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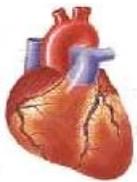
# 1D-0D modeling of coronary hemodynamics with two consecutive or parallel stenoses

**Timur Gamilov<sup>1, 2</sup>**  
**Kristina Kabanenko<sup>1</sup>**

<sup>1</sup> Moscow Institute of Physics and Technology  
<sup>2</sup> Institute of Numerical Mathematics of the Russian Academy of Sciences

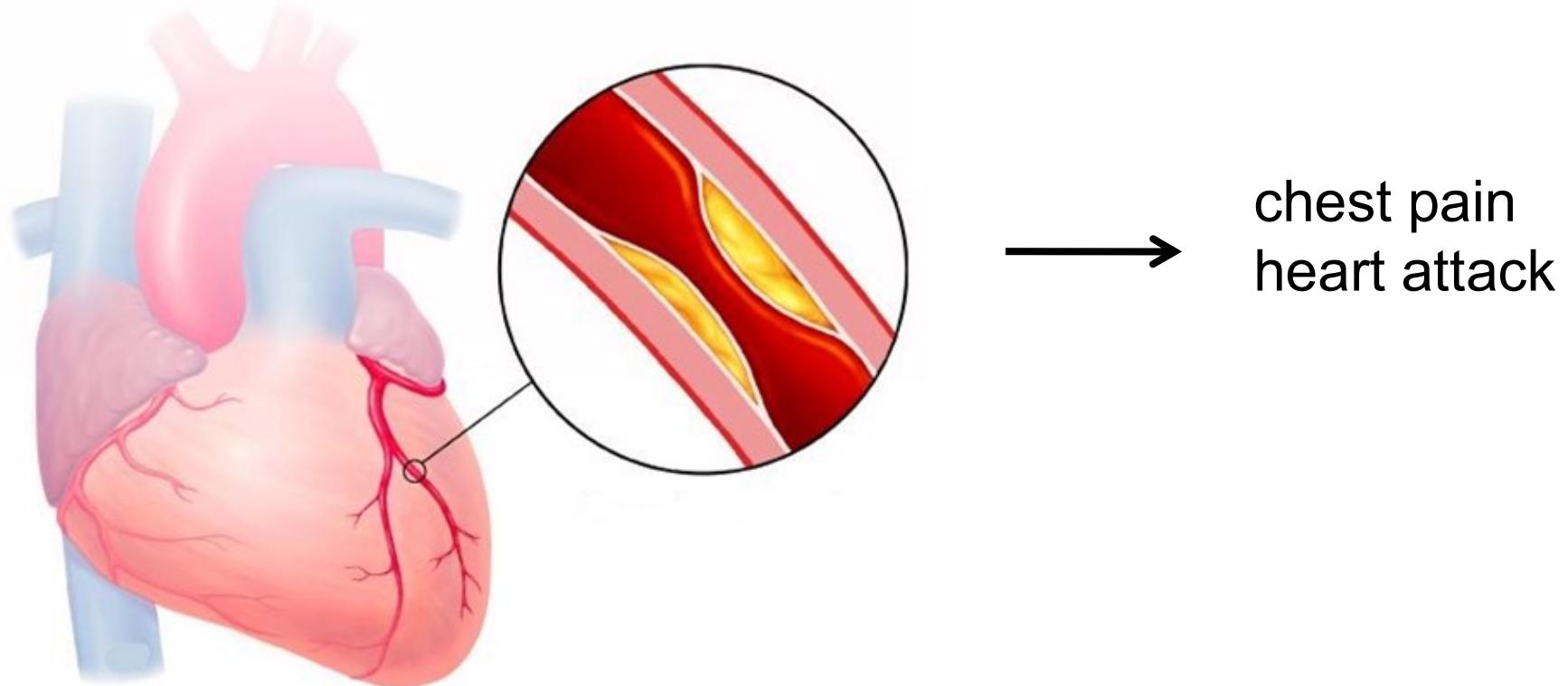
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Numerical Methods and Mathematical Modelling in Geophysical and Biomedical Sciences



# Coronary Artery Stenosis

Stenosis

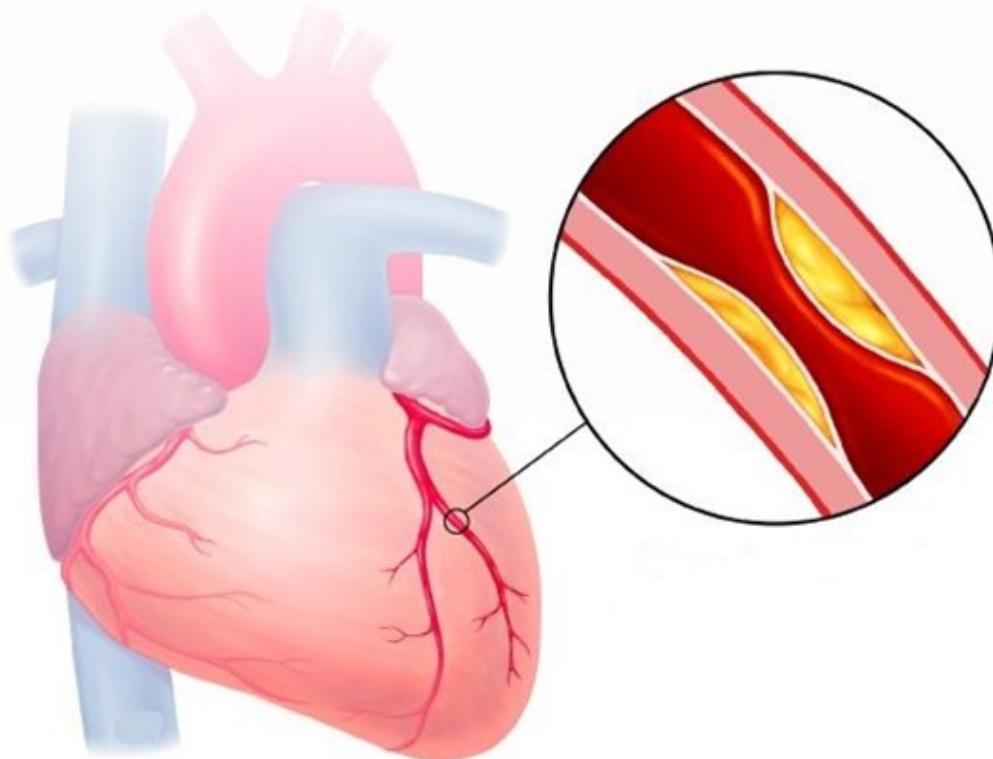


chest pain  
heart attack



# Coronary Artery Stenosis

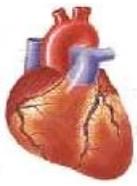
## Stenosis



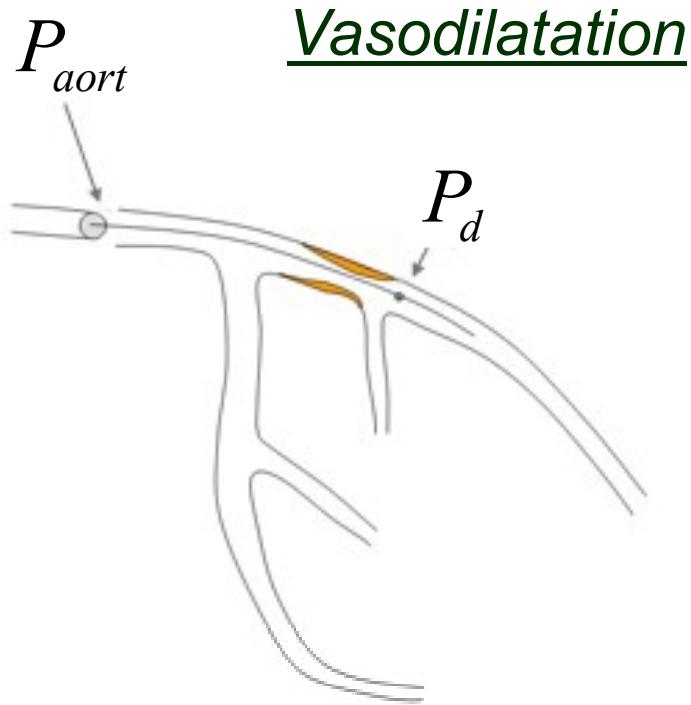
## Treatment:

- non-surgical
- surgical





# Fractional flow reserve (FFR)

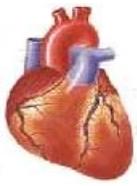


$$FFR = \frac{\text{average pressure distal to the lesion } (P_d)}{\text{average aortic pressure } (P_{aort})}$$

$FFR = 1 \longrightarrow$  healthy subject

$FFR \leq 0.75 \longrightarrow$  surgical treatment

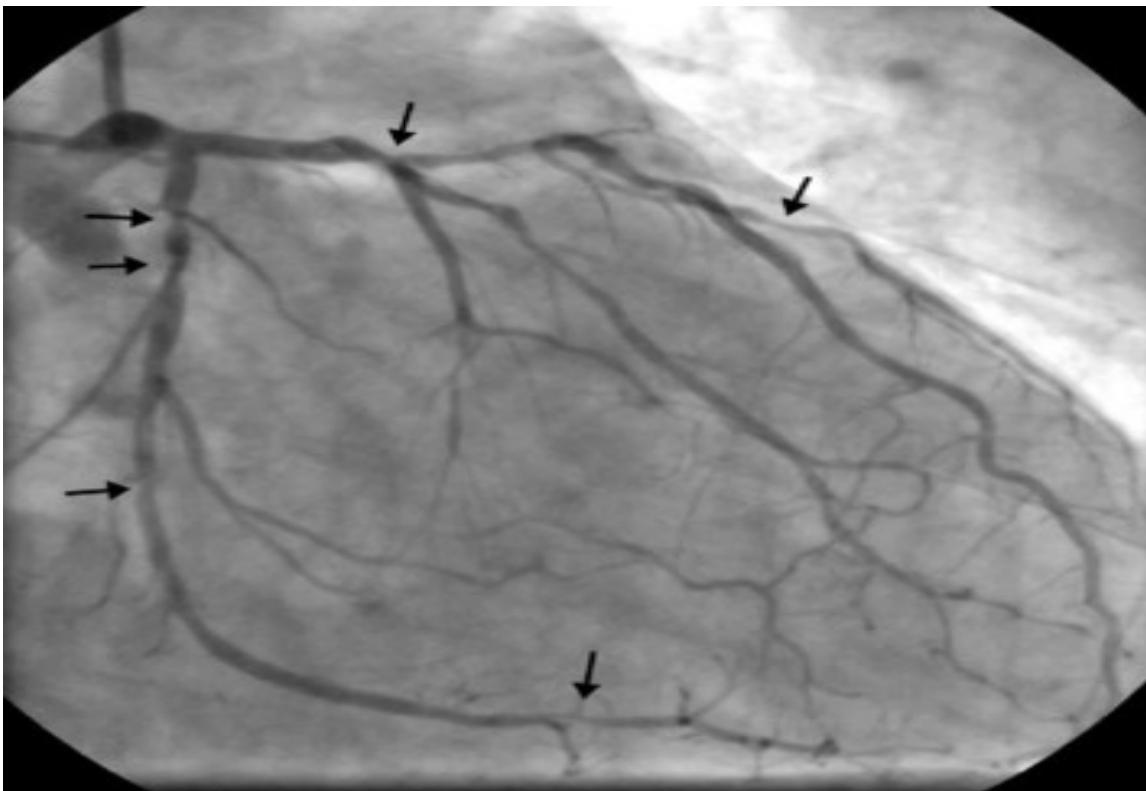


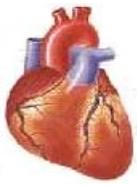


# FFR problems



- materials costs:  
~ 5000 - 6000 \$ (full treatment), ~ 1000 \$ (probe)
- multiple stenoses

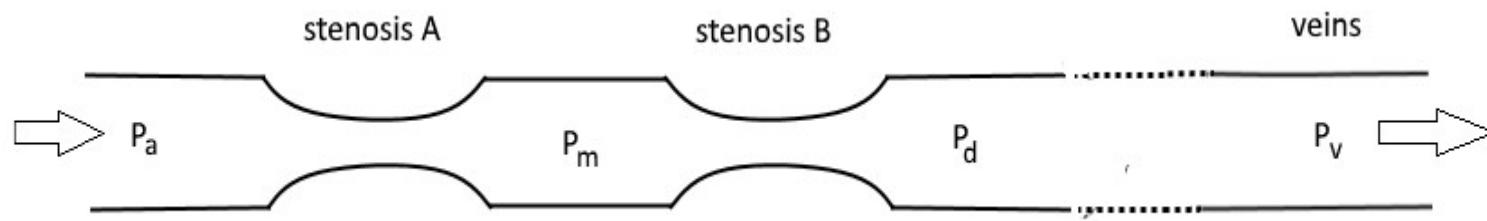




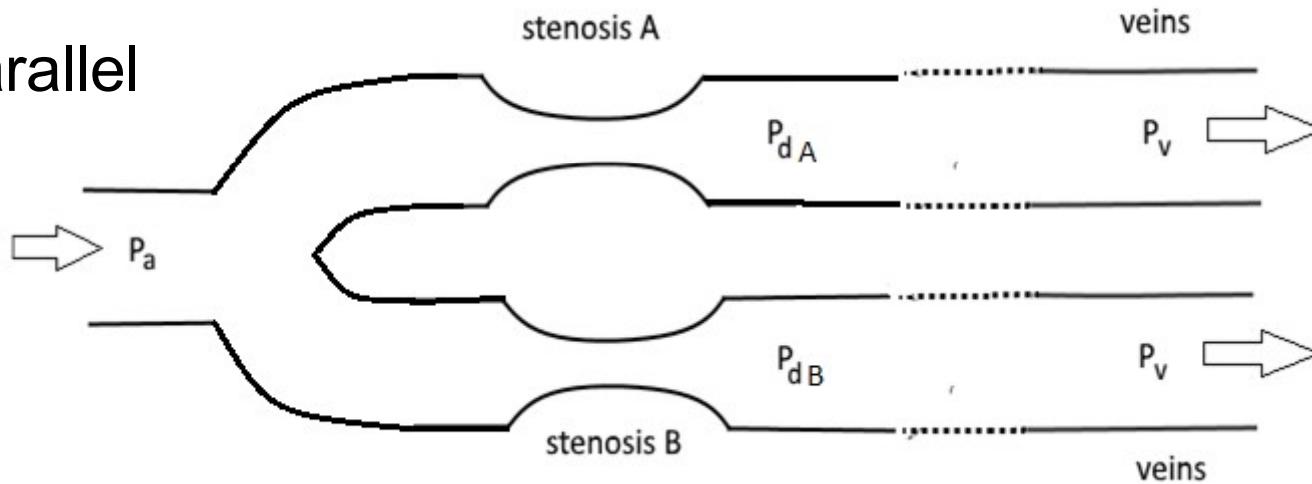
# Consecutive and parallel stenoses

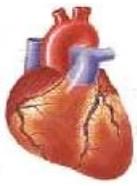


## Consecutive



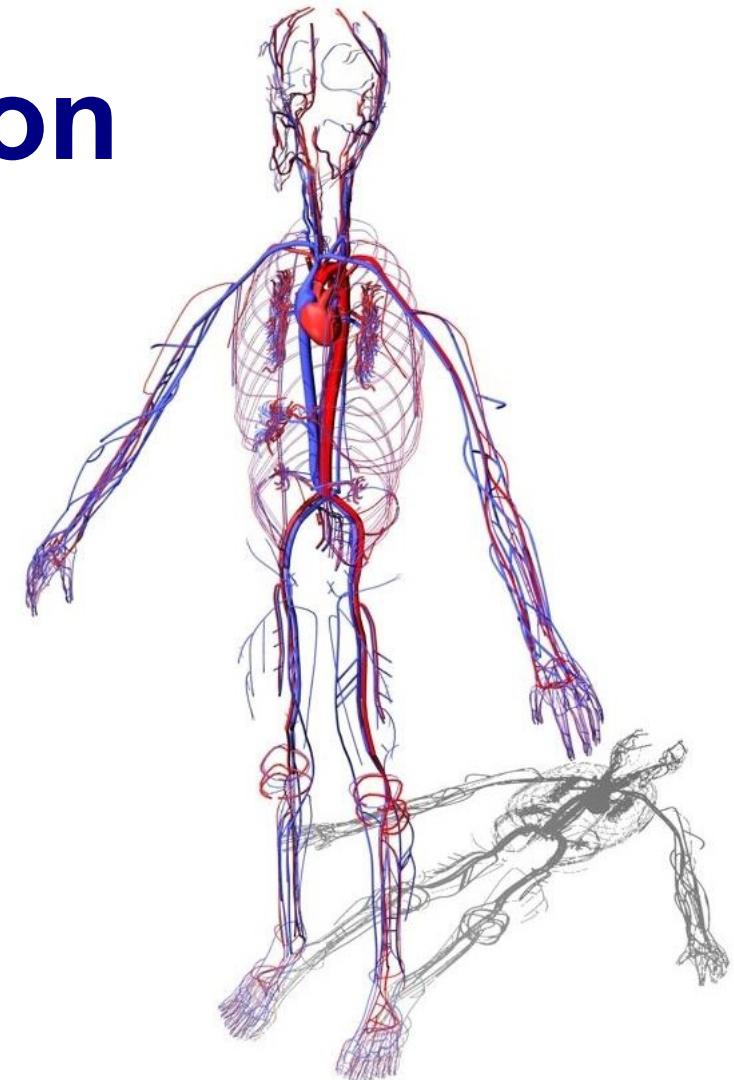
## Parallel



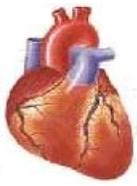


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# Blood flow circulation model



