British-Russian Workshop under the British Council Researcher Links scheme «Mathematical and Computational Modelling in Cardiovascular Problems»

Numerical simulation of blood flow in the vascular network with pathologies or implants

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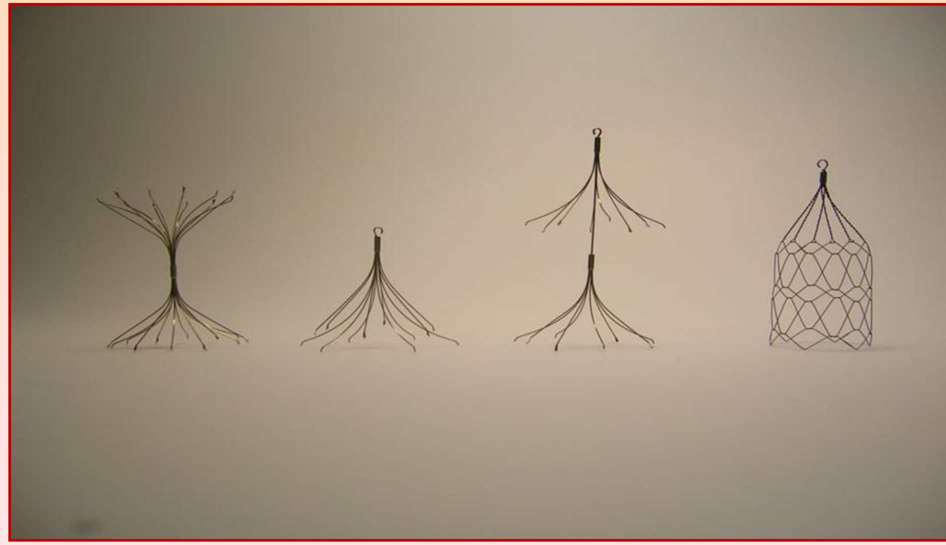
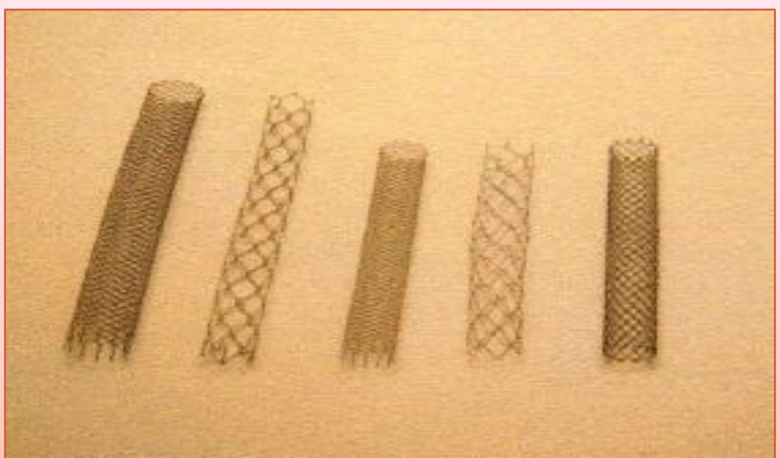
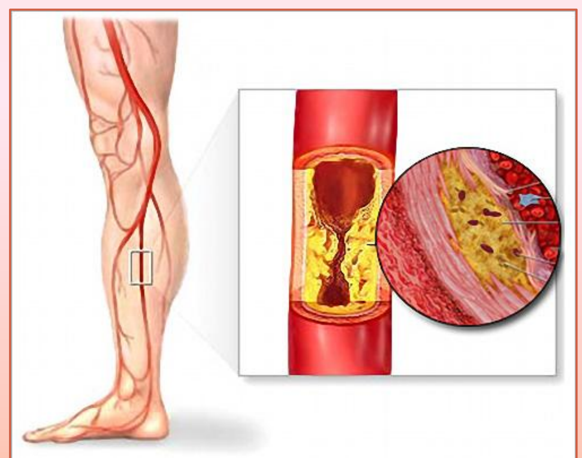
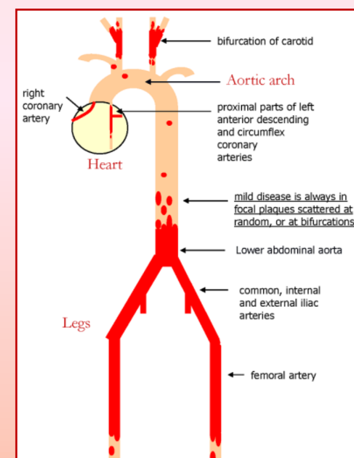
Some applications

- Reduction of side effects induced by the Cava-filter implantation
- Prediction of postsurgical conditions of patients with atherosclerotic plaques

