

# Численное моделирование задач медицинского ультразвука при помощи сеточно-характеристического метода

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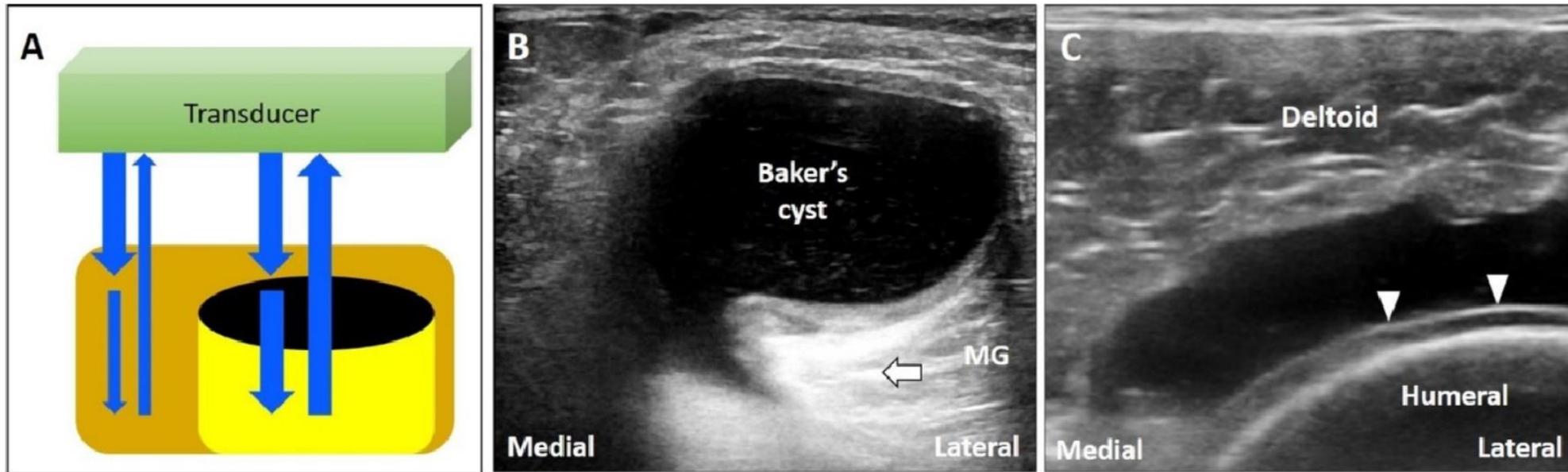
*Московский физико-технический институт,*

*кафедра вычислительной физики*

*РНФ 22-11-00142*

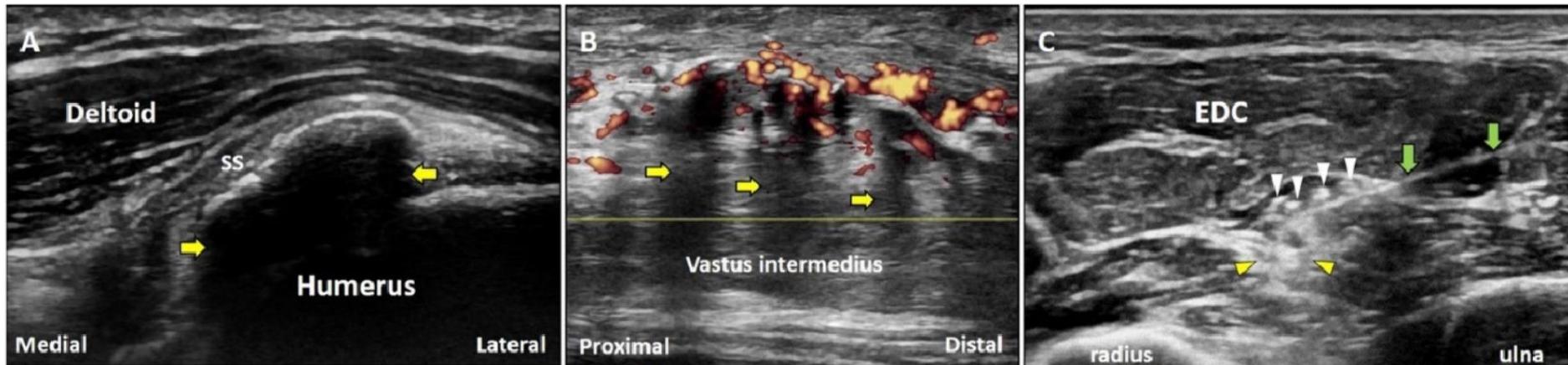
*Email: [amisto@yandex.ru](mailto:amisto@yandex.ru)*