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# Blood flow circulation model



1) Mass balance  $\frac{\partial S}{\partial t} + \frac{\partial(uS)}{\partial x} = 0$

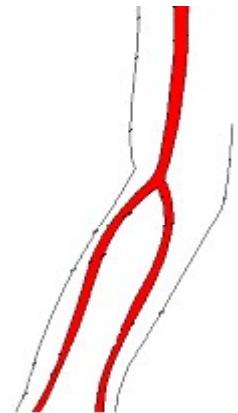
2) Momentum balance  $\frac{\partial u}{\partial t} + \frac{\partial}{\partial x} \left( \frac{u^2}{2} + \frac{P}{\rho} \right) = -16\mu u \frac{\eta(S)}{Sd^2} + \psi(\dots), \eta(S) = \begin{cases} 2, & S > S_0 \\ \frac{S}{S_0} + \frac{S_0}{S}, & S \leq S_0 \end{cases}$

## 3) Bifurcations

3.1  $\sum_{k=k_1, \dots, k_M} \varepsilon_k^m Q_k = 0, \varepsilon_k^m = \pm 1, Q_k = u_k S_k$

3.2  $p_k + \frac{\rho u_k^2}{2} = p_j + \frac{\rho u_j^2}{2}, \quad \forall j, k$

## 3.3 Compatibility conditions



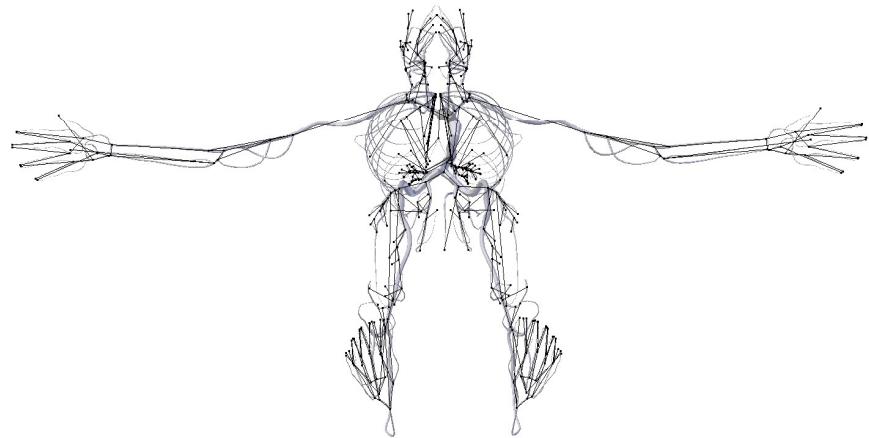
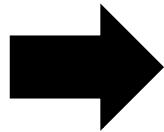


# Vessels structure

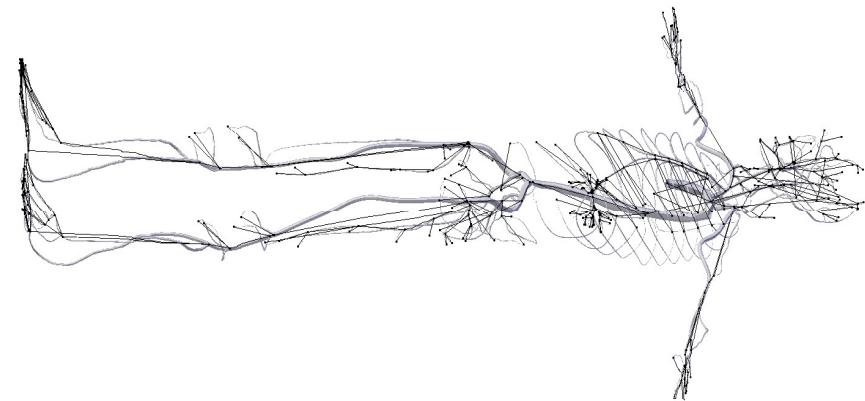


- General physiologically correct anatomical model

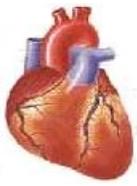
Sources: books, post mortem data,  
e.g. Visible Human Project  
[www.nlm.nih.gov/research/visible](http://www.nlm.nih.gov/research/visible)



- Patient-specific (individual) anatomical model



MRI/CT, Dopplerography



# Wall state equation



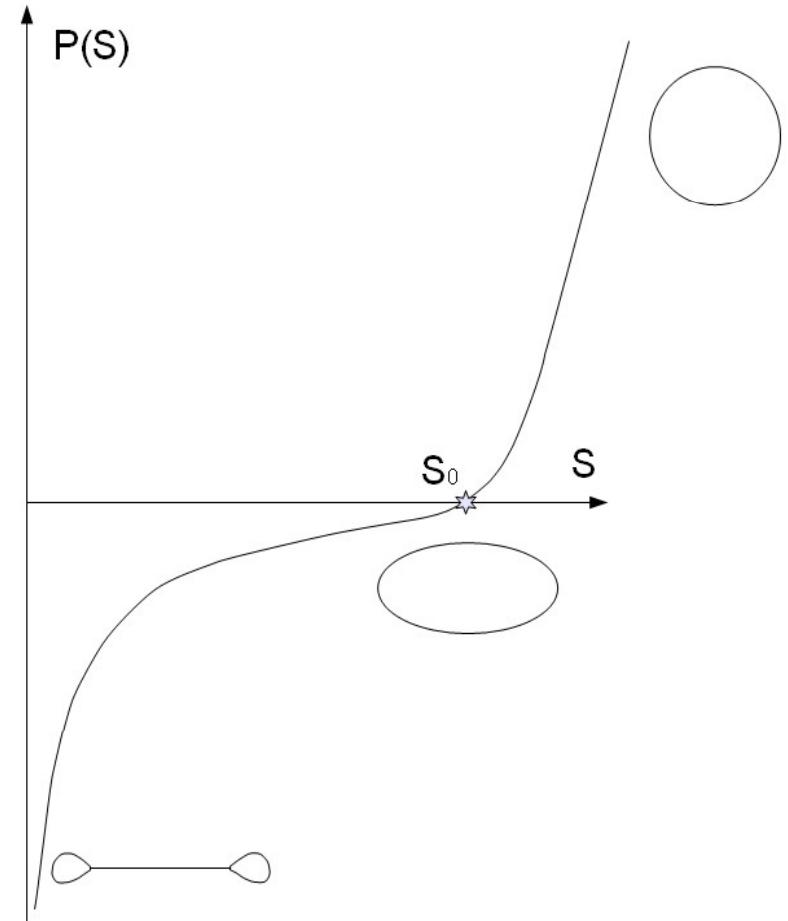
## 4) Vessel wall elasticity

Analytic approximation

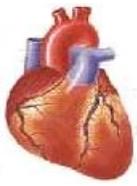
$$P(S) = P^{ext}(t, x) + \rho c^2 f(S)$$

$$f(S) = \begin{cases} \exp(S/S_0 - 1) - 1, & S > S_0 \\ \ln(S/S_0), & S \leq S_0 \end{cases}$$