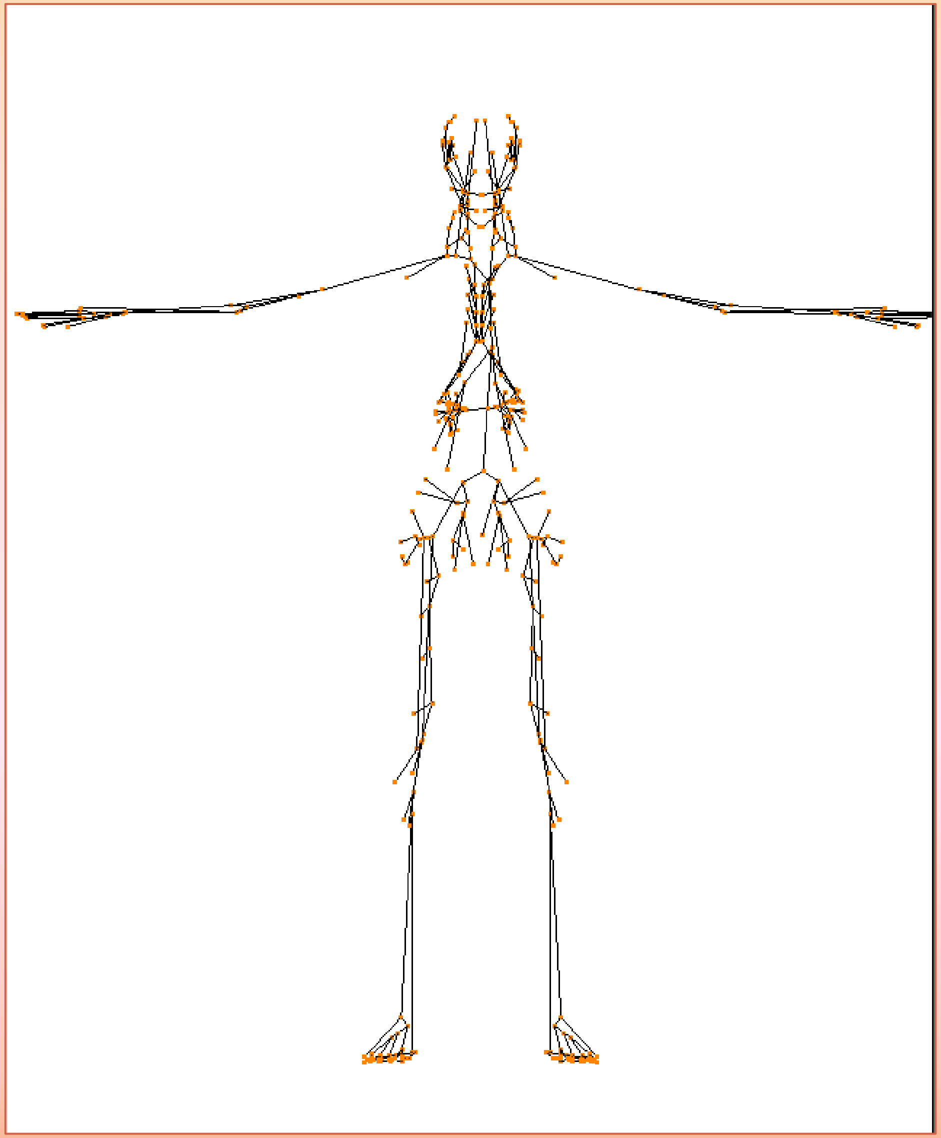


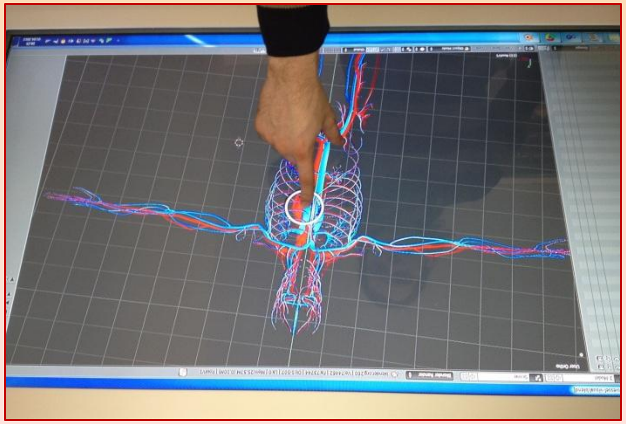






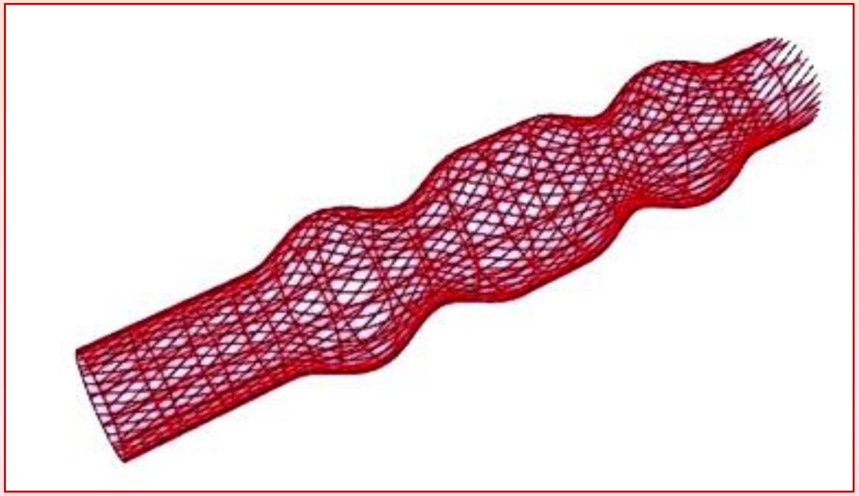
Technology





Modelling of blood flow in the vessel network with implants or pathologies

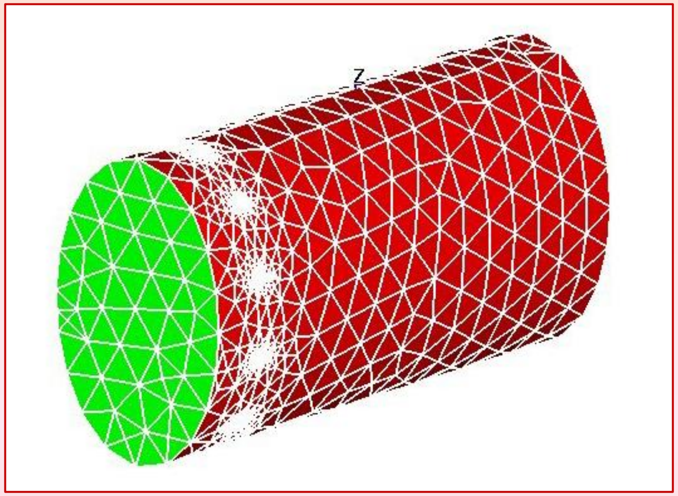
- Modification of the state equation (description of elastic properties of the vessel wall)



