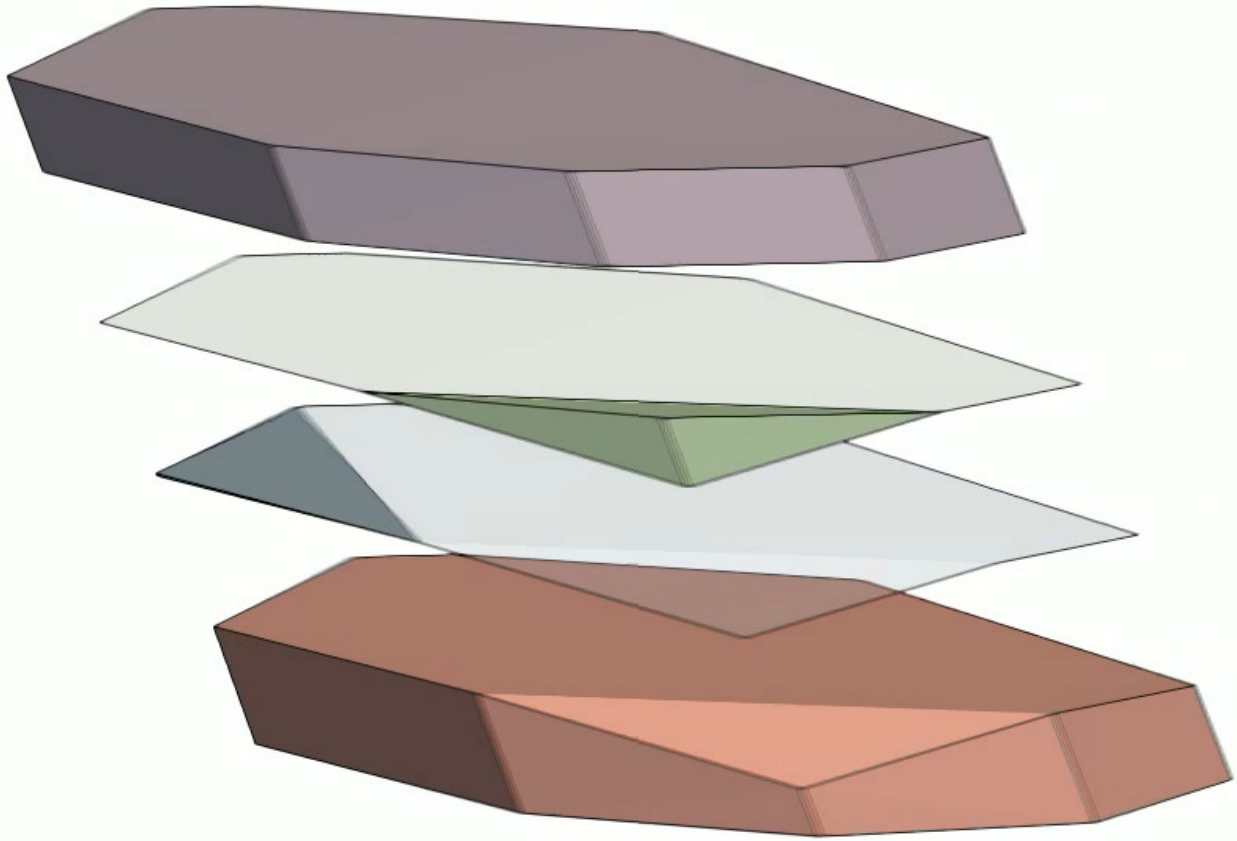


Complex cases



Non-planar faces

Complex cases



Degenerate elements

Important notes

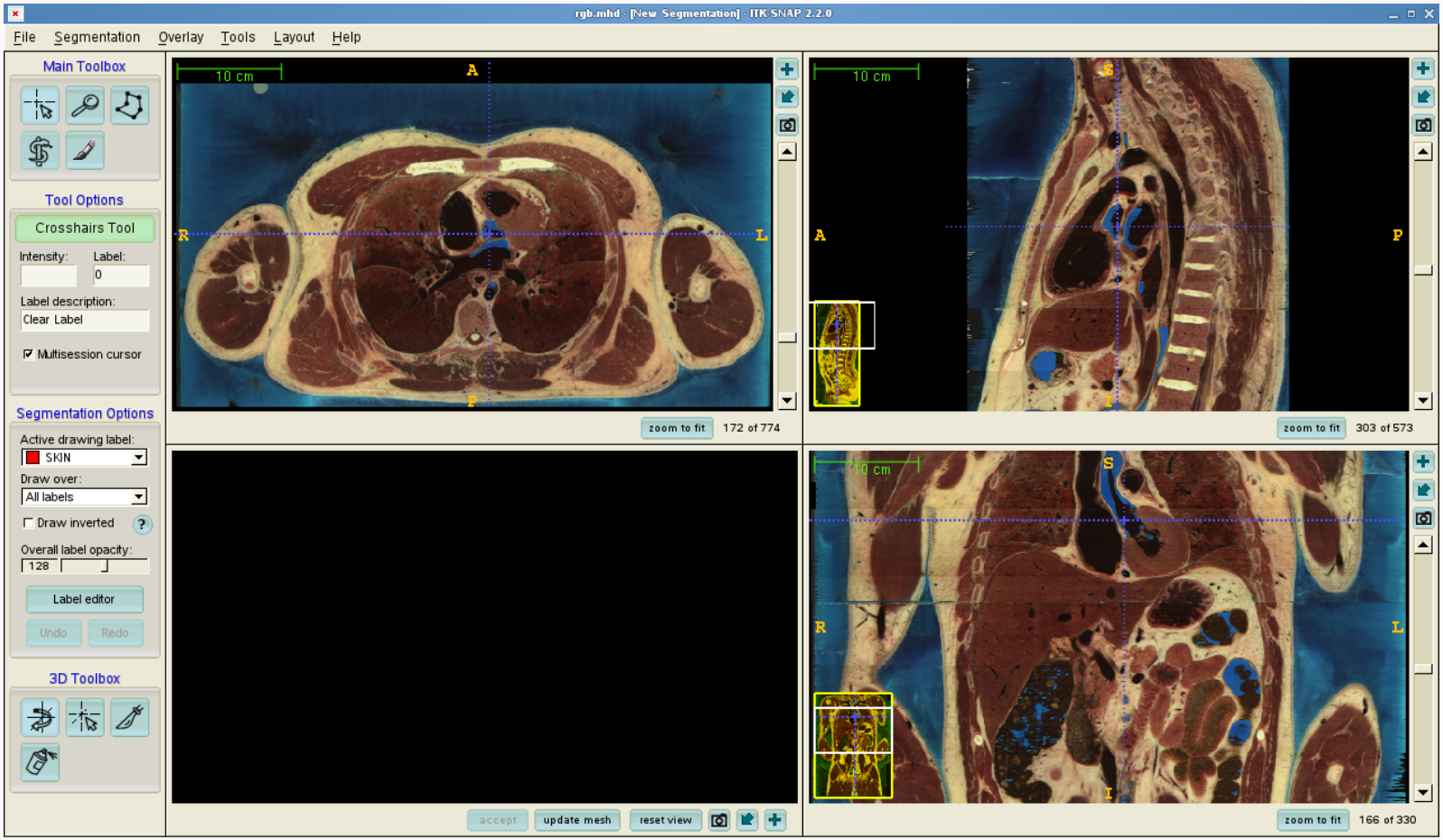
- Extrusion direction may be altered across layers and across domain – check tangling of the elements
- Intersection of layers (pinch-outs) should be treated carefully – it is a good idea to resolve their traces in the base mesh
- Faults may produce nonconforming mesh – conformity may be restored by producing polyhedral elements or splitting them into tetrahedra
- Always check the planarity of the faces and detect degenerate elements