$P^{ext}(t, x)$  – external pressure



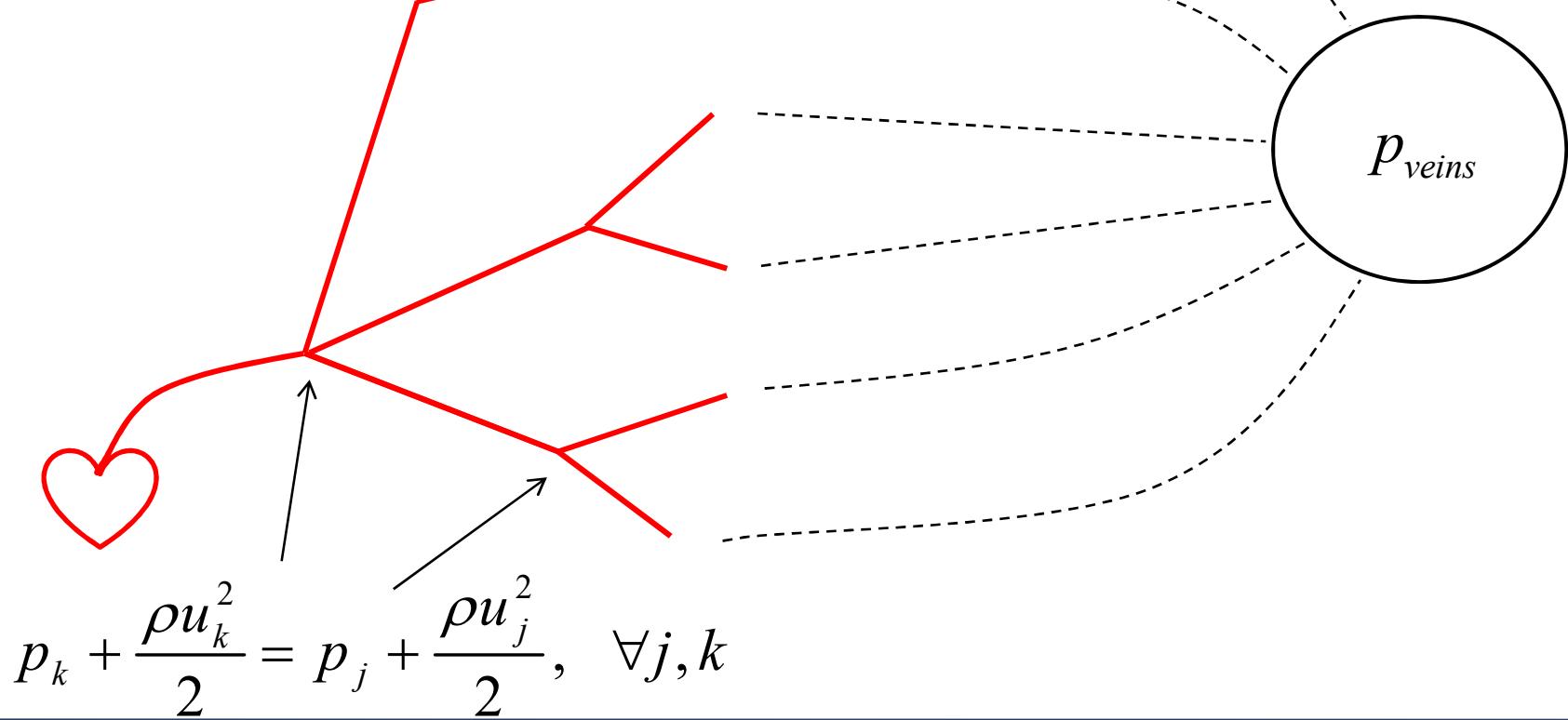
Pedley, Luo, 1998



# Bifurcations. Resistance

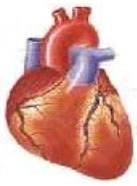


arteries

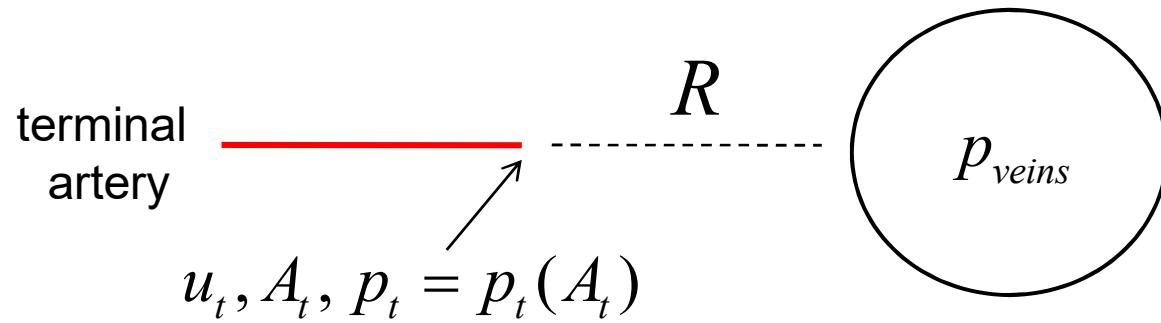


$$p_k - p_{veins} = R_k Q_k$$

$$R_k = R_k(t)$$

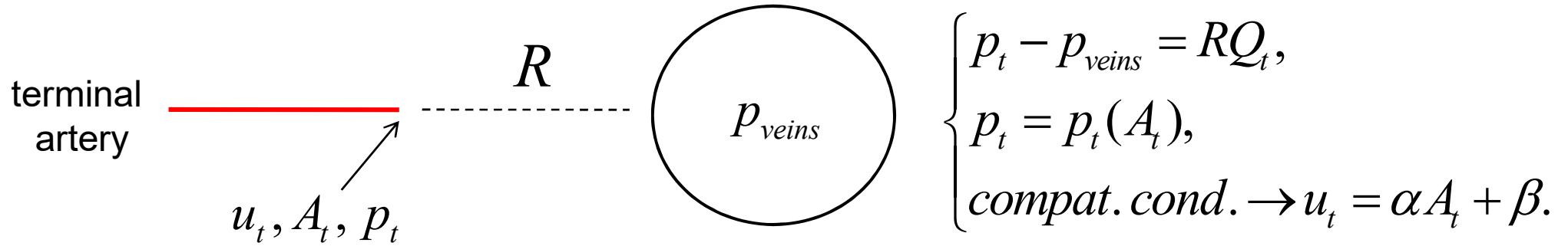


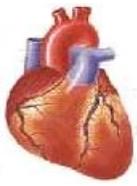
# Bifurcations. Terminal vessel



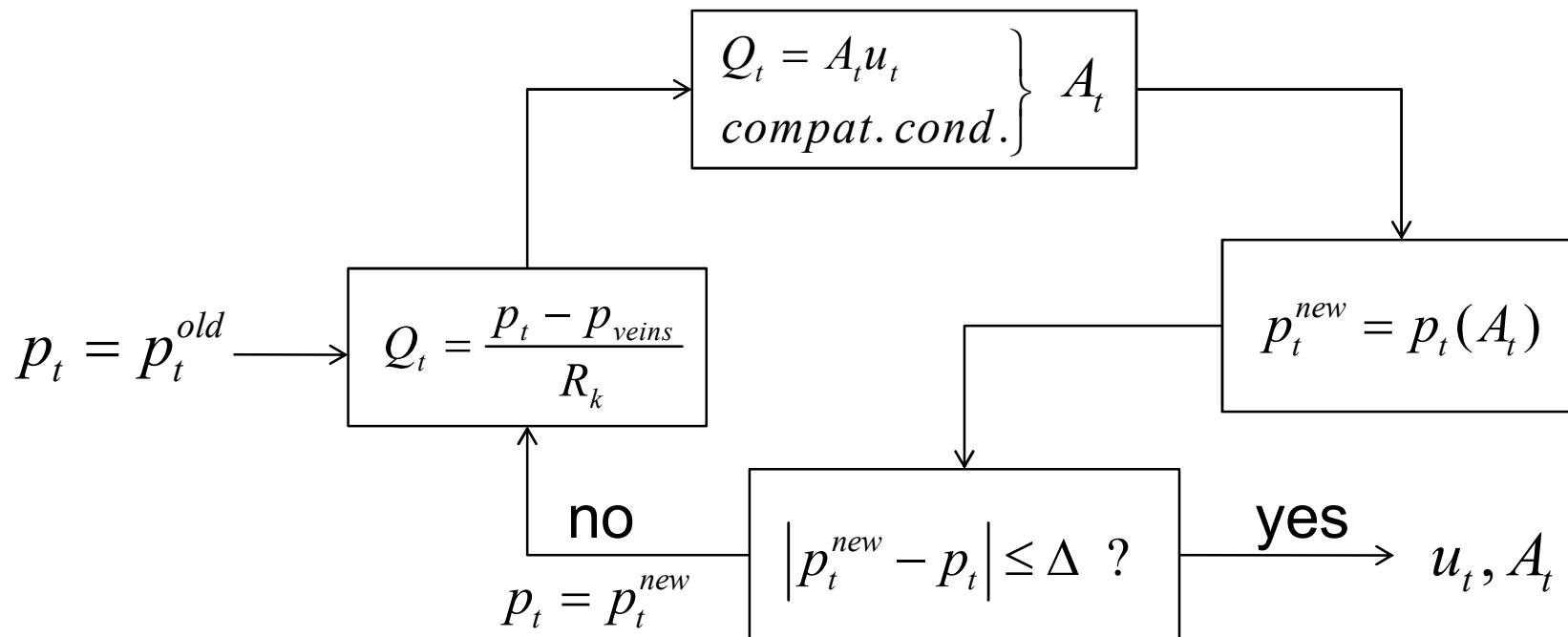
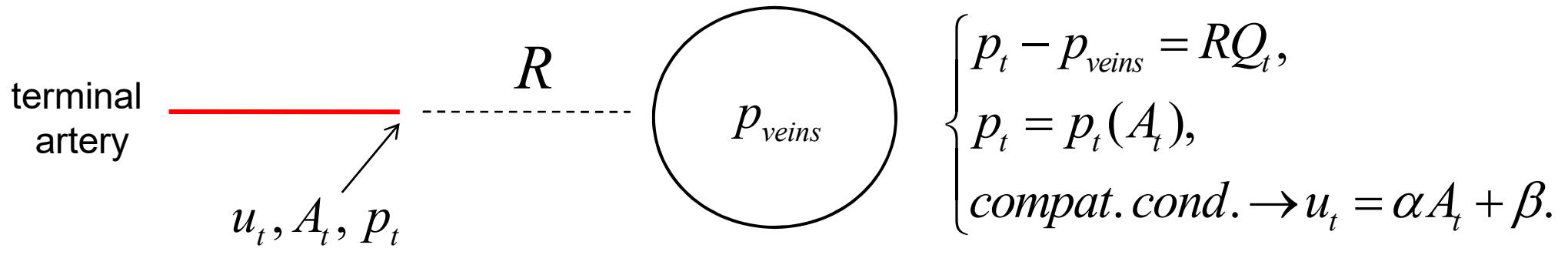


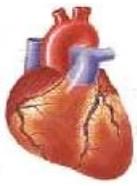
# Bifurcations. Terminal vessel





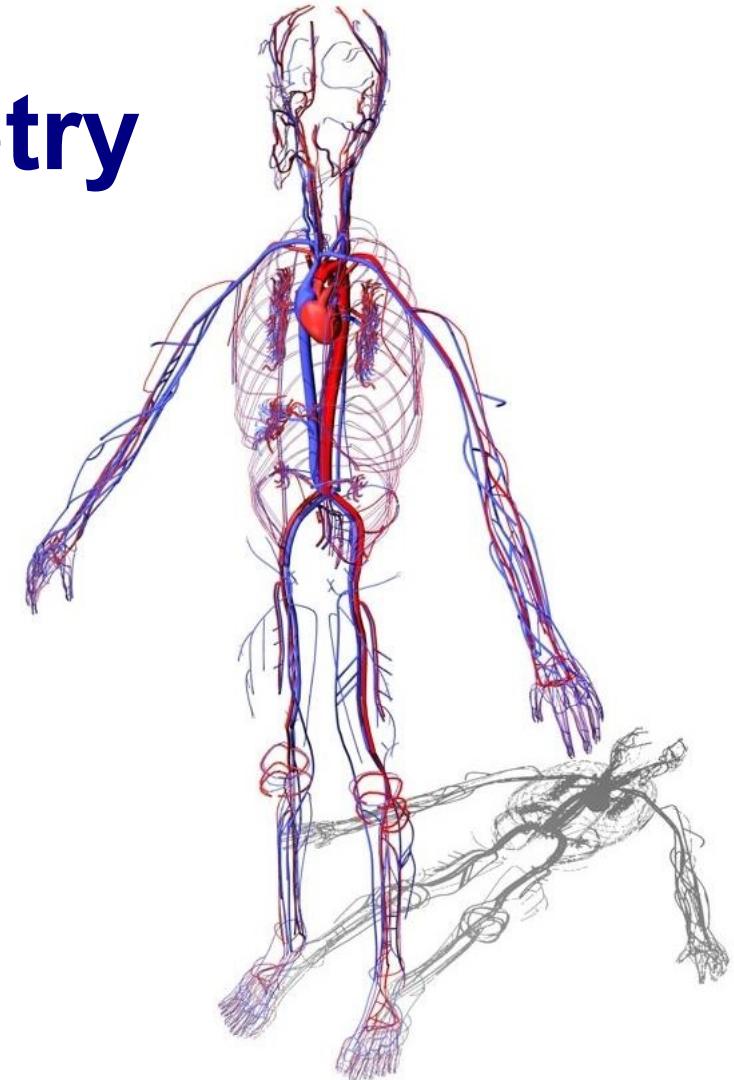
# Bifurcations. Terminal vessel

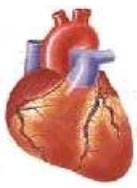




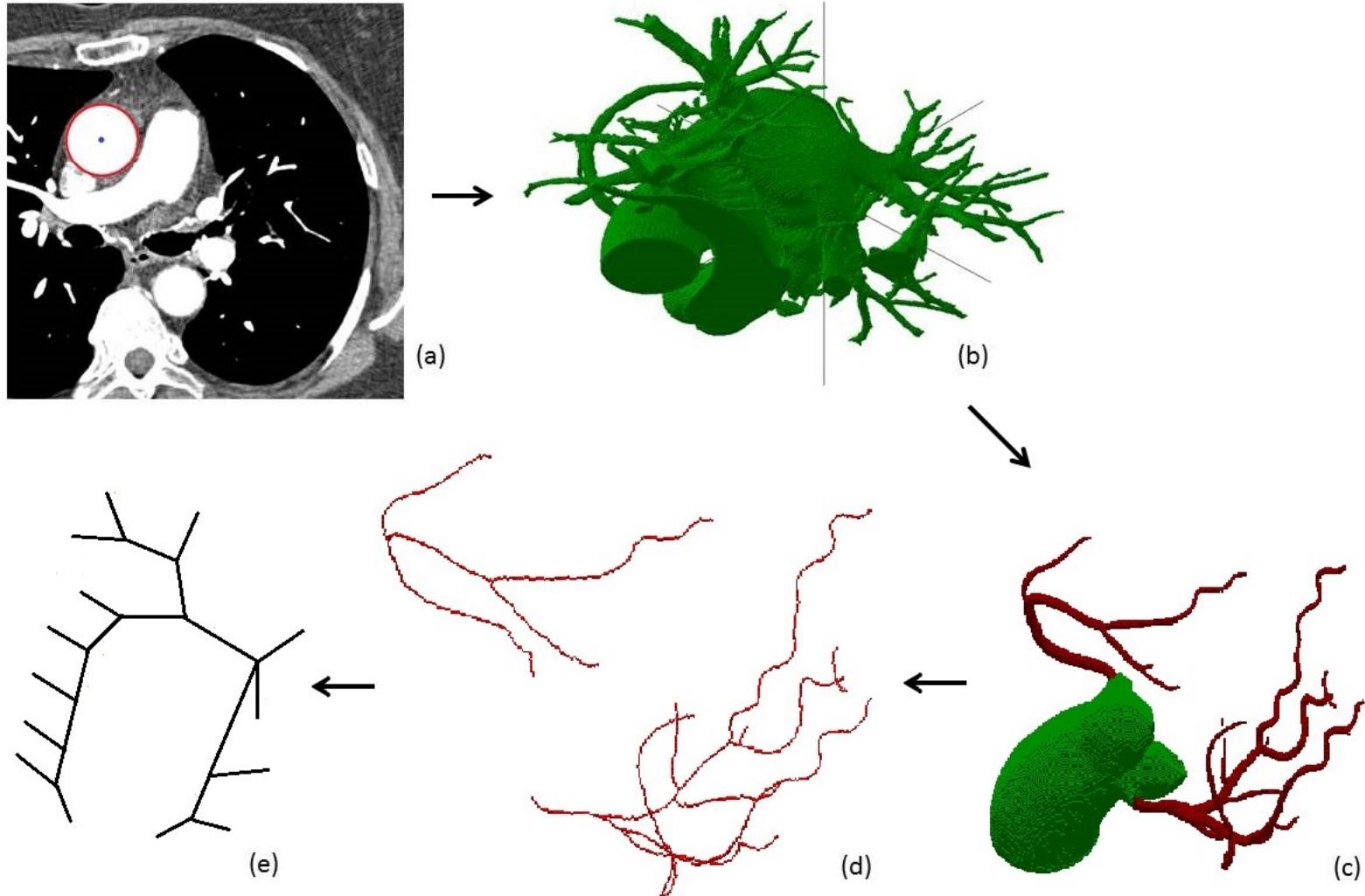
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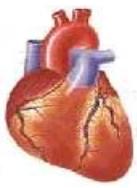
# Patient-specific geometry Results



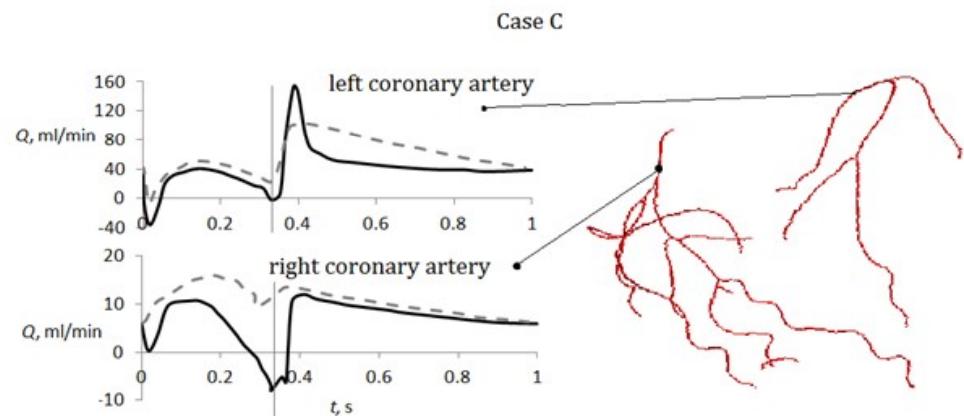
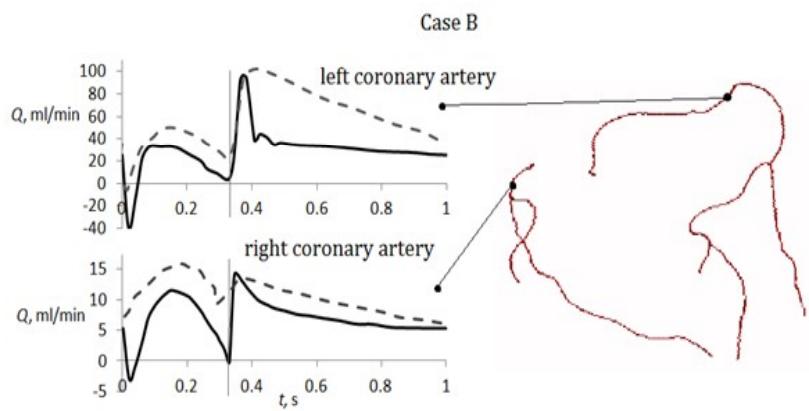
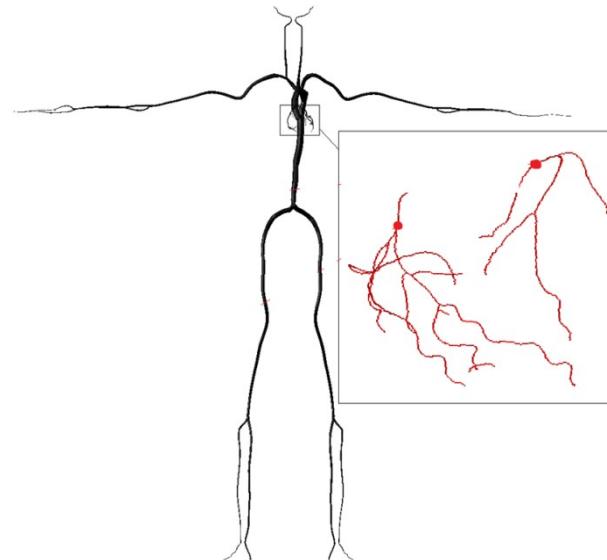
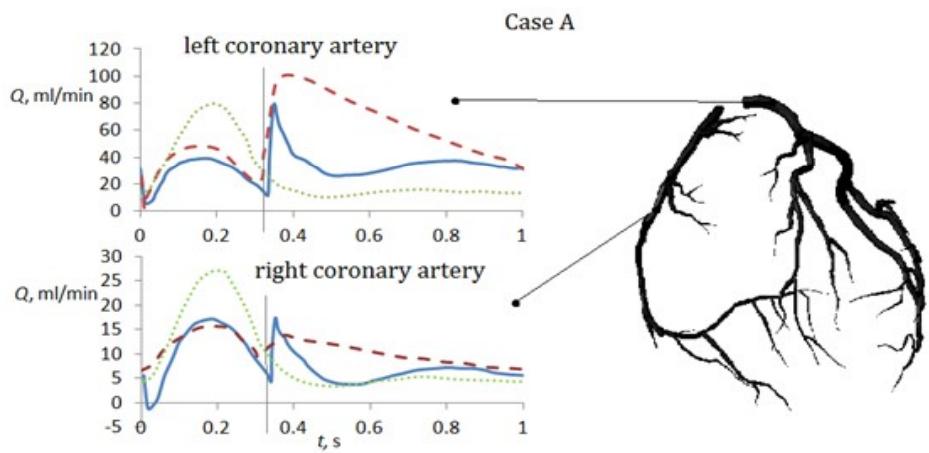


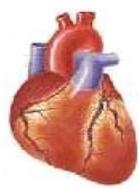
# Patient geometrics





# Results. Blood flow

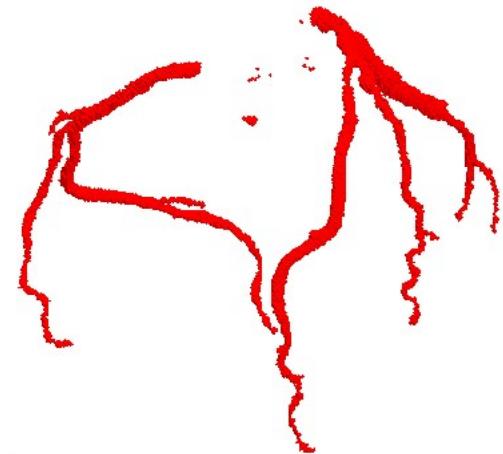




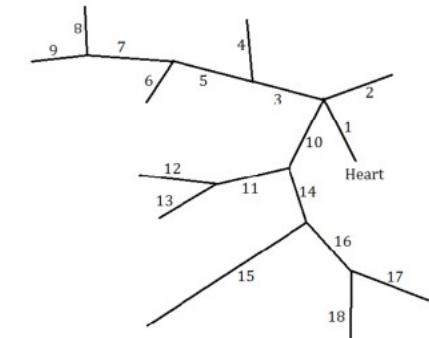
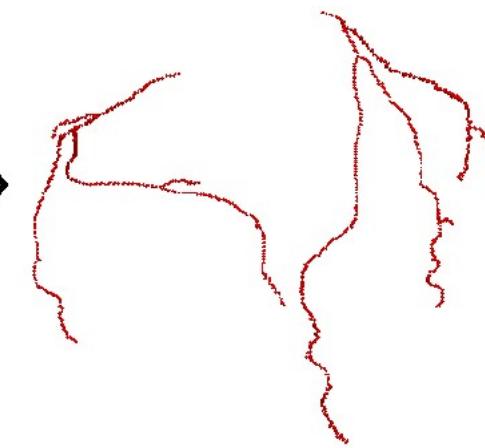
# FFR. Geometry