# Аберрации при наличии неоднородностей



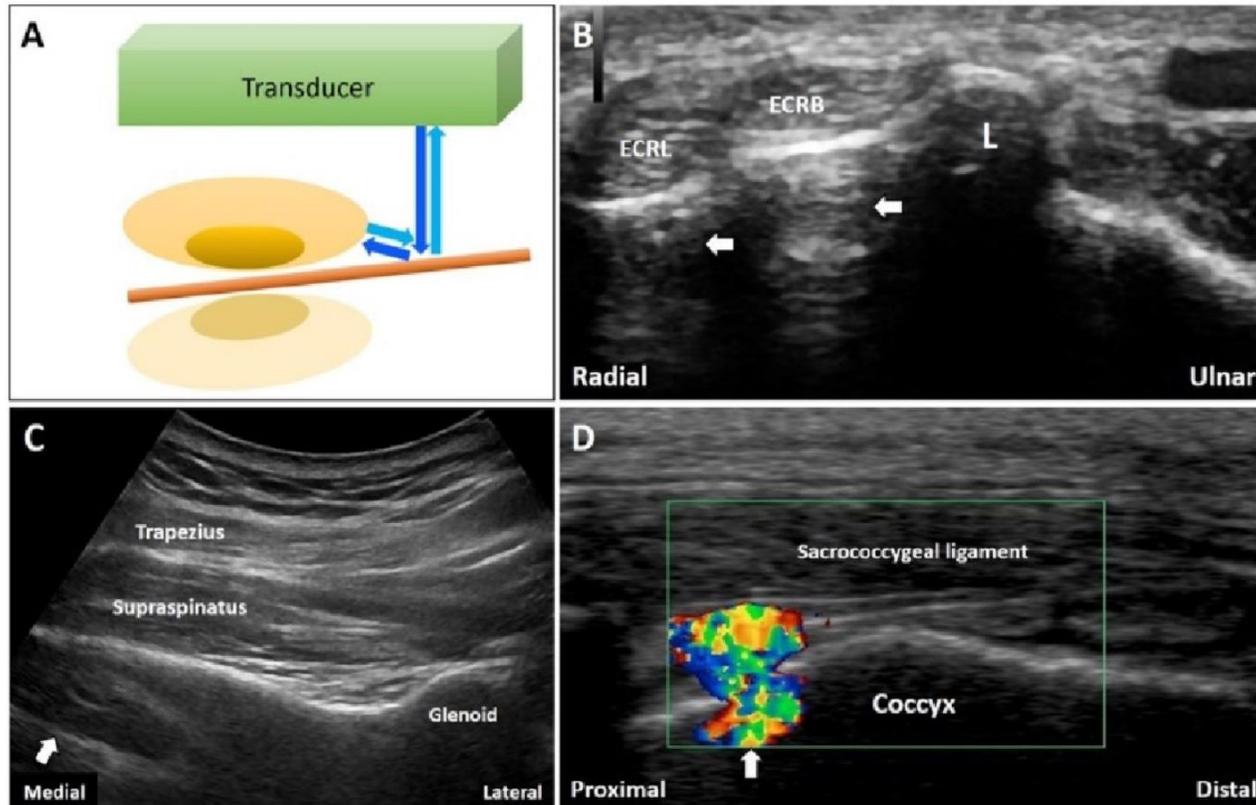
**Figure 3.** If the US beam passes through a low-attenuating structure, the signals reflected from the deep structure increase in relation to the surrounding tissues (A). The areas deep to a Baker's cyst (B) and a complete tear of the supraspinatus tendon (C) become hyperechoic because of the posterior acoustic enhancement (and discrepancy of the acoustic impedance between two different tissues). White arrow: artifact due to posterior acoustic enhancement; white arrowhead: cartilage interface sign. MG: medial gastrocnemius muscle. Blue arrows: the projected and reflected US beams. Black circle: the low-attenuation structure.