Fiber model of elastic vessel wall

(Peskin, Rosar)

- Consider a part of 1D vessel with implant or pathology as a 3D domain

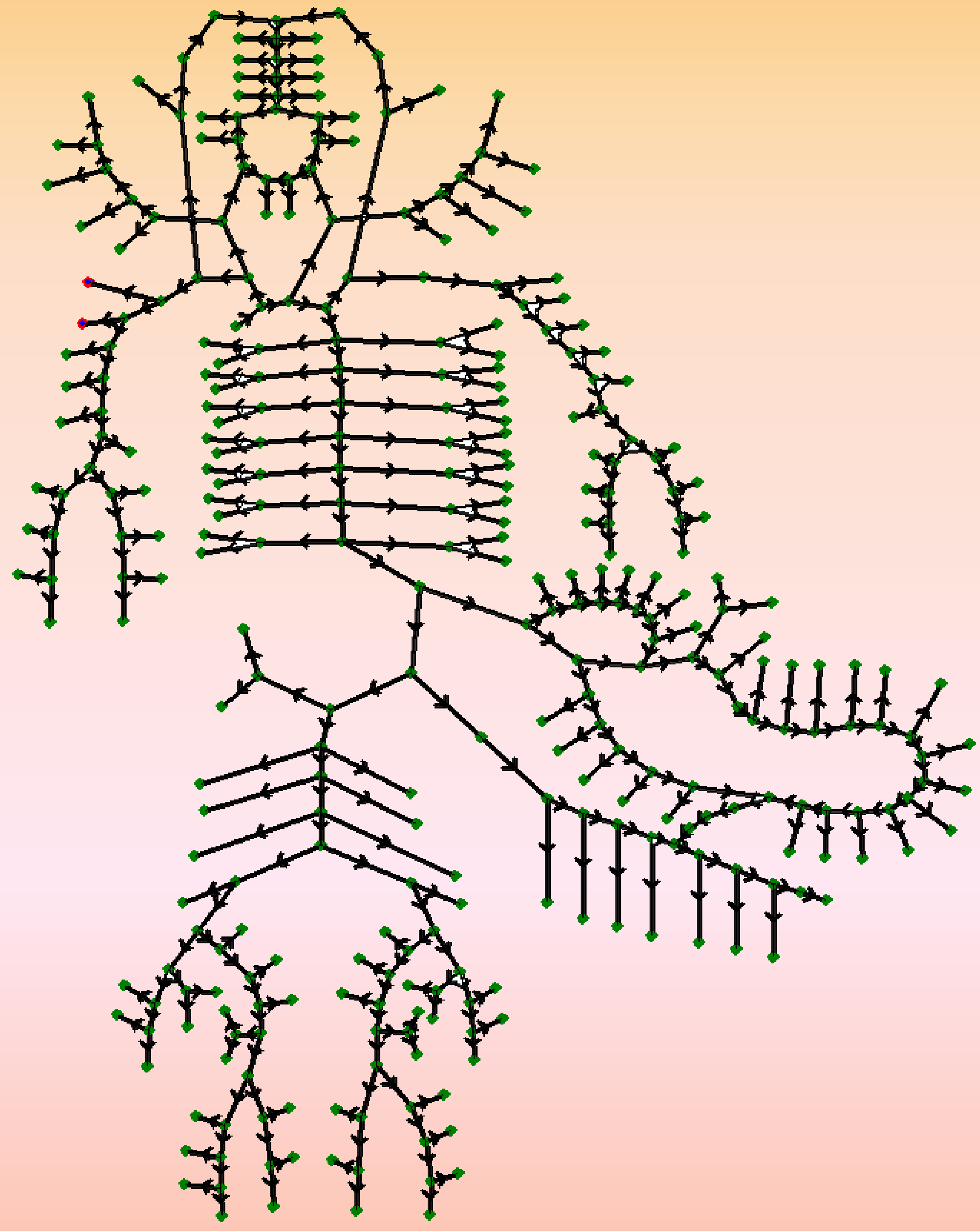


3D Model of the fluid flow

(Navier-Stokes equations)

Ani3D Package

Model of global blood circulation



System of equations

1. Mass conservation law

$$\frac{\partial S}{\partial t} + \frac{\partial (uS)}{\partial x} = 0$$

2. Momentum conservation law

$$\frac{\partial u}{\partial t} + \frac{\partial}{\partial x}\left(\frac{u^2}{2} + \frac{p}{\rho}\right) = F_{\text{тр}}$$

3. The state equation

$$p = \rho c^2 f(S),\ f(S) = \begin{cases} \exp\left(\frac{S}{S_0} - 1\right) - 1, S > S_0 \\ \ln\left(\frac{S}{S_0}\right), S \le S_0 \end{cases}$$