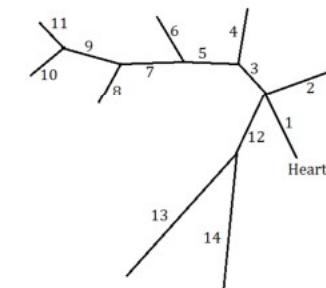
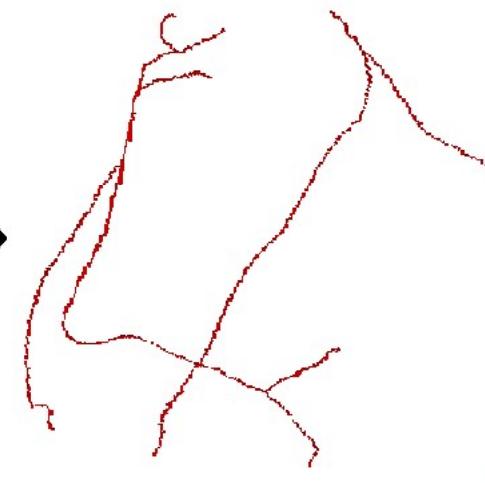
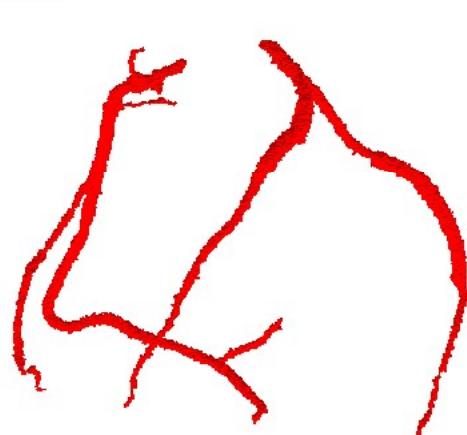
Patient 1



Center-lines

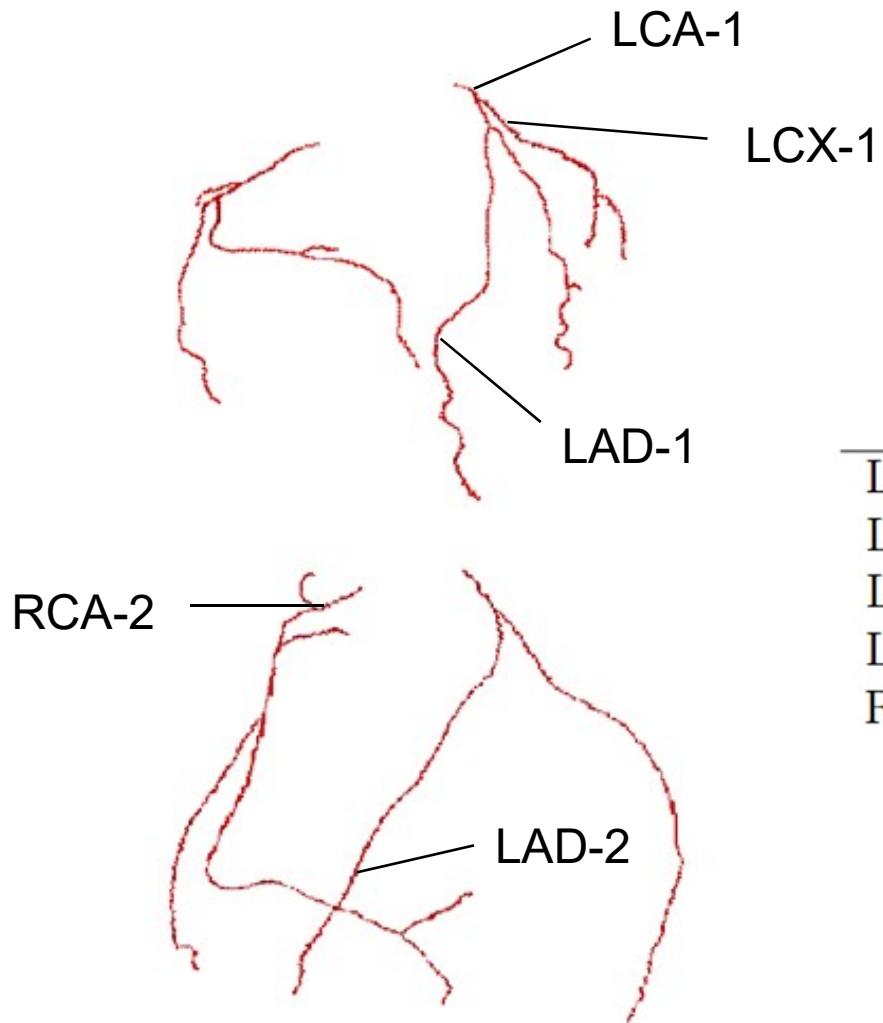


Patient 2

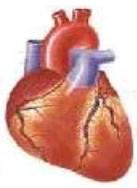




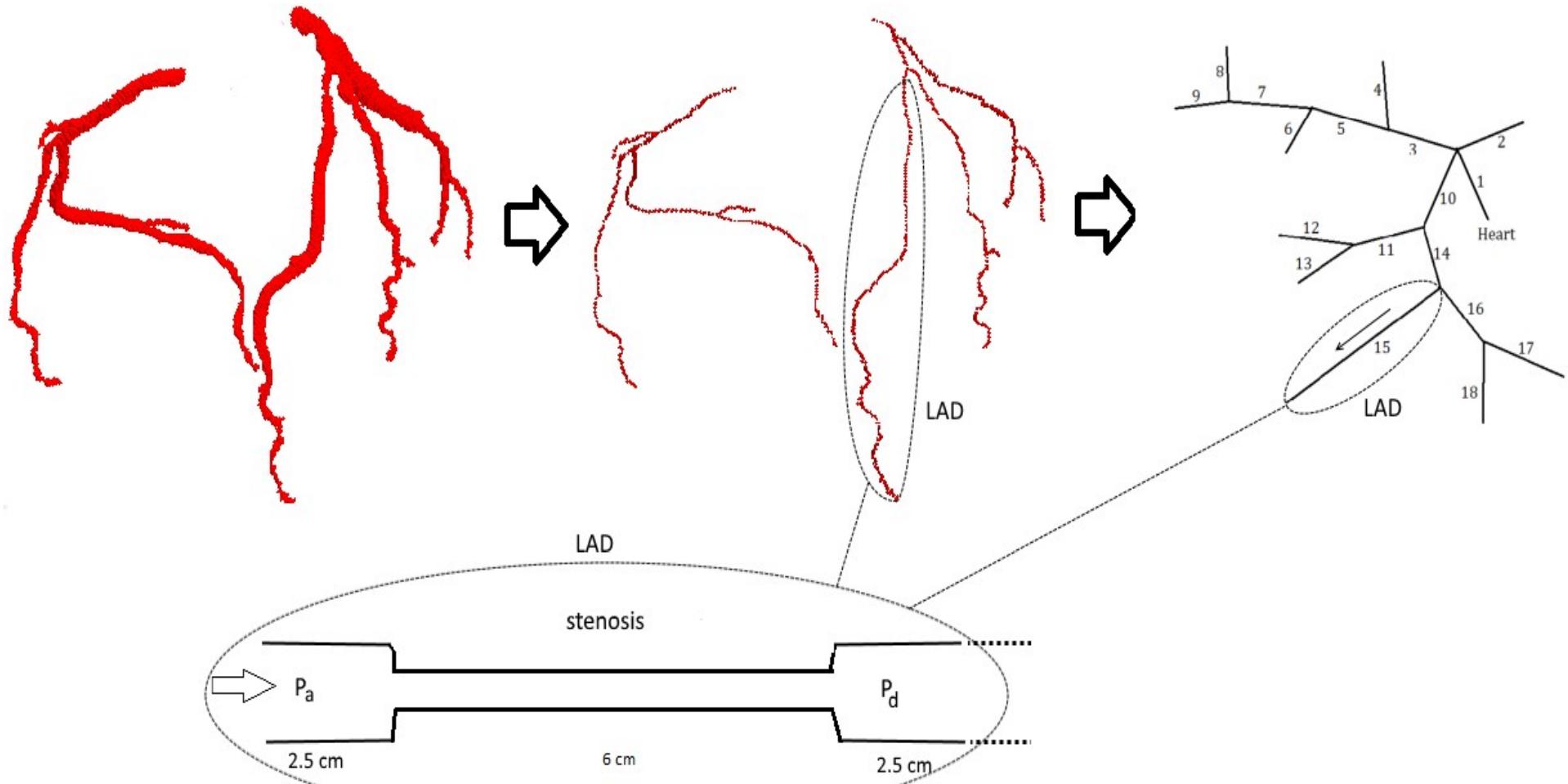
# FFR. Results

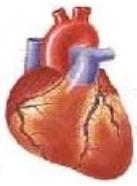


	Measured FFR	Calculated FFR	
LAD-1	0.51	0.58	+14%
LCA-1	0.72	0.84	+17%
LCX-1	0.59	0.61	+3%
LAD-2	0.74	0.78	+5%
RCA-2	0.93	0.87	-5%