Biomedical applications – preview

Segmentation of medical images

ITK-SNAP – free software for Visualization and Segmentation

`www.itksnap.org`



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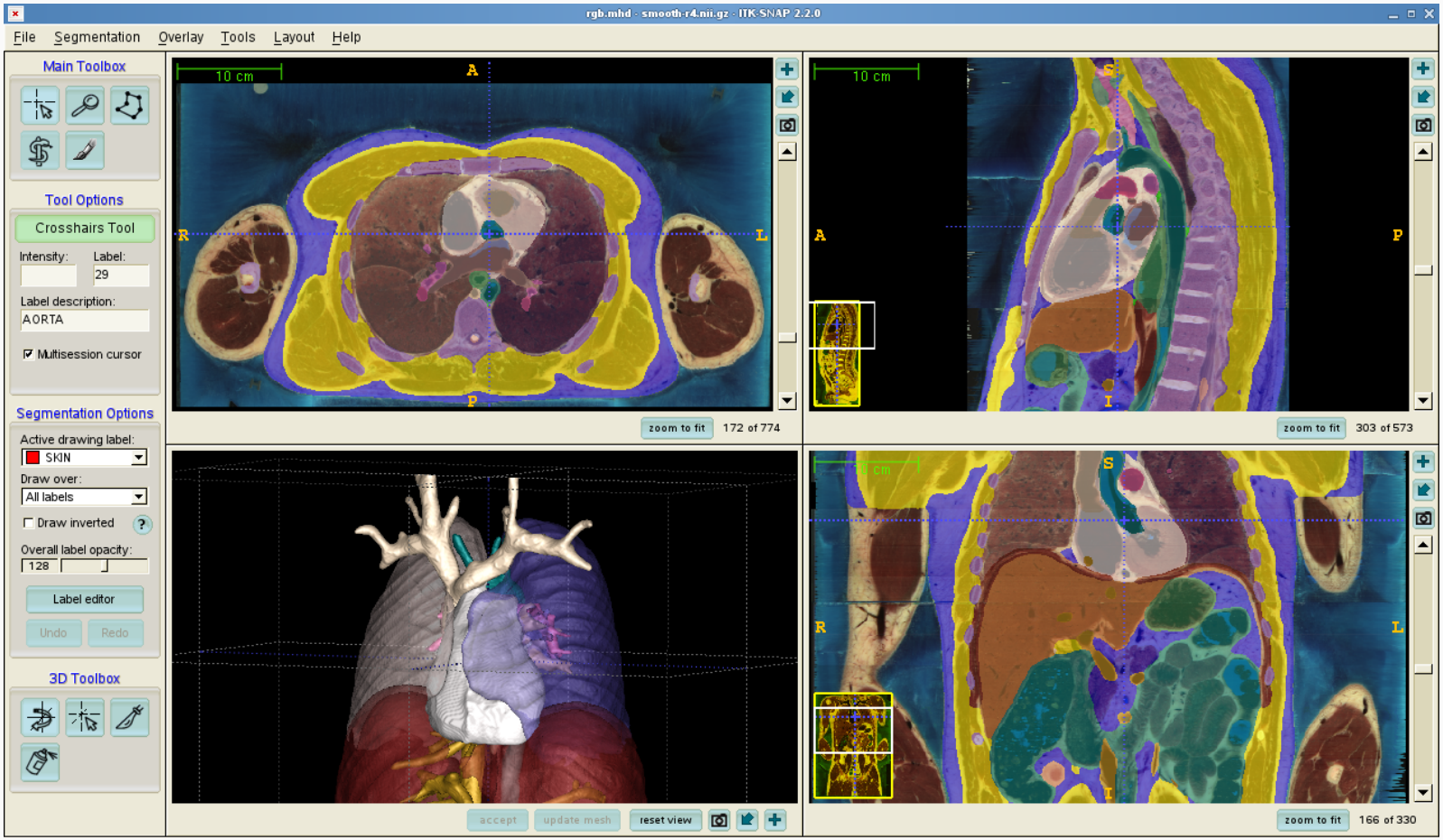
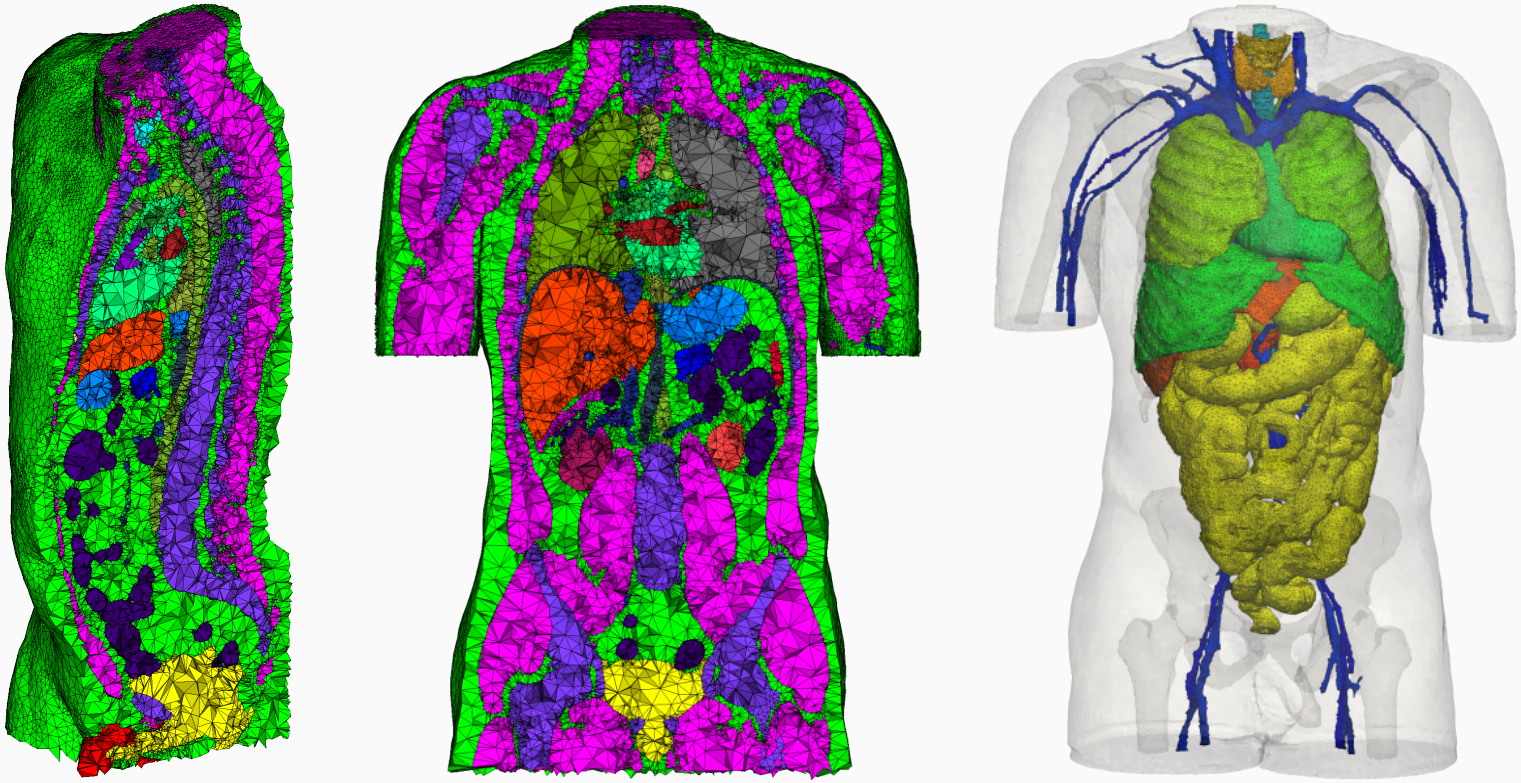