# Аберрации при наличии неоднородностей



**Figure 6.** Below the large or egg-shelled calcifications, clean acoustic shadowing can be observed (A). Partial acoustic shadowing is observed deep to fragmented calcifications resulting from the heterotrophic ossification in rectus femoris muscle (B). Dirty acoustic shadowing can be observed behind the gas bubbles during an US-guided injection (C). Yellow arrow: acoustic shadowing artifact; white arrowhead: gas bubbles; yellow arrowhead: dirty acoustic shadowing; green arrow: needle. SS: supraspinatus tendon; EDC: extensor digitorum communis muscle.

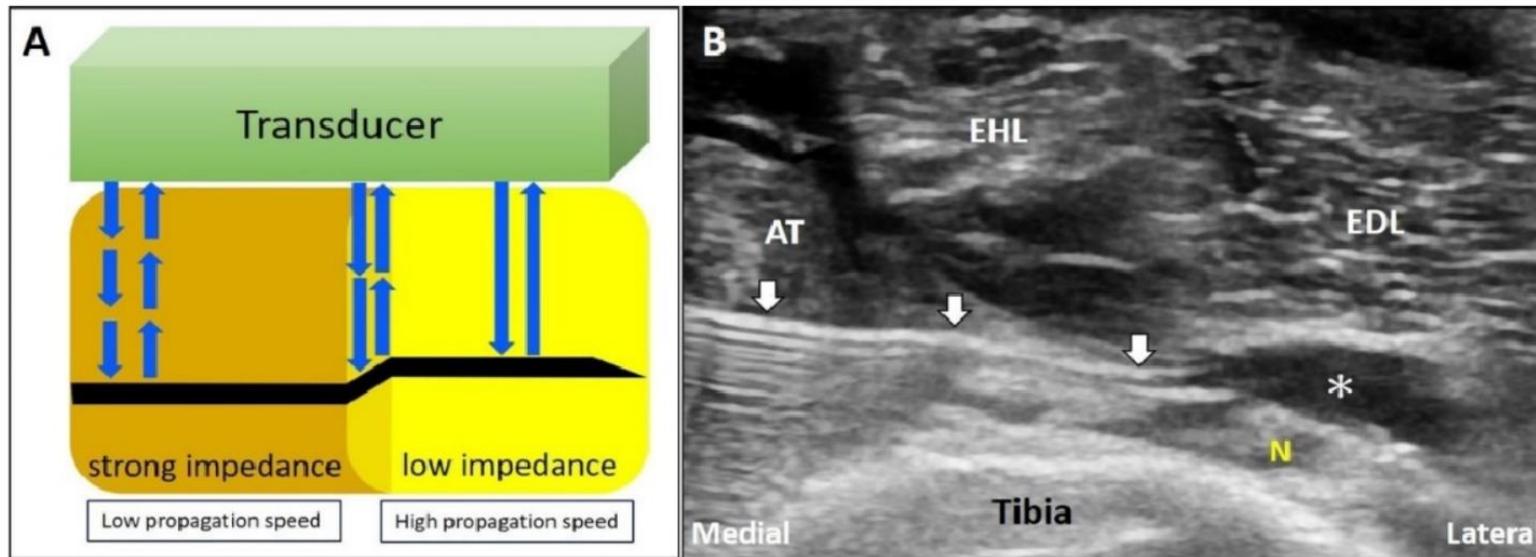
# Аберрации при наличии неоднородностей



**Figure 9.** When the sound waves encounter a reflective interface, the reflected beam would cast a mirror image at the opposite side of the interface (A). The mirror artifacts can be seen at many body regions like the forearm (B), supraspinatus fossa (C), and coccyx (D). White arrow: mirror artifact. ECRL: extensor carpi radialis longus tendon; ECRB: extensor carpi radialis brevis tendon; L: Lister's tubercle. Blue arrows: the projected and reflected US beams. Brown pillar: the reflective interface. Green box: the color box for detection of Doppler signals.

W.T. Wu, K.V. Chang, Y. C. Hsu, P.S. Hsu, V. Ricci, L. Ozcakar, Artifacts in Musculoskeletal Ultrasonography: From Physics to Clinics, *Diagnostics (Basel)*, 10 (9), (2020).