# FFR. Isolated vessel



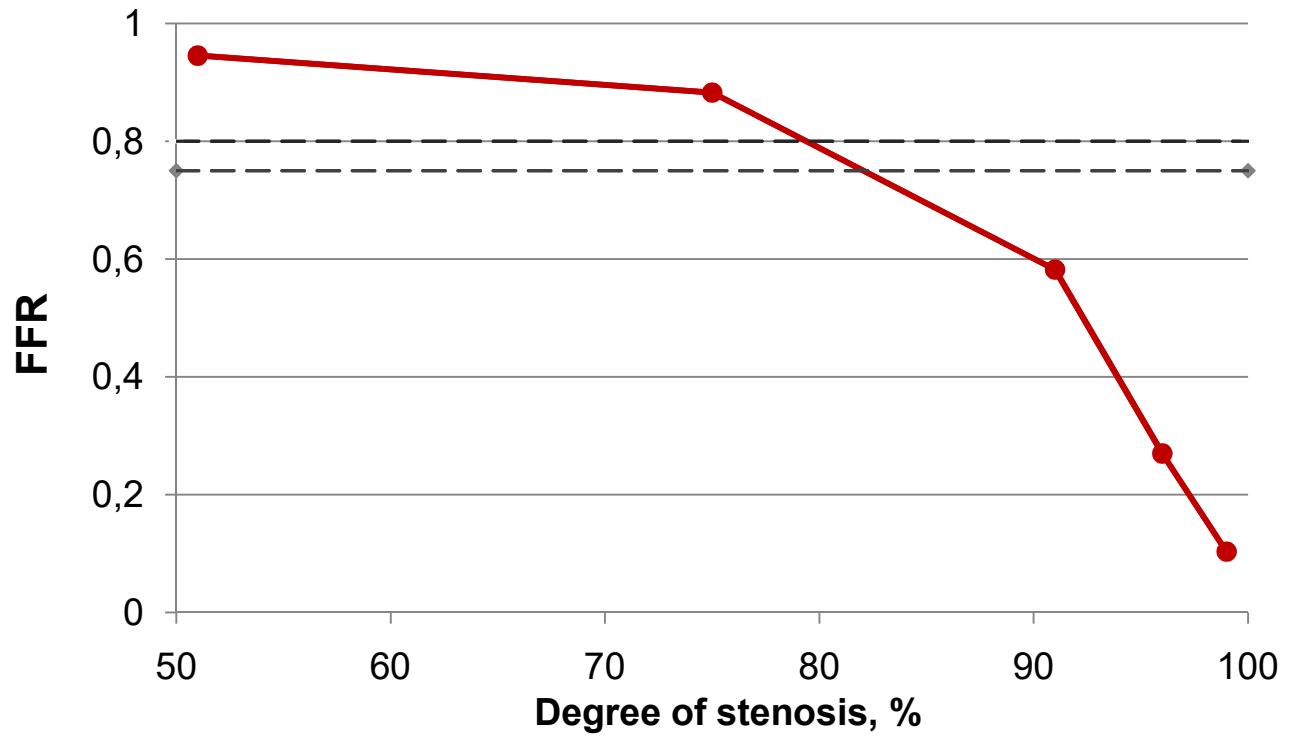


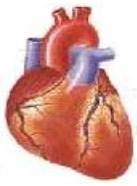
# FFR. Isolated vessel



LAD

$d = 3\text{mm}$





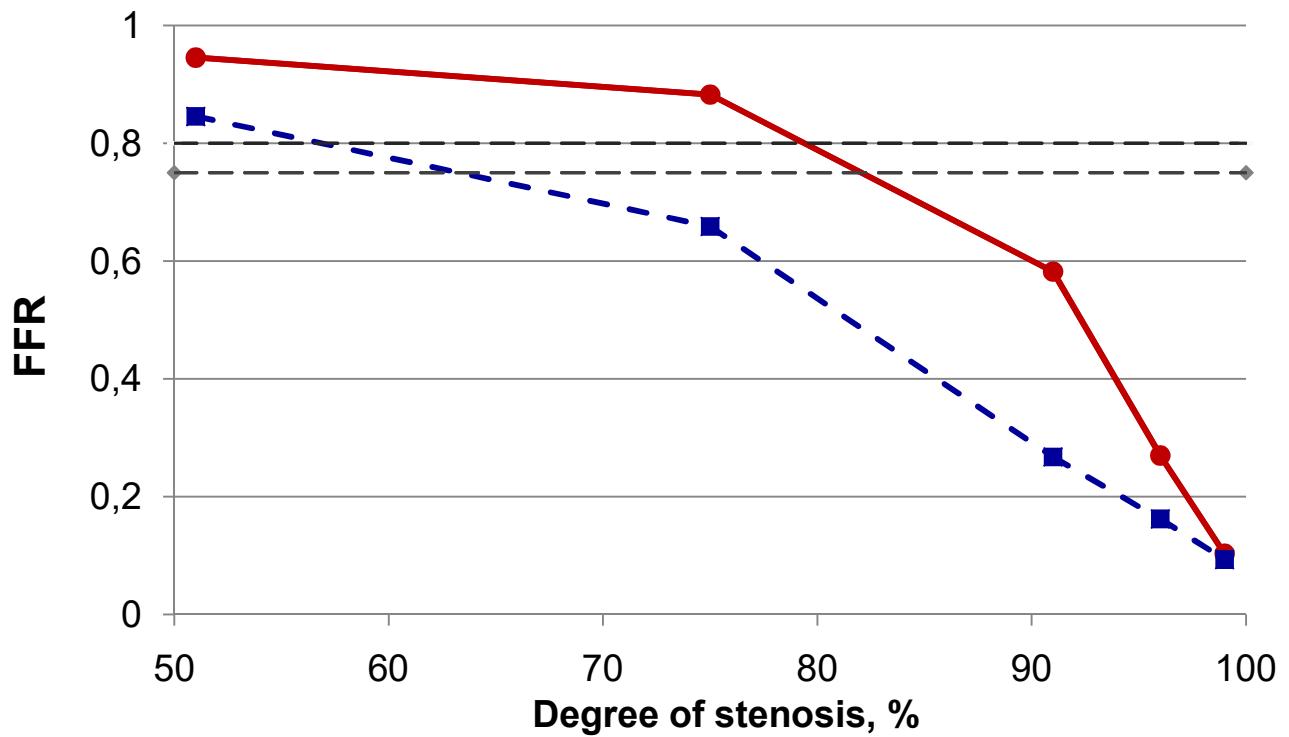
# FFR. Isolated vessel



LAD

$d = 3\text{mm}$

$d = 2\text{mm}$





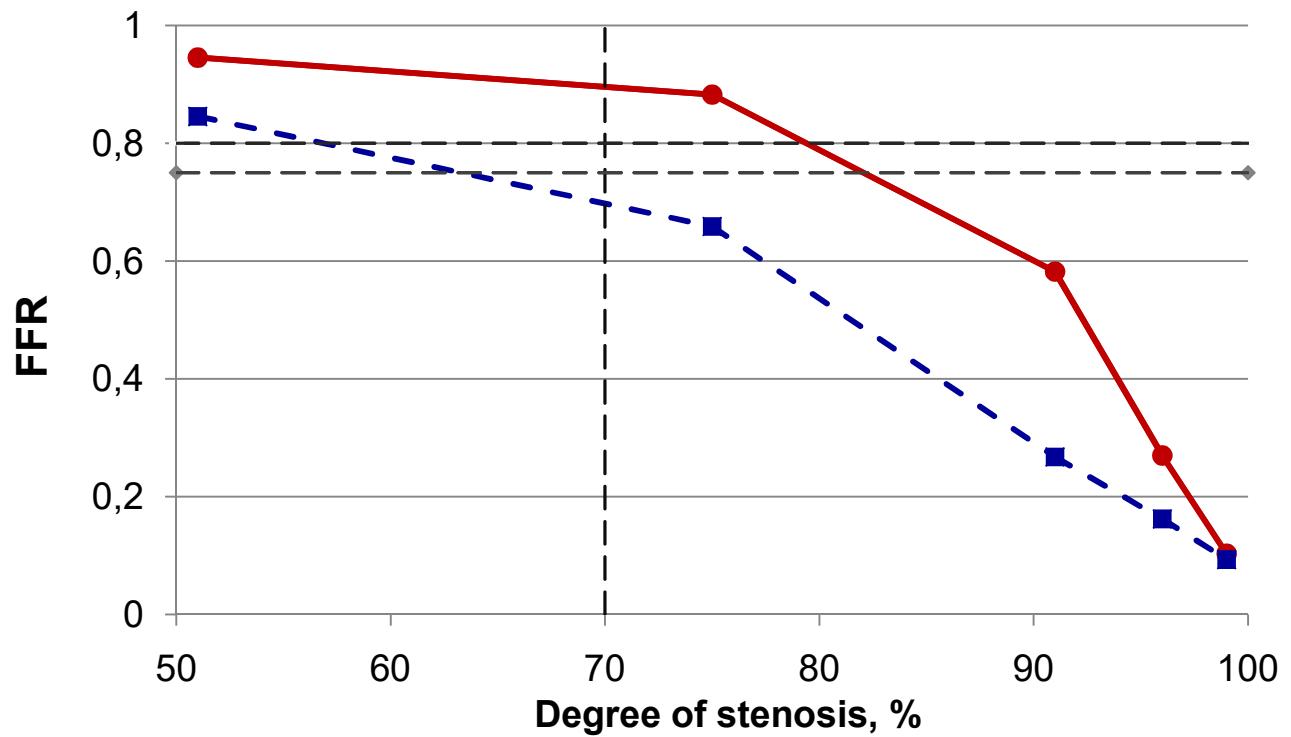
# FFR. Isolated vessel



LAD

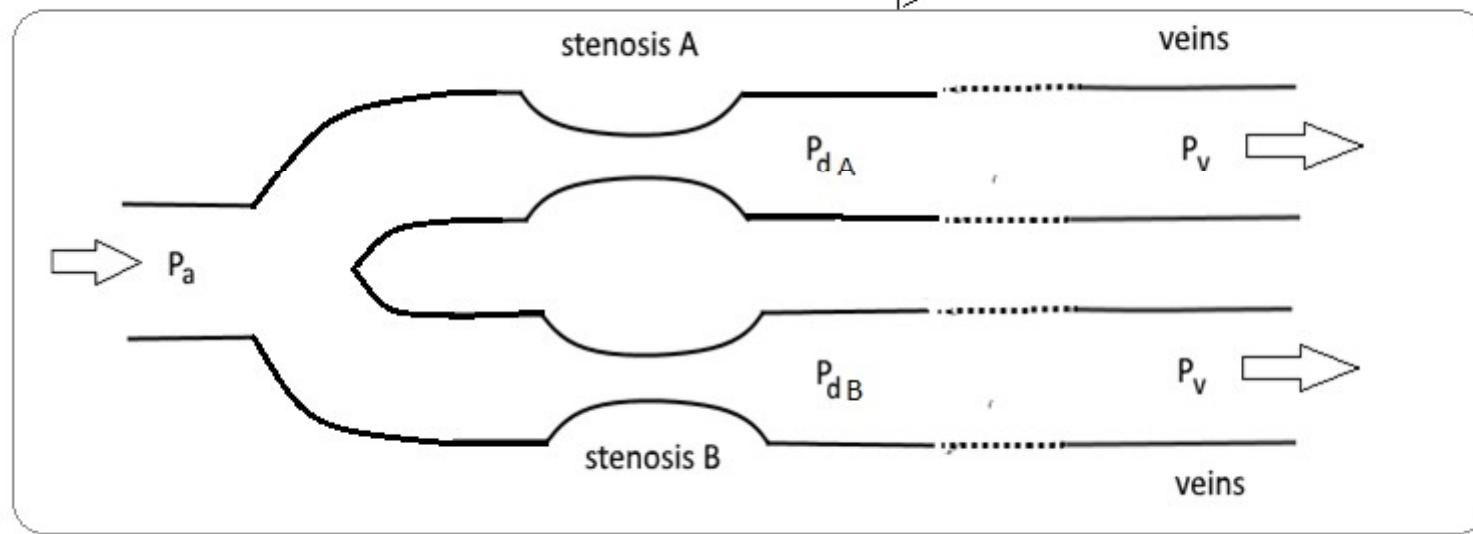
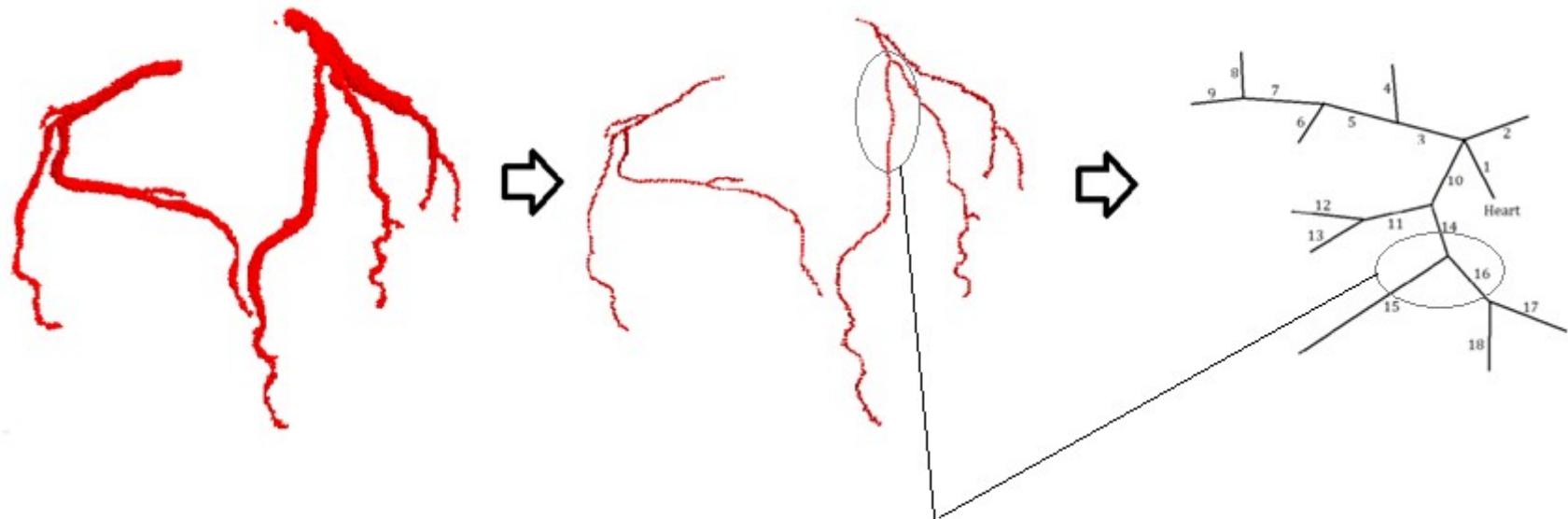
$d = 3\text{mm}$

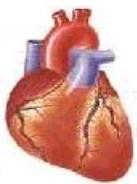
$d = 2\text{mm}$



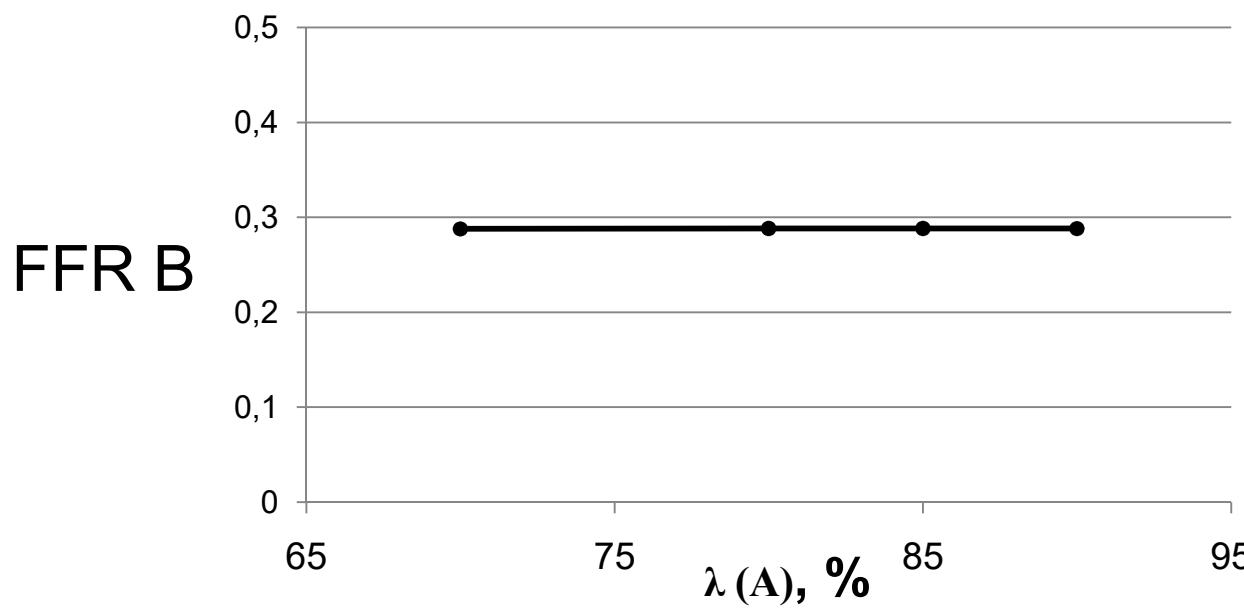
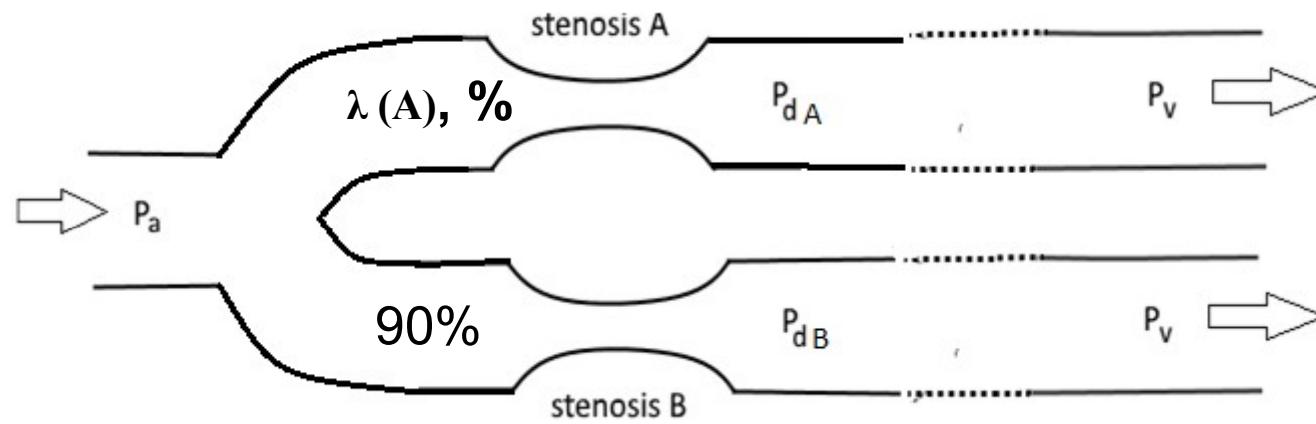


# FFR. Two parallel stenoses



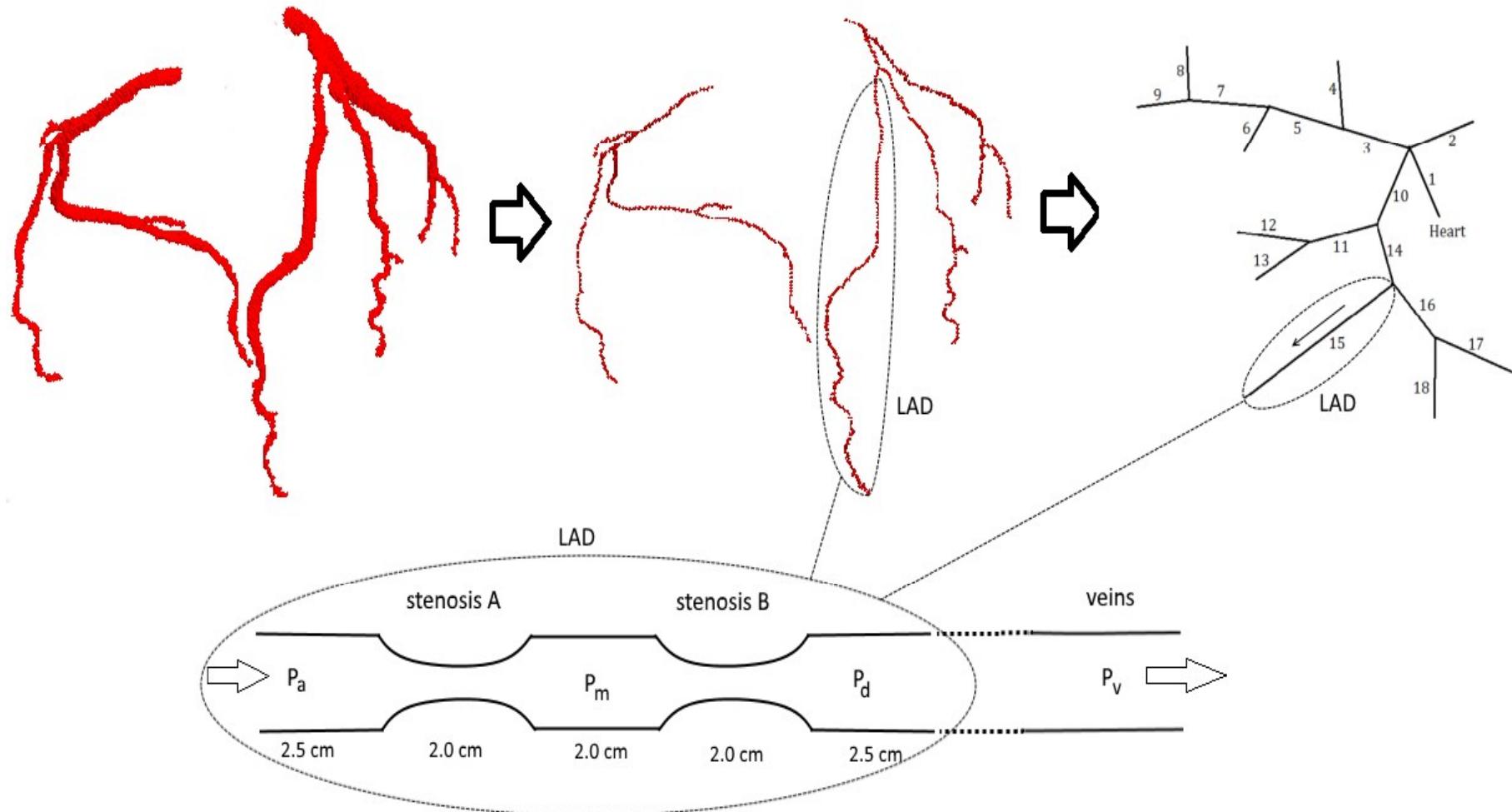


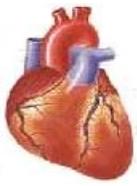
# FFR. Two parallel stenoses



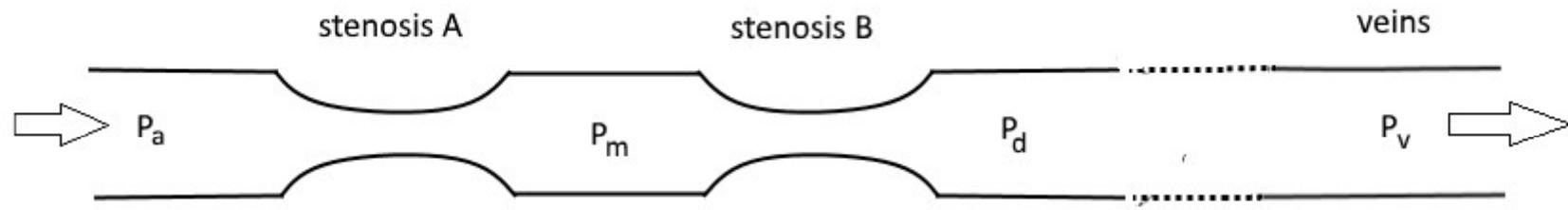


# FFR. Two sequential stenoses

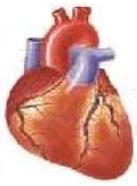




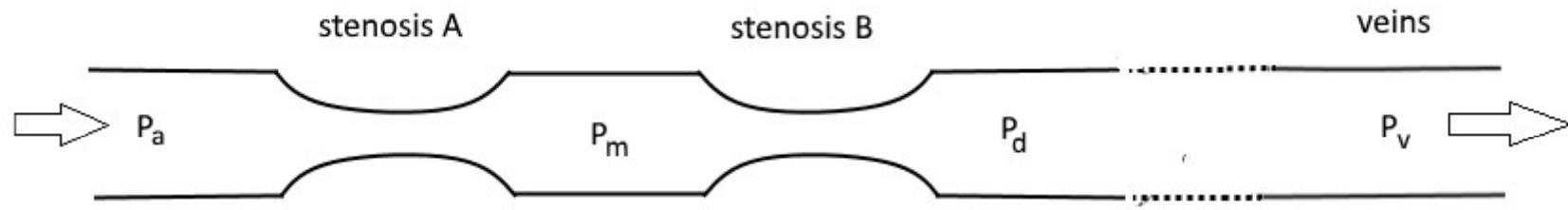
# FFR. Two sequential stenoses



$$1) FFR = \frac{\text{pressure distal to the lesion } (P_{m,d})}{\text{aortic pressure } (P_{aort})}$$