Boundary conditions

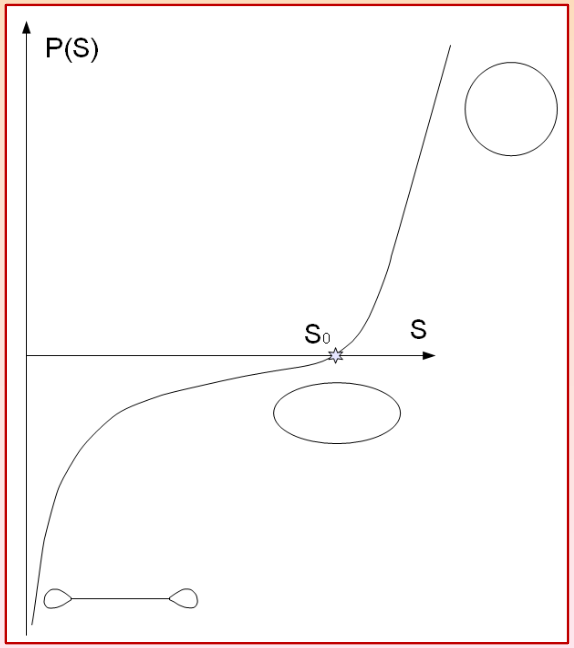
1. Mass balance condition:
$$\sum_{k=k_1,\ldots,k_M} \alpha_k^m Q_k = 0, \alpha_k^m = \pm 1, Q_k = u_k S_k$$
2. Poiseuille's pressure drop conditions:
$$p_k\left(t, x_k\right) - p_m^{node}\left(t\right) = \alpha_k R_k^m Q_k, x_k = 0, L_k$$
3. Compatibility conditions

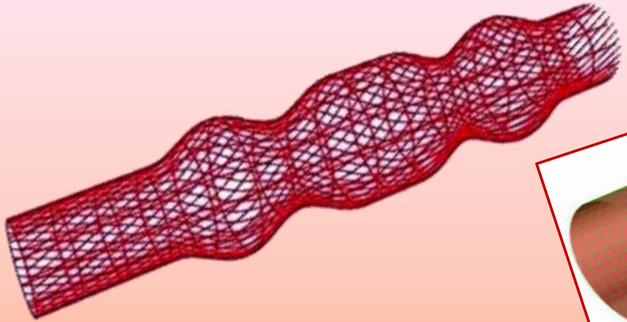
The state equation

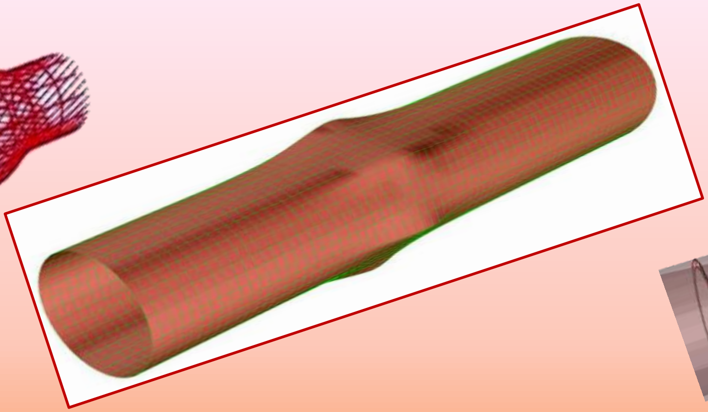
$$p = \rho c^2 f(S)$$

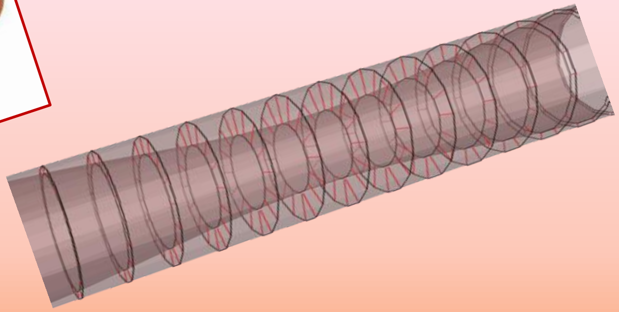
An example of analytical function $f(S)$:

$$f(S) = \begin{cases} \exp\left(\frac{S}{S_0} - 1\right) - 1, S > S_0 \\ \ln\left(\frac{S}{S_0}\right), S \leq S_0 \end{cases}$$