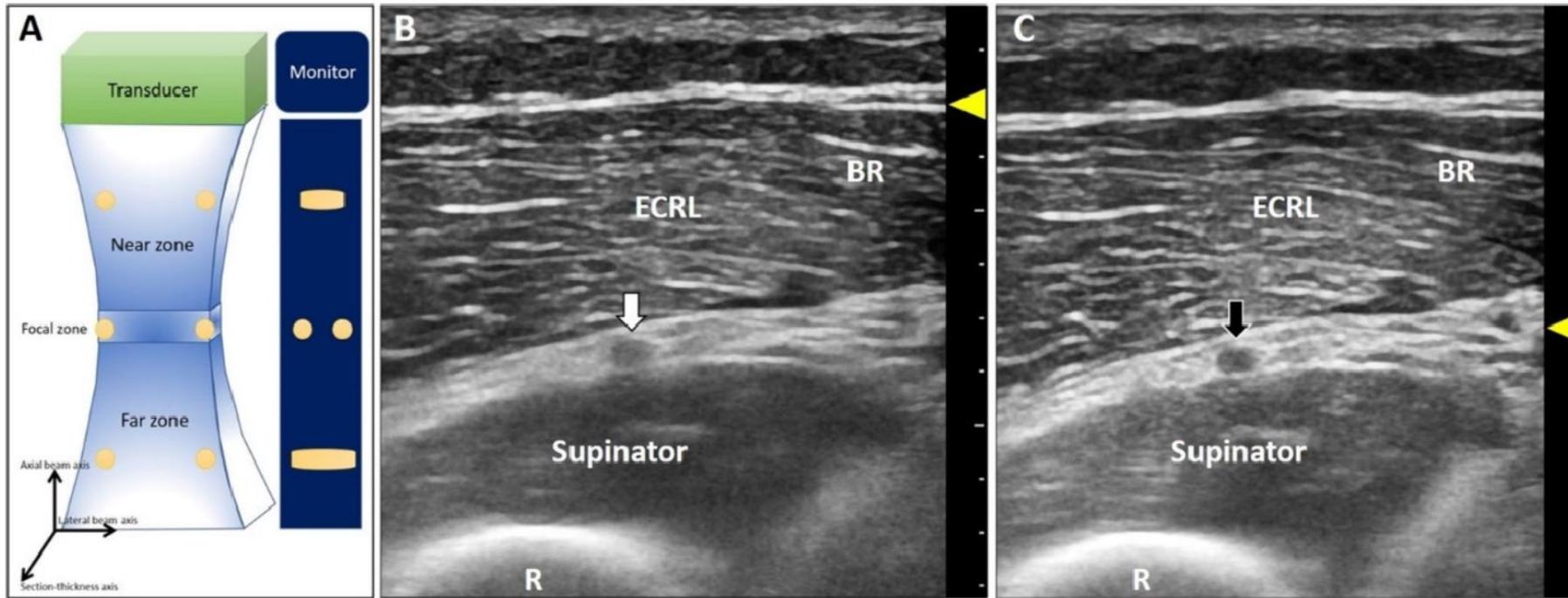
# Аберрации при наличии неоднородностей



**Figure 12.** When the US beam travels through an area with strong impedance, the delayed return of the US signals to the transducer would lead to overestimation of the depth of the object. In contrast, if the target is located in the area of low acoustic impedance, the object shown on the monitor would appear shallower than its actual depth (A). During deep peroneal (fibular) nerve block; as the propagation speed of sound waves in the muscle is faster than that in the fat, the needle shaft in the fat pad will be seen bended toward the tibia bone (B). White arrow: refraction artifact; asterisks: injectate. AT: tibialis anterior muscle; EHL: extensor hallucis longus muscle; EDL: extensor digitorum longus muscle; N: deep peroneal (fibular) nerve. Blue arrows: the projected and reflected US beams.

W.T. Wu, K.V. Chang, Y. C. Hsu, P.S. Hsu, V. Ricci, L. Ozcakar,  
Artifacts in Musculoskeletal Ultrasonography: From Physics to Clinics, *Diagnostics (Basel)*, 10 (9), (2020).