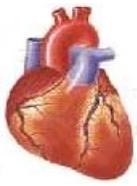


# FFR. Two sequential stenoses

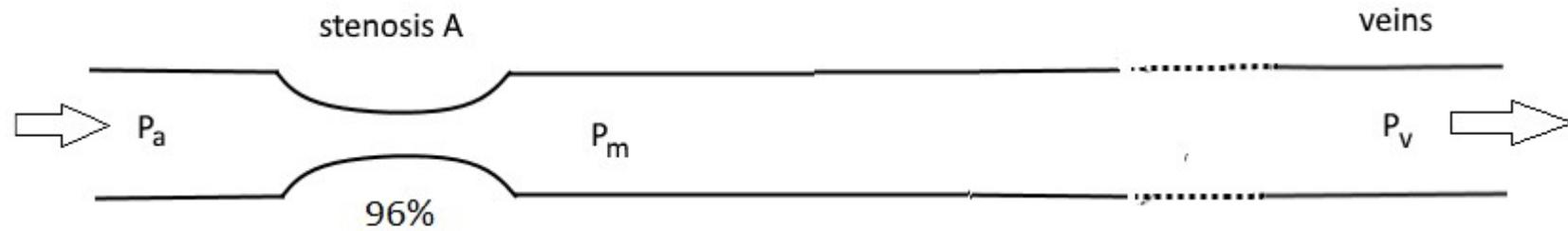


$$1) FFR = \frac{\text{pressure distal to the lesion } (P_{m,d})}{\text{aortic pressure } (P_{aort})}$$

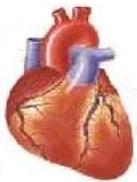
$$2) FFR(A) = \frac{P_d - P_v (P_m / P_a)}{P_a - P_m + P_d - P_v} \quad FFR(B) = 1 - \frac{(P_a - P_v)(P_m - P_d)}{P_a(P_m - P_v)}$$