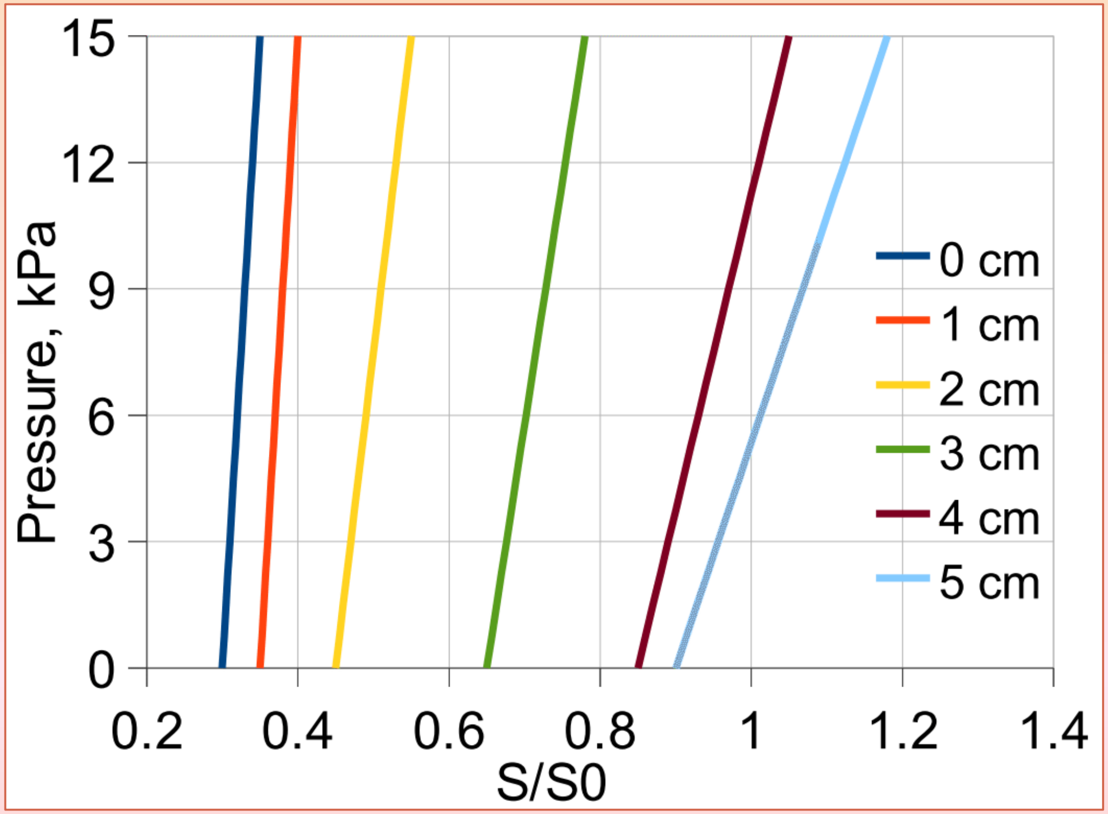


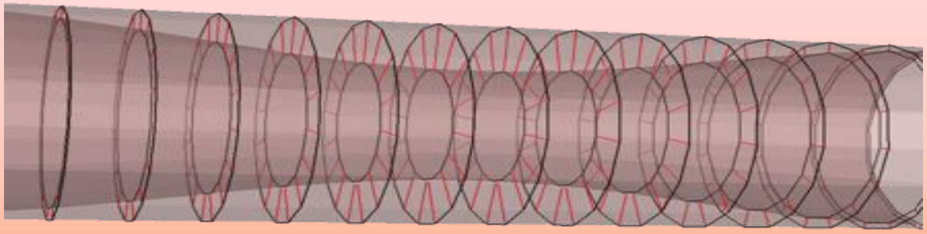




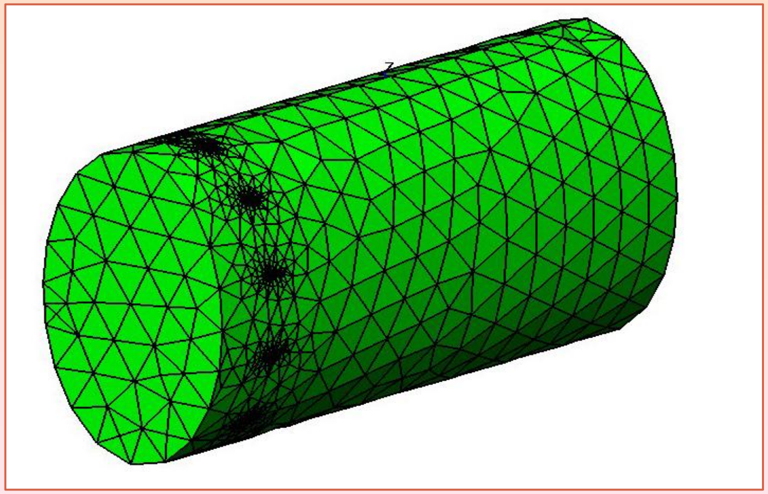


The state equation for atherosclerotic artery





3D Model of Fluid Flow



3D Model of Fluid Flow

- Navier-Stokes equations:

$$\frac{\partial \mathbf{u}}{\partial t} - \nu\Delta\mathbf{u} + (\mathbf{u} \cdot \nabla)\mathbf{u} + \nabla p = \mathrm{f} \text{ in } \Omega \times [0, T],$$

$$div\,\mathbf{u} = 0 \text{ in } \Omega \times [0, T]$$

- Boundary conditions:

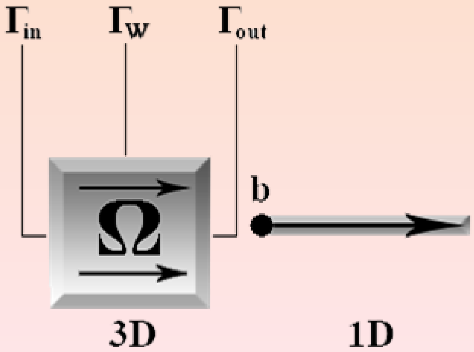
$$\mathbf{u} = \mathbf{g} \text{ on } \Gamma_{in} \times [0, T],$$

$$\mathbf{u} = \mathbf{0} \text{ on } \Gamma_{W} \times [0, T],$$

$$-\nu\frac{\partial \mathbf{u}}{\partial \mathbf{n}} + p\mathbf{n} = \mathbf{h} \text{ on } \Gamma_{out} \times [0, T]$$

- **Not assumed:** $\mathbf{u} \bullet \mathbf{n} < 0$ on Γ_{in} , $\mathbf{u} \bullet \mathbf{n} > 0$ on Γ_{out}

Downstream Coupling Boundary Conditions (x=b)



1D: Compatibility condition.