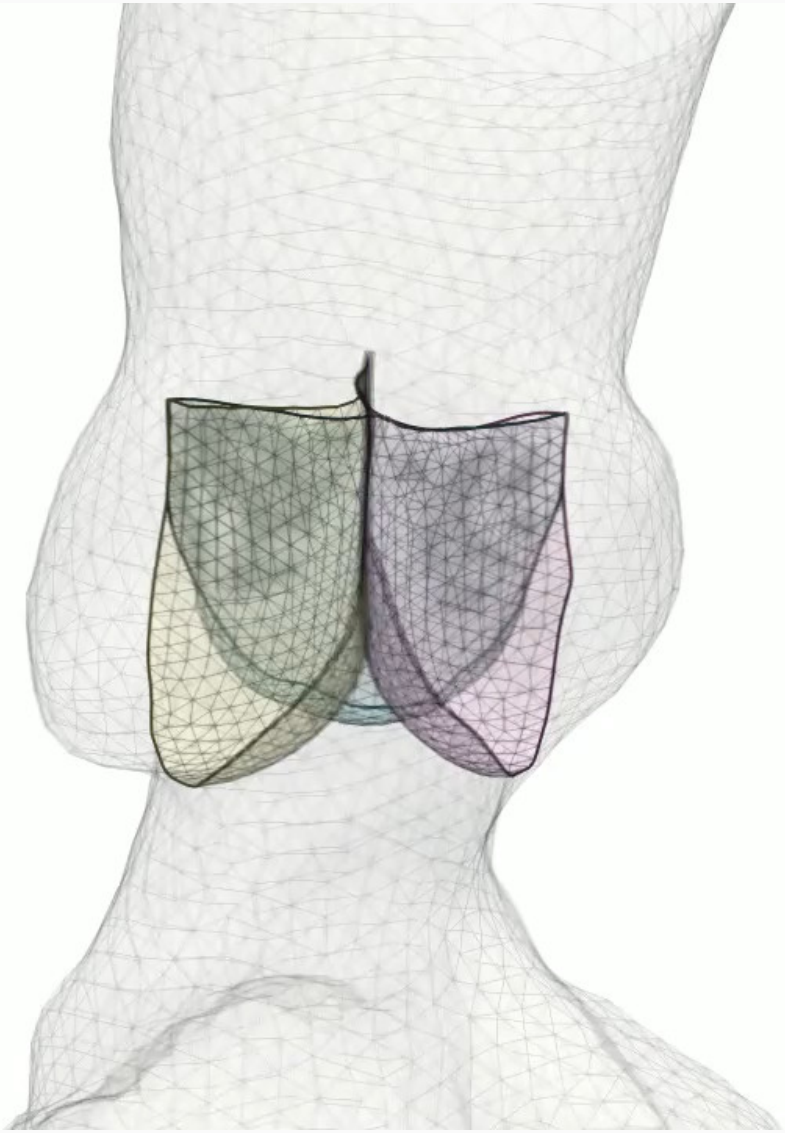
Unstructured tetrahedral meshes

CGAL Mesh (`www.cgal.org`) – Delaunay mesh generation
Ani3D (`sf.net/p/ani3d`) – mesh cosmetics