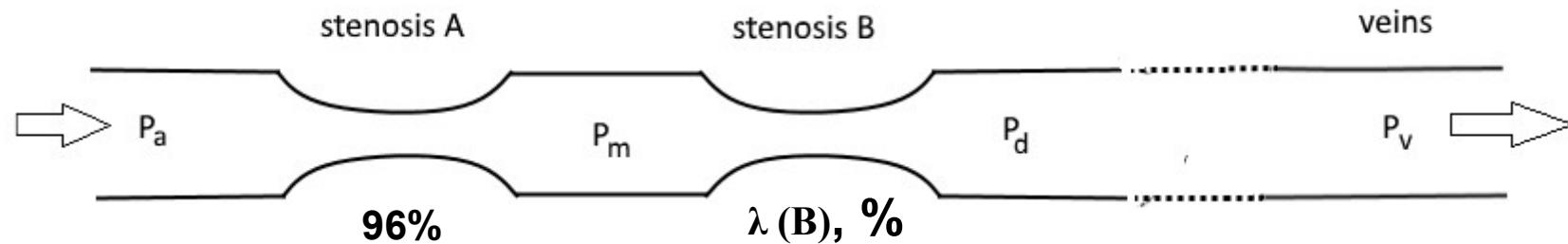
# FFR. Two sequential stenoses



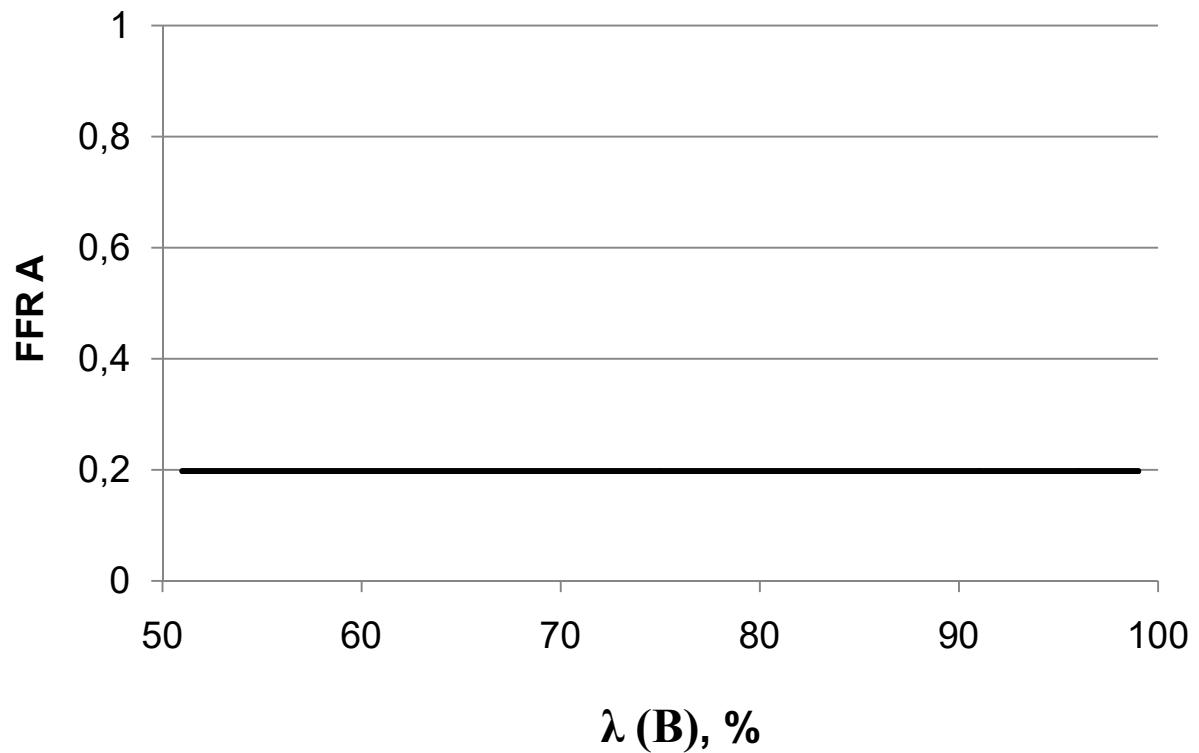
$$FFR(A)_{isolated} = 0.2$$



# FFR. Two sequential stenoses

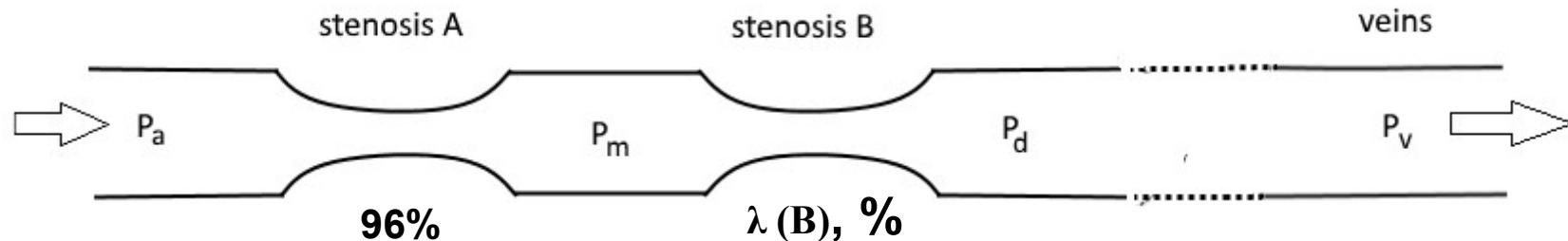


$$FFR(A)_{isolated} = 0.2$$



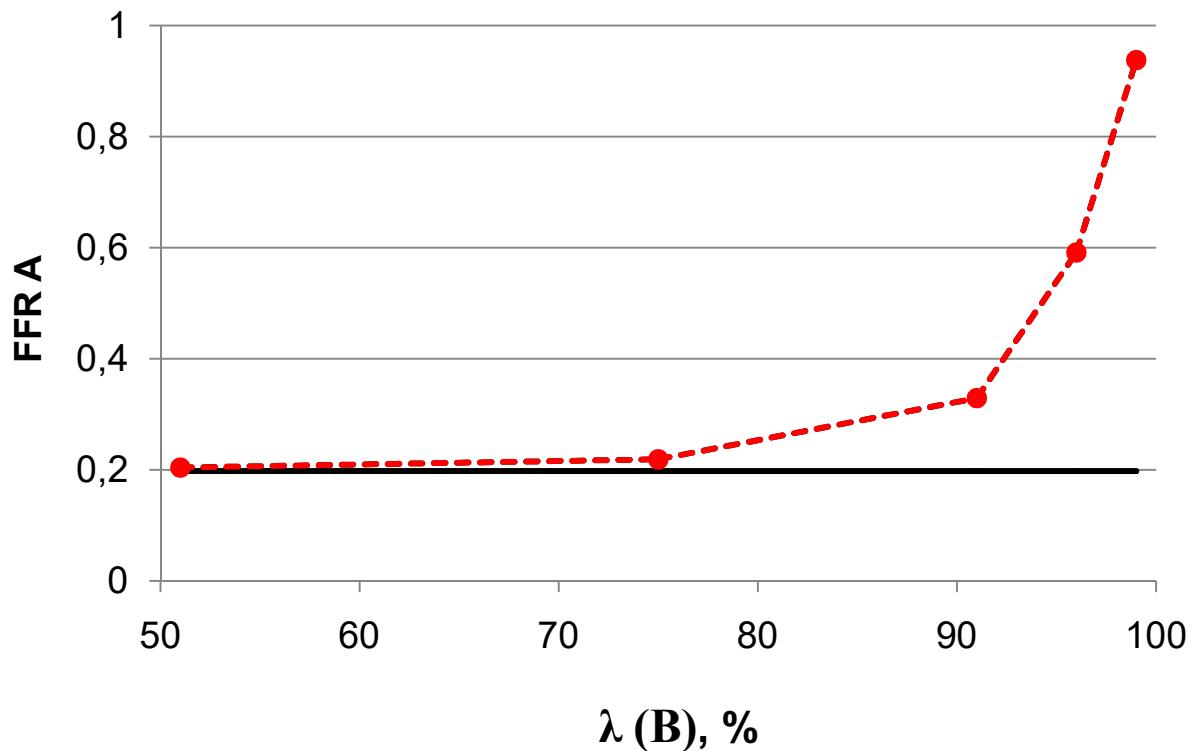


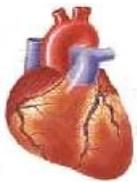
# FFR. Two sequential stenoses



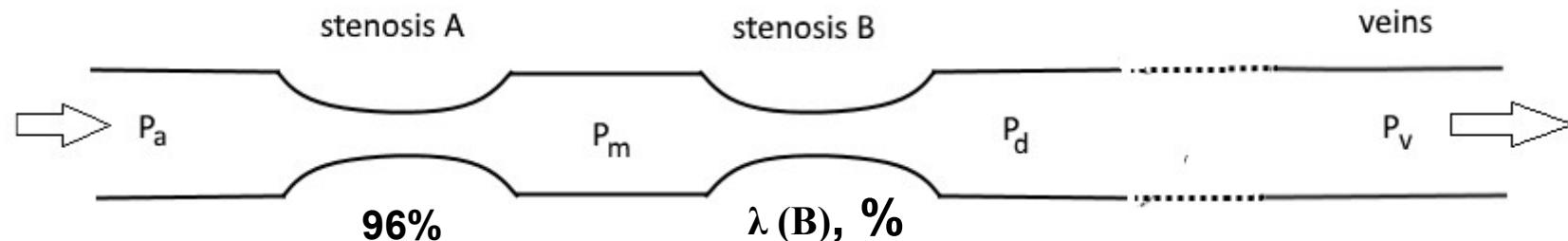
$$FFR(A)_{isolated} = 0.2$$

$$FFR(A)_{apparent} = \frac{P_m}{P_a}$$





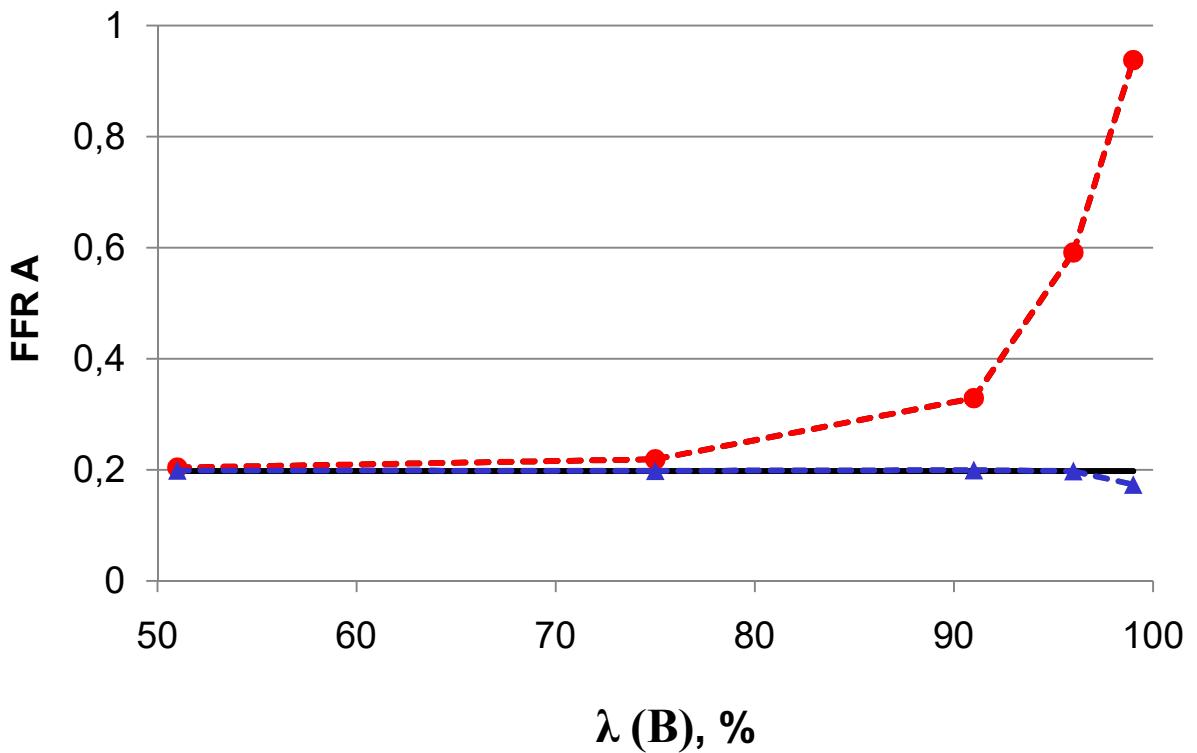
# FFR. Two sequential stenoses



$$FFR(A)_{isolated} = 0.2$$

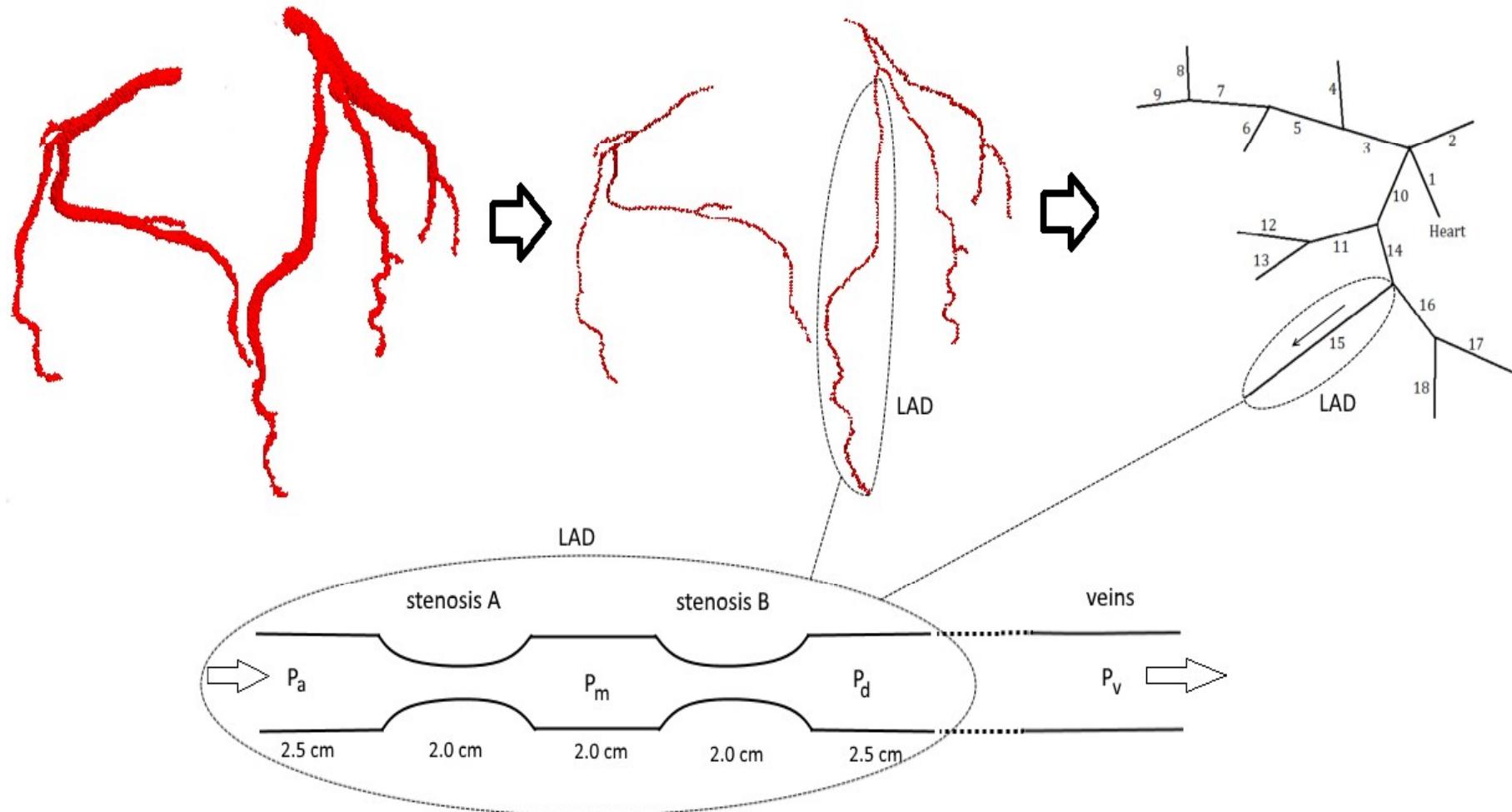
$$FFR(A)_{apparent} = \frac{P_m}{P_a}$$

$$FFR(A)_{true} = \frac{P_d - P_v (P_m / P_a)}{P_a - P_m + P_d - P_v}$$



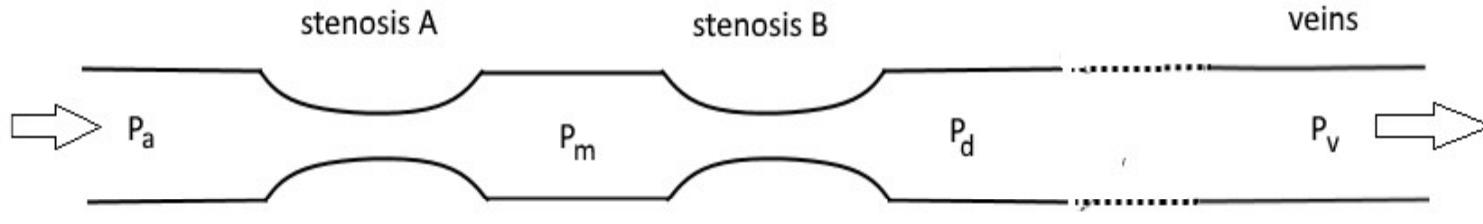


# FFR. Two sequential stenoses





# FFR. Two sequential stenoses



$$FFR = \frac{P_d}{P_a}$$

Options

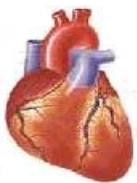


$$FFR = ?$$

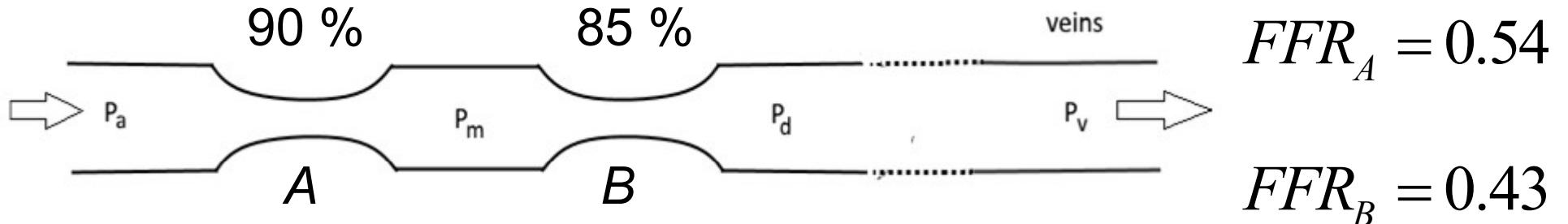


$$FFR = ?$$

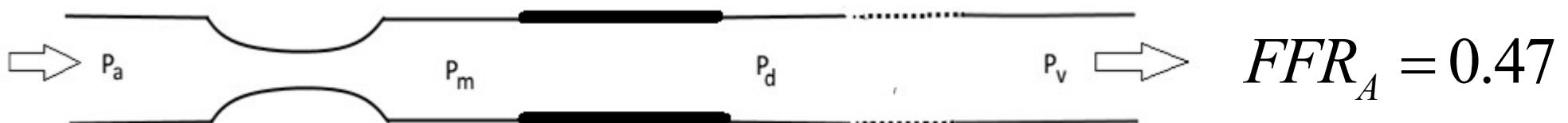
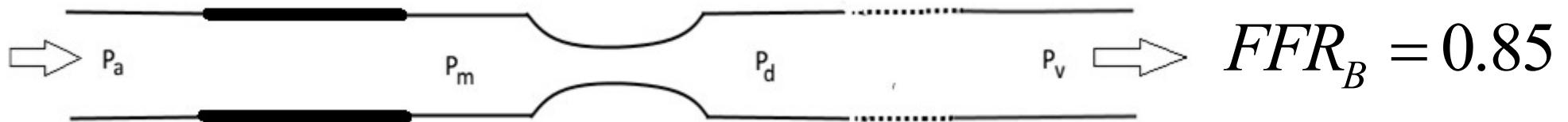




# FFR. Two sequential stenoses. Calculations

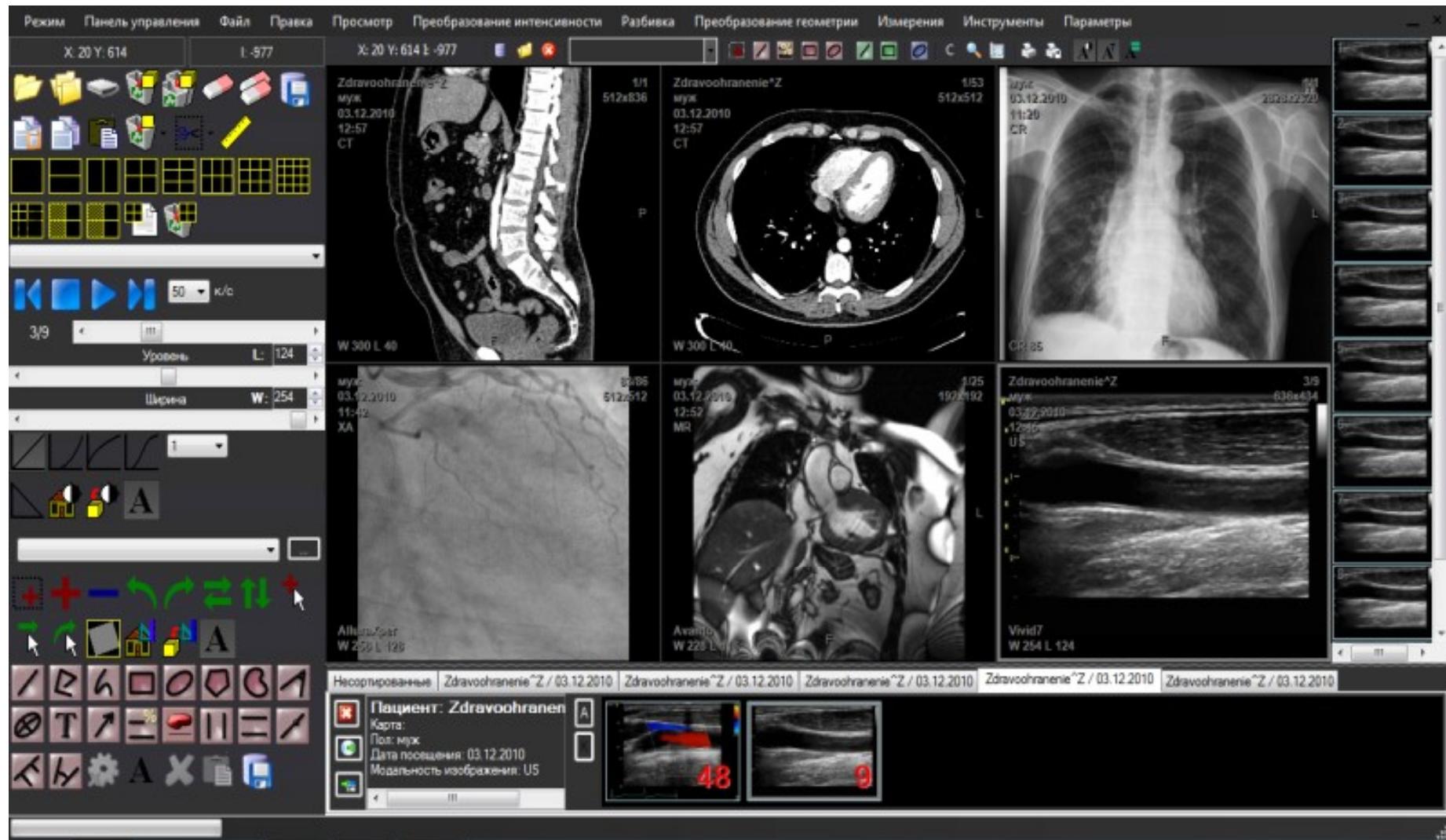


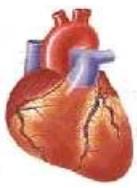
Options





# Multivox





# Multivox



Режим Файл База данных Правка Просмотр Преобразование интенсивности Измерения Преобразование геометрии Инструменты Параметры

1/1 [BGRA] 964x461

ANON  
AN: 1  
ID: 1  
жен  
13.03.2017 [21:54:28]  
HEART [CT]

Визуализация Анализ

Стандартный: Realistic Ред.

Освещение: Уровень: Порог: Прозрачность: X границы: Y границы:

3D Сегментация

Отрезать Внутри Снаружи Дополнительные

Выделение объектов Выделение объектов

Действия с объектами Действия с объектами

Загрузка/Сохранение объектов в БД Сохранить Сохраненные измерения

Менеджер объектов Корзина Удалить

История корзины << >>

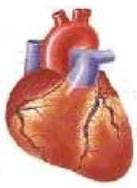
Aorta preview Parameters Cut: OK

W:256 H:118 111%

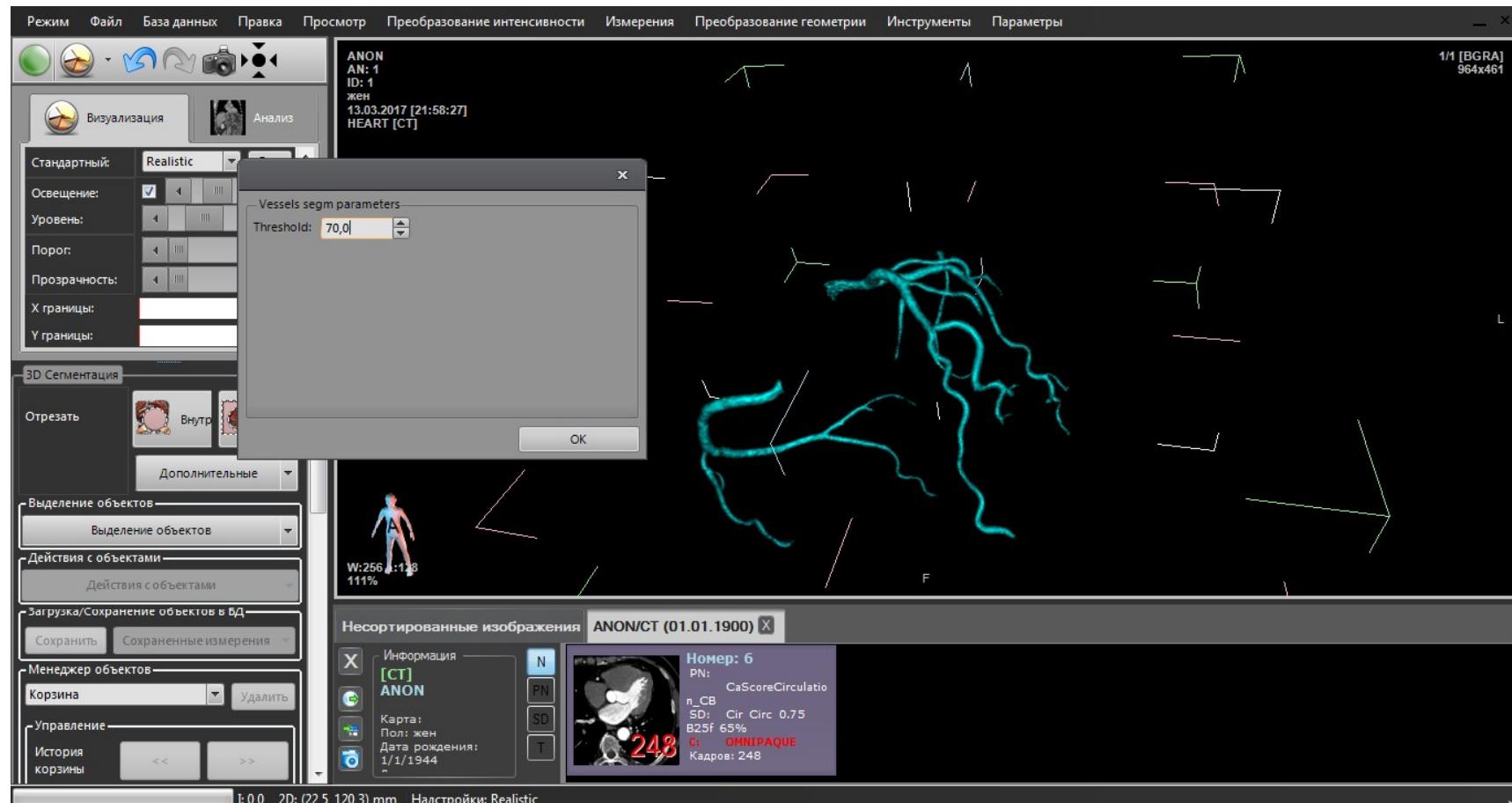
Несортированные изображения ANON/CT (01.01.1900) X

Информация [CT] ANON  
Карта: Пол: жен Дата рождения: 1/1/1944  
Номер: 6 PN: CaScoreCirculatio  
n\_CB SD: Cir Circ 0.75  
B25f 65% C: OMNIPACQUE  
Кадров: 248

I: 0,0 2D: (802,1,171,6) mm Надстройки: Realistic



# Multivox





# Multivox



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Визуализация Анализ

Стандартный: Realistic Ред.