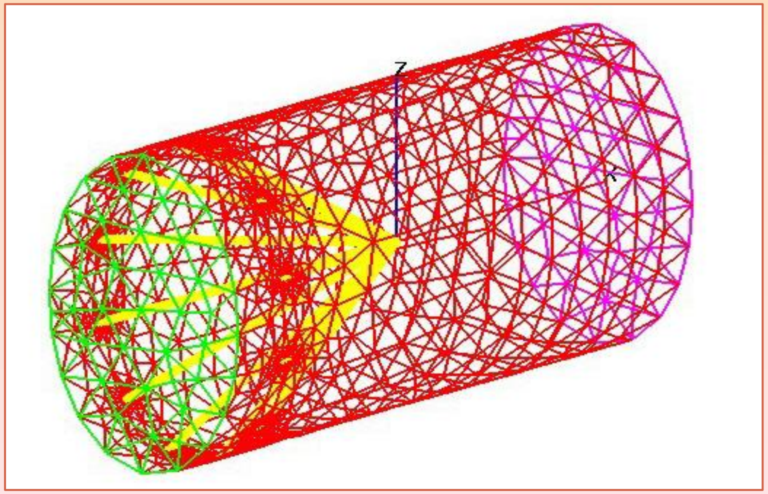
- $\int_{\Gamma_{out}} \mathbf{u} \bullet \mathbf{n}$ ds = $\bar{u}_b\, S_b$
  $(-\nu \frac{\partial \mathbf{u}}{\partial \mathbf{n}} + p\mathbf{n}) = p_b \mathbf{n}$
- $\int_{\Gamma_{out}} \mathbf{u} \bullet \mathbf{n}$ ds = $\bar{u}_b\, S_b$
  $-\nu \frac{\partial \mathbf{u}}{\partial \mathbf{n}} + (p + \frac{\rho}{2}|\mathbf{u}|^2)\mathbf{n} = (\bar{p} + \frac{\rho}{2}\bar{u}^2)|_b\, \mathbf{n}$
- $(-\nu \frac{\partial \mathbf{u}}{\partial \mathbf{n}} + p\mathbf{n}) = p_b \mathbf{n}$
  $\bar{p} \int \mathbf{u} \bullet \mathbf{n}$ ds+$\frac{\rho}{2} \int |\mathbf{u}|^2 \mathbf{u} \bullet \mathbf{n}$ ds =$(\bar{p} S\bar{u} + \frac{\rho}{2} S\bar{u}^3)|_b$

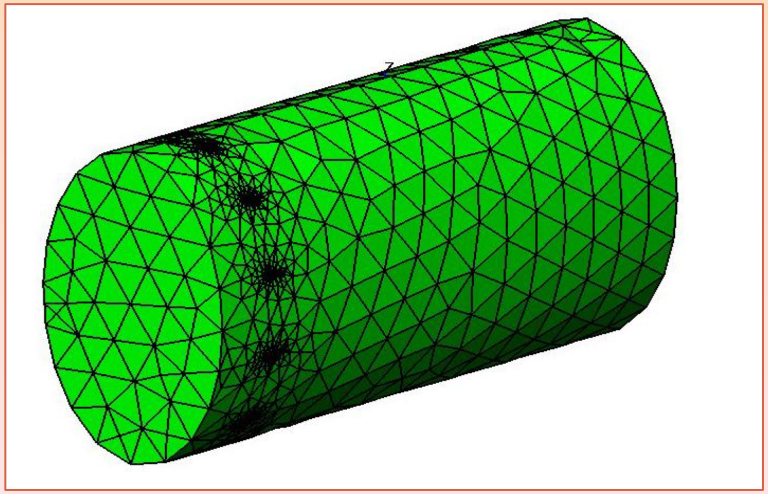
Energy equality for 1D-3D model:

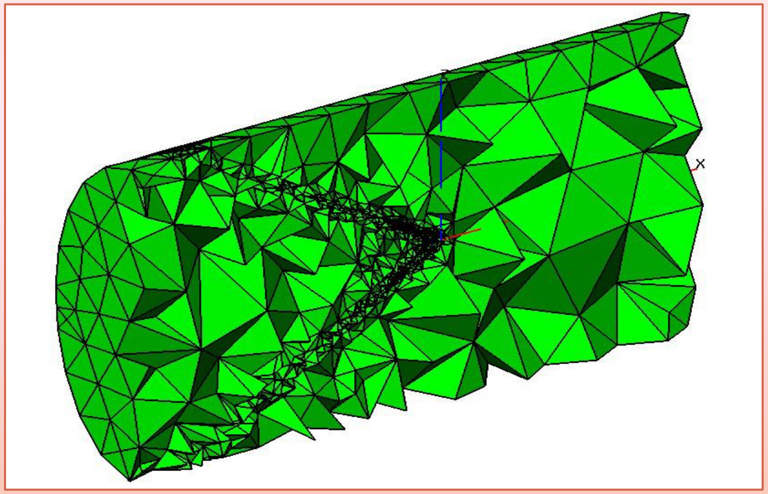
$$\frac{d}{dt}(E_{1D}(\mathrm{t}) + E_{3D}(\mathrm{t})) + \nu \int_0^T \|\nabla \mathbf{u}\|^2\, \mathrm{dt} + 16\nu \int_0^T \int_{1\mathrm{D}} \eta(\tilde{S})\hat{S} d^{-2}\bar{u}^2\, \mathrm{dx\,dt} =$$
$$-\int_{\Gamma_{out}} \left((p + \frac{\rho}{2}|\mathbf{u}|^2)\mathbf{I} - \nu\, \nabla \mathbf{u}\right) \mathbf{n} \bullet \mathbf{u}\,\mathrm{ds} + S\bar{u}(\bar{p} + \frac{\rho}{2}\bar{u}^2)|_b$$

T. Dobroserdova, M. Olshanskii. A finite element solver and energy stable coupling for 3D and 1D fluid models. Computer Methods in Applied Mechanics and Engineering.