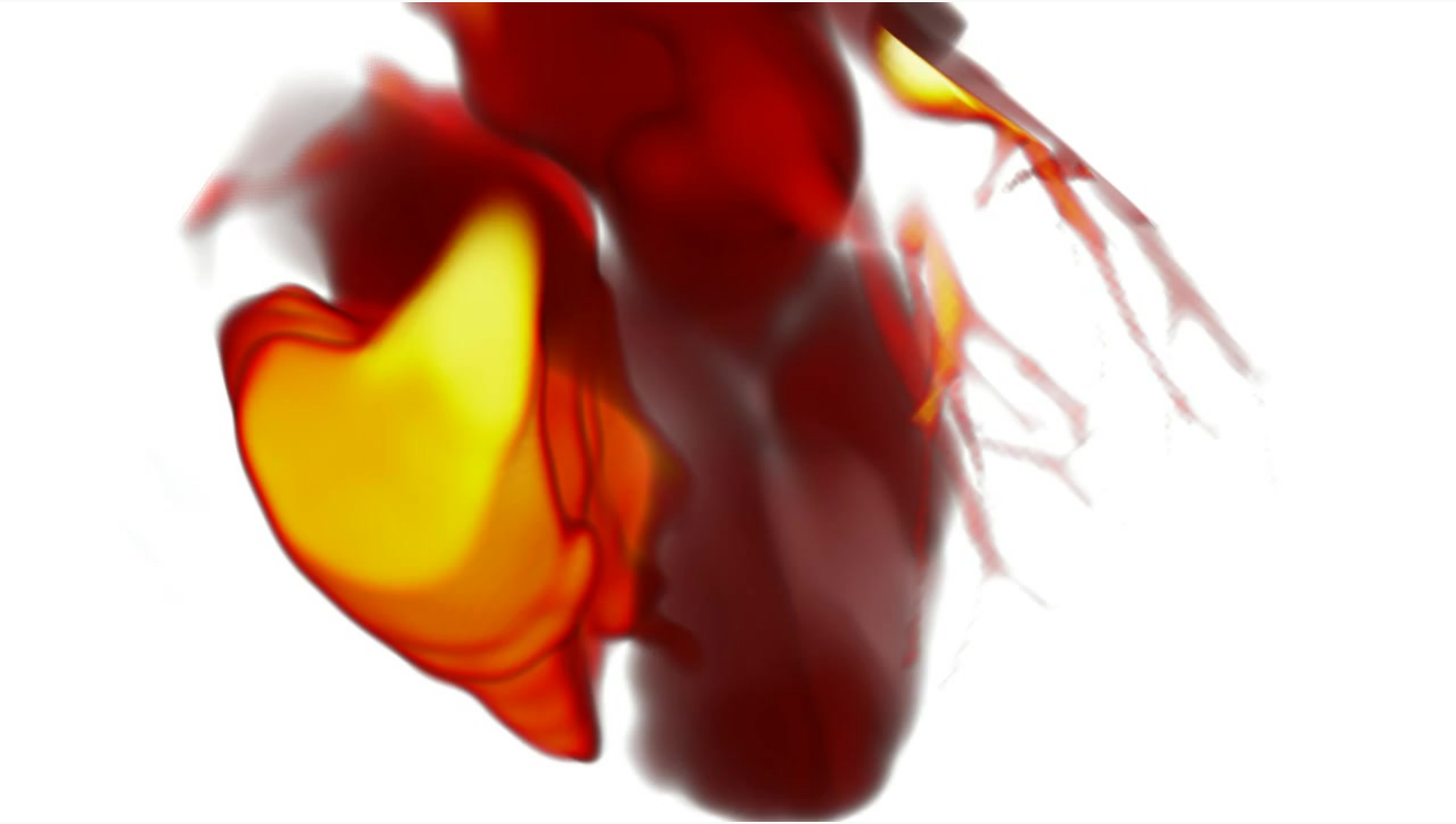


413 508 vertices, 2 315 329 tetrahedra, 84 430 boundary faces

Aortic valve replacement



Left ventricle dynamics



Interested in biomedical applications? See you on Thursday at 14:50

Thank you!

This presentation includes images from Alexey Chernyshenko, Kirill Nikitin, Hang Si, Stefano Paoletti, Steve Owen