Освещение:

Уровень:

Порог:

Прозрачность:

Х границы: 55%

Y границы:

3D Сегментация

Отрезать Внутрь Снаружи

Дополнительные

Выделение объектов:

Выделение объектов

Действия с объектами:

Действия с объектами

Загрузка/Сохранение объектов в БД:

Сохранить Сохраненные измерения

Менеджер объектов:

Vessel\_1 Удалить

Управление:

Цвет/Прозр. << >>

Несортированные изображения ANON/CT (01.01.1900) X

Информация [CT] ANON

Карта: Пол: жен Дата рождения: 1/1/1944

Номер: 6 РН: CaScoreCirculatio

n\_CB SD: Cir Circ 0.75 B25f 65% C: OMNIPACQUE Кадров: 248

1/1 [BGRA] 964x461

ANON AN: 1 ID: 1 жен 13.03.2017 [21:59:36] HEART [CT]

W: 256 L: 13 55%

1/1 [sY2] 511x247 SW: 0,00 mm

HEART [CT]

-125,5 mm

CaScoreCirculation\_CB W: 600 L: 200 96%

L

F

Анализ сосуда

Измерения Цветовая Карта Графики

Редактирование

left vessel: 3  
left vessel: 4  
left vessel: 5  
left vessel: 6  
left vessel: 7  
left vessel: 8  
left vessel: 9

Удалить сосуд

Масштаб M Bo

0,0 MIP

C=6,03 mm<sup>2</sup> (Fr=8,19 mm<sup>2</sup> R=5,34 mm<sup>2</sup>) | A%=89,08% L=32,1 mm

Пороги детектора Мин: 160 Макс: 650

Надстройки: Realistic

1/1 [BGRA] 964x461

ANON AN: 1 ID: 1 жен 13.03.2017 [21:59:36] HEART [CT]

W: 256 L: 13 55%

1/1 [sY2] 511x247 SW: 0,00 mm

HEART [CT]

-125,5 mm

CaScoreCirculation\_CB W: 600 L: 200 96%

L

F

Анализ сосуда

Измерения Цветовая Карта Графики

Редактирование

left vessel: 3  
left vessel: 4  
left vessel: 5  
left vessel: 6  
left vessel: 7  
left vessel: 8  
left vessel: 9

Удалить сосуд

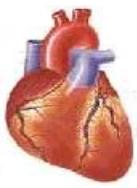
Масштаб M Bo

0,0 MIP

C=6,03 mm<sup>2</sup> (Fr=8,19 mm<sup>2</sup> R=5,34 mm<sup>2</sup>) | A%=89,08% L=32,1 mm

Пороги детектора Мин: 160 Макс: 650

Надстройки: Realistic



# Multivox



Режим Файл База данных Правка Просмотр Преобразование интенсивности Измерения Преобразование геометрии Инструменты Параметры

1/1 [BGRA] 964x461

ANON  
AN: 1  
ID: 1  
жен  
13.03.2017 [21:53:01]  
HEART [CT]

Визуализация Анализ

Стандартный: Realistic Ред.

Освещение: Уровень: Порог: Прозрачность:

Х границы: Y границы:

3D Сегментация

Отрезать Внутри Снаружи Дополнительные

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Менеджер объектов Корзина Удалить

История корзины << >>

I: 0,0 2D: (916,3, 210,3) mm Надстройки: Realistic

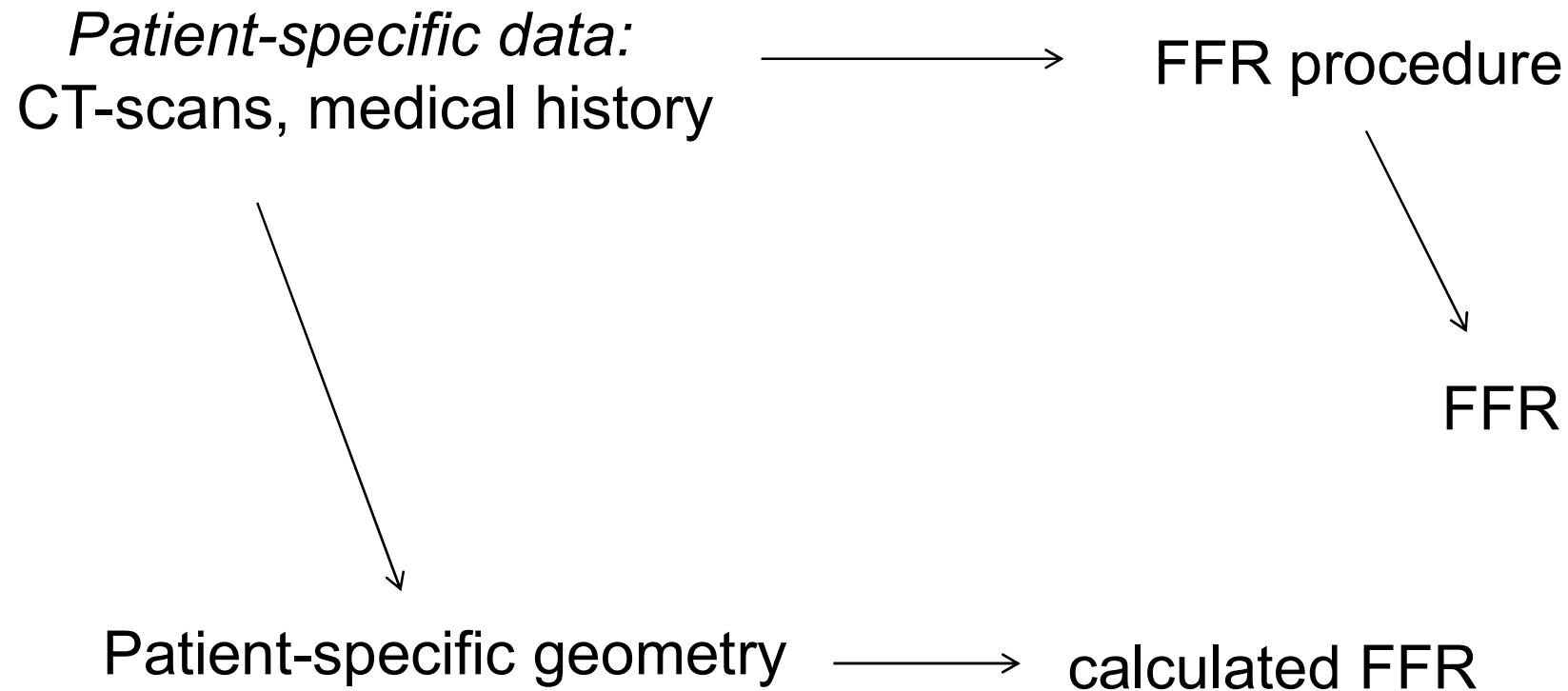
Несортированные изображения ANON/CT (01.01.1900) X

Информация [CT] ANON N  
Карта: Пол: жен Дата рождения: 1/1/1944 PN SD T

Номер: 6 PN: CaScoreCirculatio  
n\_CB SD: Cir Circ 0.75  
B25f 65% C: OMNIPACQUE  
Кадров: 248

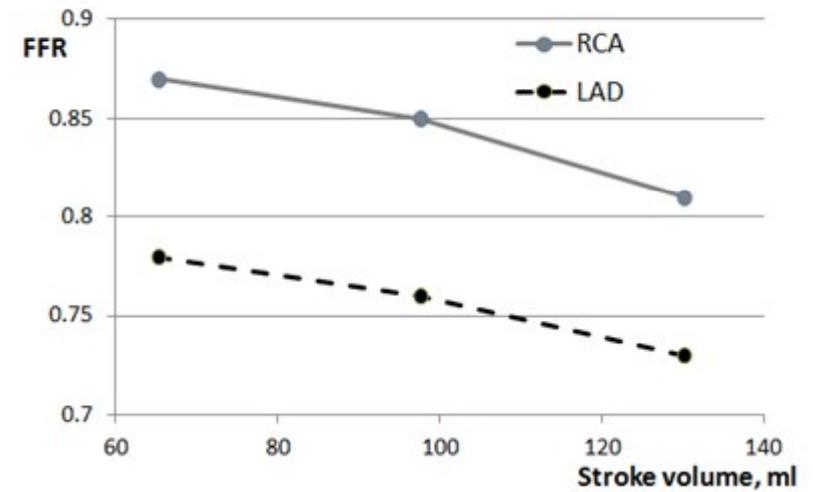
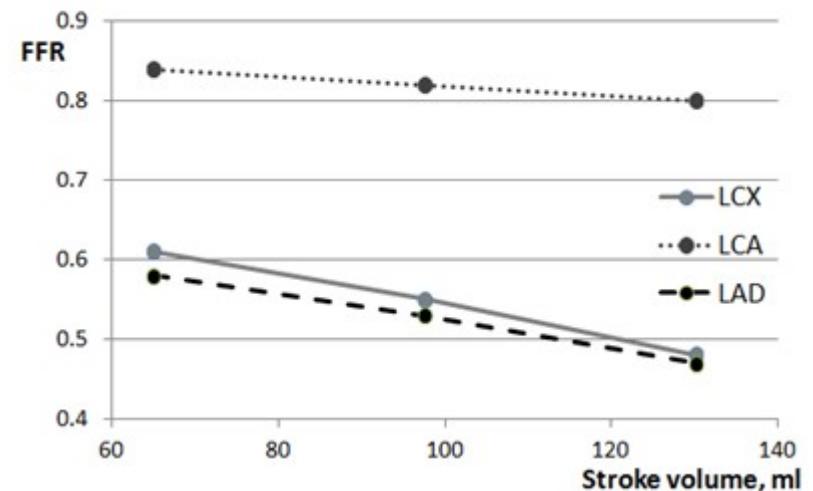
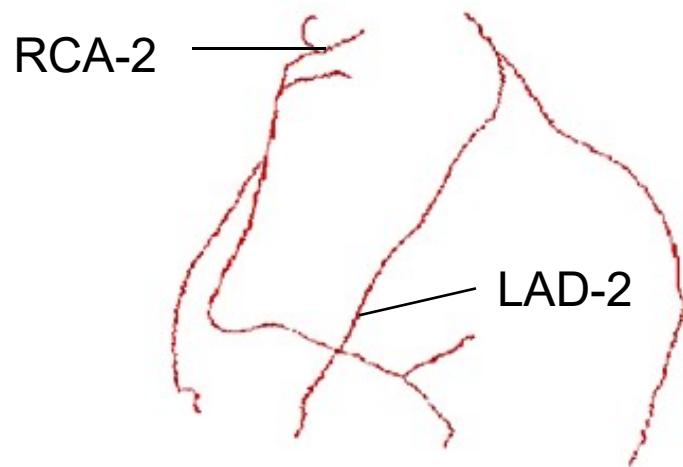
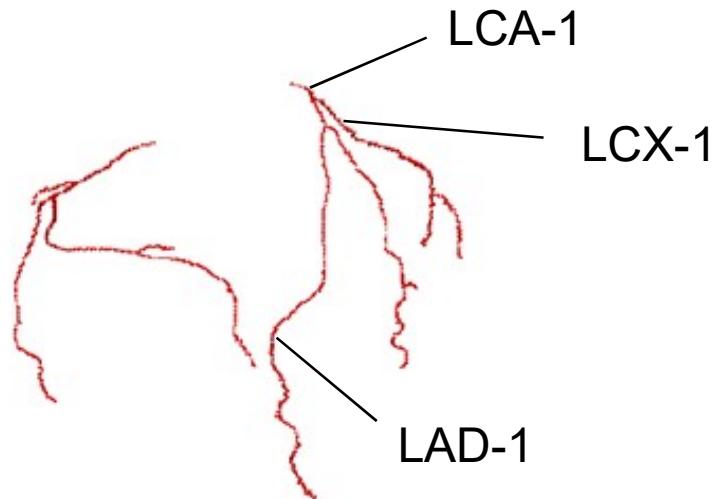


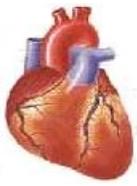
# Discussion



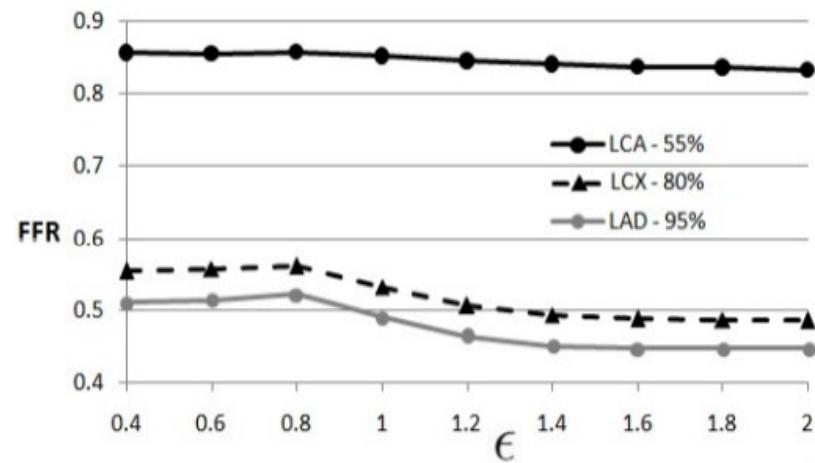
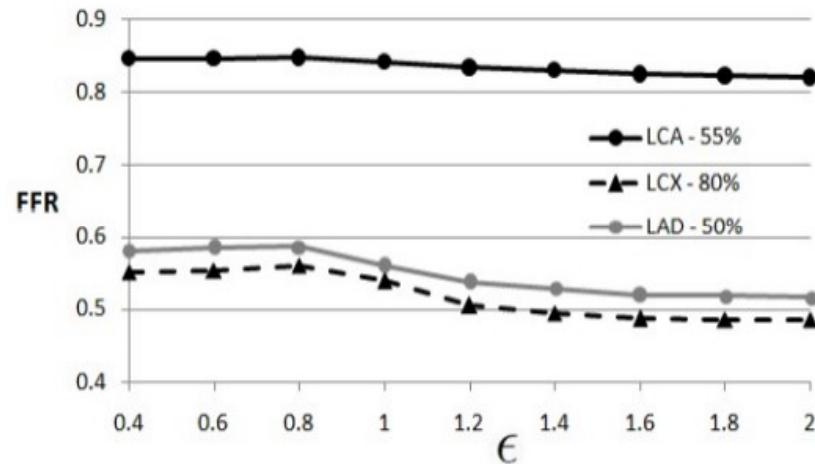
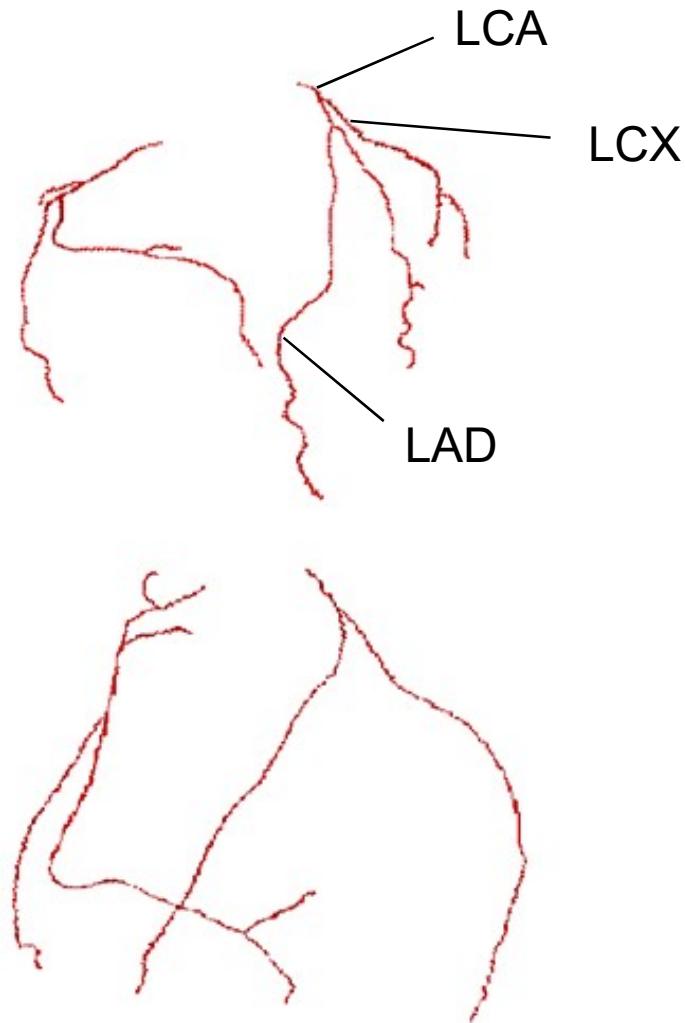


# FFR. Increased stroke volume





# FFR. Elasticity of vessel walls



stiffness ↑