The visualization of the adaptive mesh for the flow over a model IVC filter problem





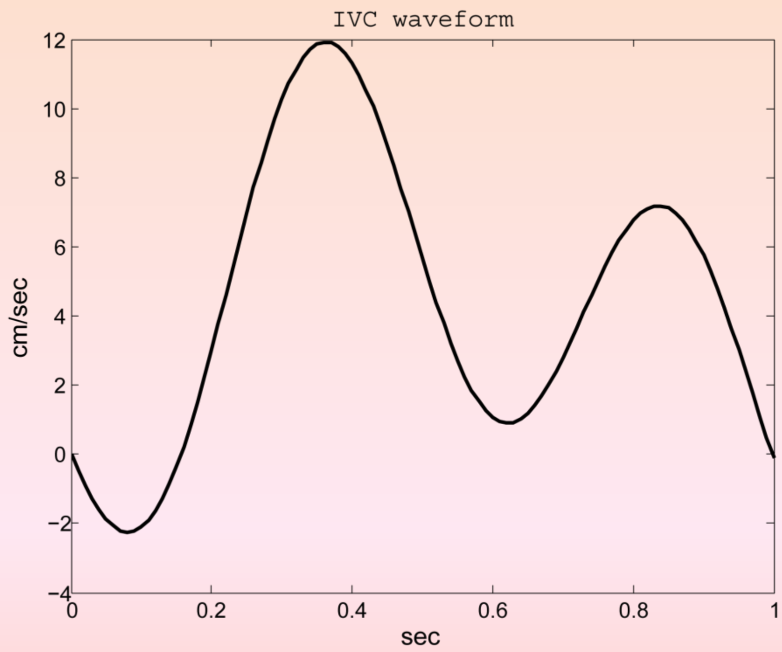


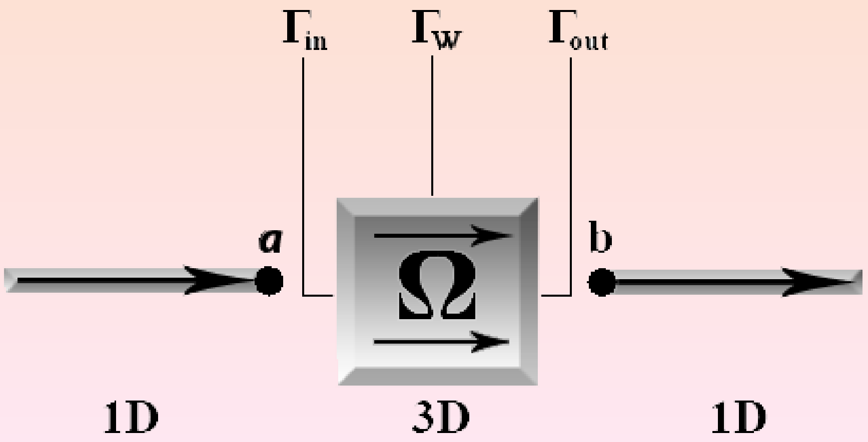
The ratio of largest and smallest element diameters was $1.1e+2$

Ani3D Package

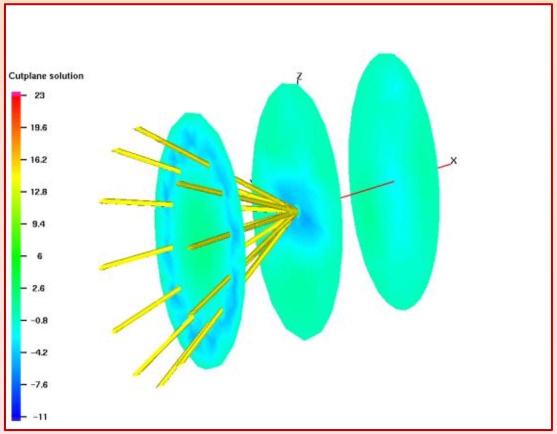
http://sourceforge.net/projects/ani3d

Calculation of haemodynamics in the simple vessel network with cava-filtr

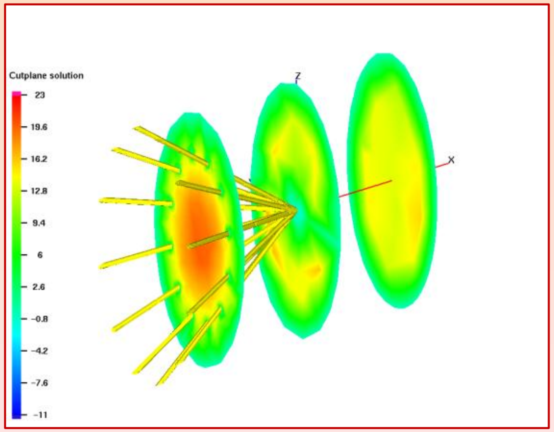




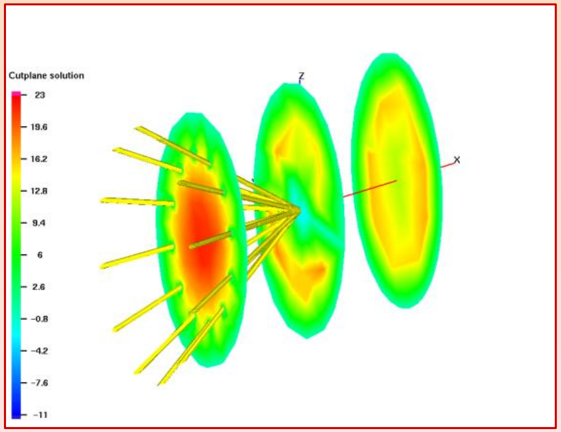
The visualization of the velocity x-component in several cutplanes orthogonal to x-axis



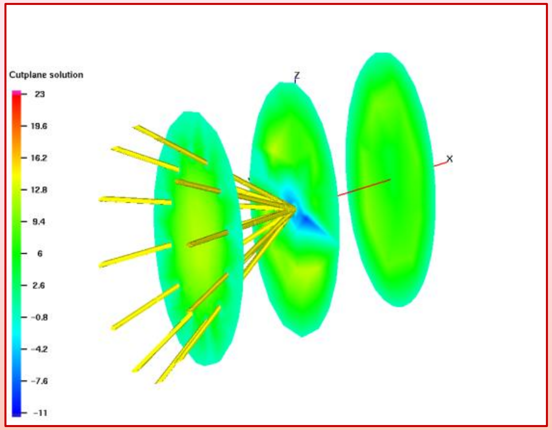
t=3.06



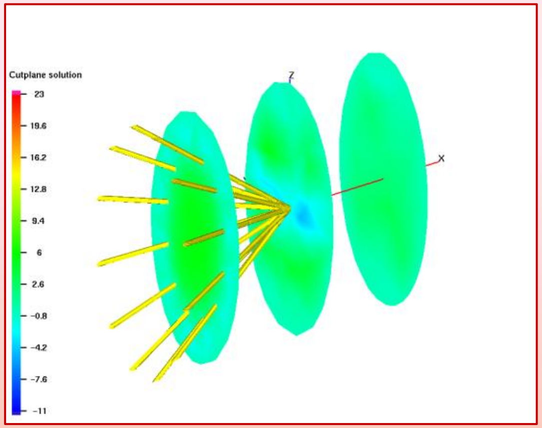
t=3.34



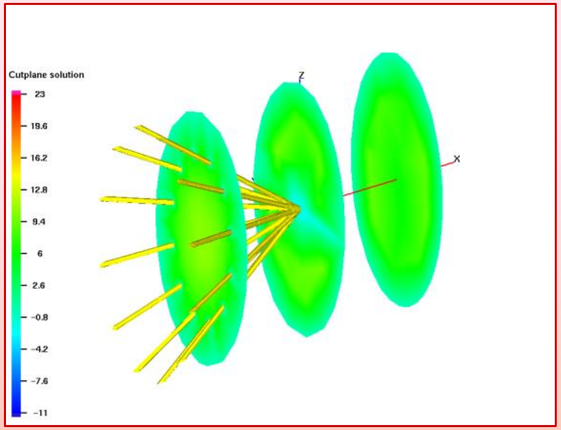
t=3.39



t=3.52

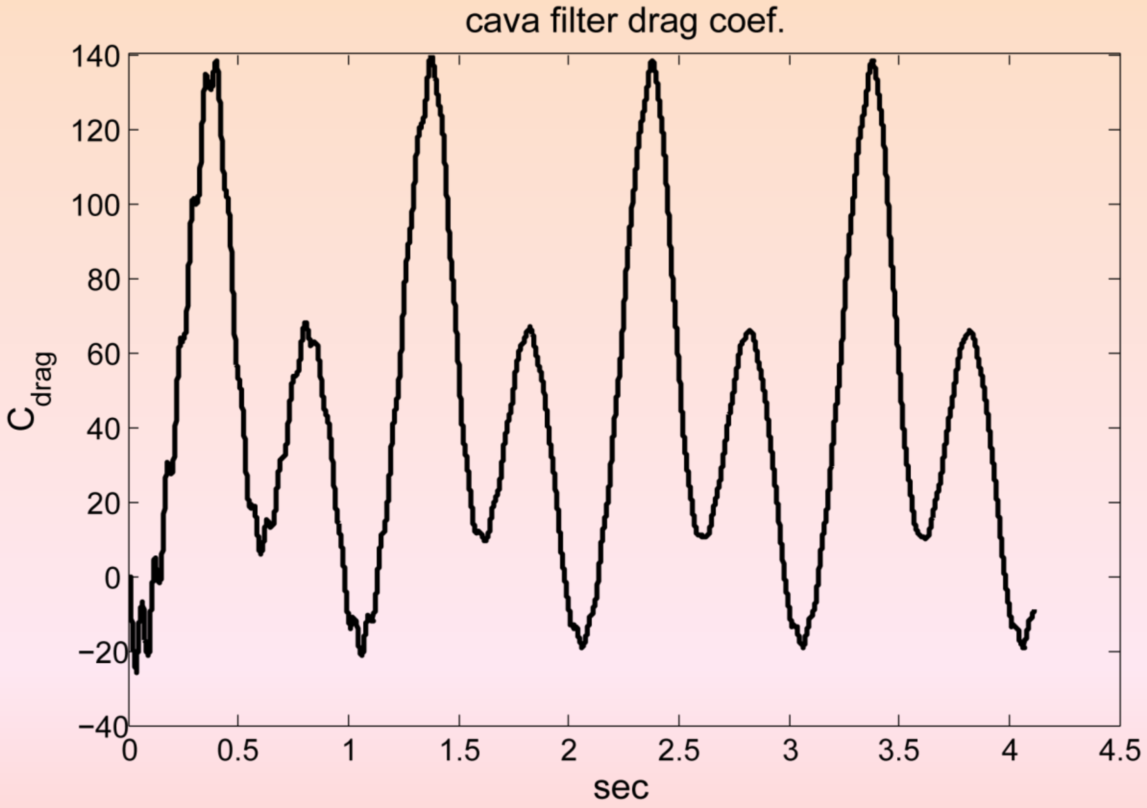


t=3.66



t=3.92

The evolution of the drag force for the IVC filter



$$F = \int (\rho \nu \frac{\partial v_t}{\partial \mathbf{n}} n_{\boldsymbol{y}} - p n_x) dS \quad [10^{-5}\mathrm{N}]$$

Conclusions

- Modification of the state equation by the fiber or fiber-spring model of elastic vessel wall to model the blood flow in the vessel network with vascular pathologies or implants through
- New boundary condition for the 3D–1D coupling:
  - It ensures the energy balance for 1D-3D model
  - The inequalities $\mathbf{u} \bullet \mathbf{n} < 0$ on Γ_{in} , $\mathbf{u} \bullet \mathbf{n} > 0$ on Γ_{out} are not assumed.
  - It can be easy decoupled with splitting methods into the separate 1D problems and the 3D problem with usual inflow-outflow boundary conditions on every time step.

Thank you for your attention!