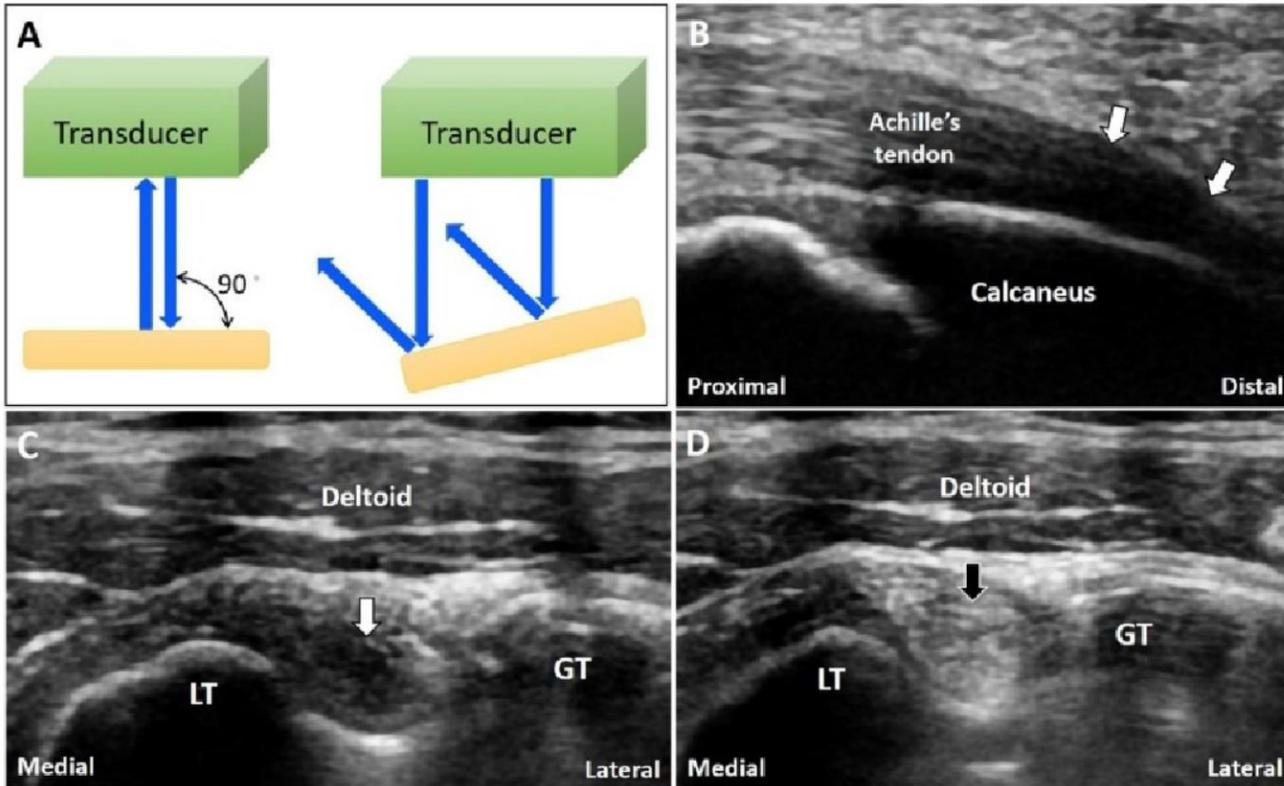
# Аберрации при наличии неоднородностей



**Figure 2.** The US beam width is the narrowest at the focal zone with the best lateral resolution of two objects perpendicular to the beam (A). The deep radial nerve is blurred at the far zone (B) and becomes clearer when the focus has been set at the same depth as the nerve (C). White arrow: the blurred image of the deep radial nerve due to the improper location of the focal zone; black arrow: the clear image of the deep radial nerve after adjustment of the focal zone; yellow arrowheads: focal zone. ECRL: extensor carpi radialis longus muscle; BR: brachioradialis muscle; R: radius.

W.T. Wu, K.V. Chang, Y. C . Hsu, P.S . Hsu, V. Ricci, L. Ozcakar,  
Artifacts in Musculoskeletal Ultrasonography: From Physics to Clinics, Diagnostics (Basel), 10 (9), (2020).

# Аберрации при наличии неоднородностей



**Figure 10.** The US beam is totally reflected back when the transducer is perpendicular to the target while they are reflected away from the transducer if the object is not perpendicular to the US beam (A). The anisotropy is seen at the insertion of the Achilles tendon (B). A normal biceps long head tendon can be misinterpreted as tendinopathy due to anisotropy (C) and tilting the transducer to make it perpendicular to the bicipital groove can eliminate the artifact (D). White arrow: anisotropy artifact; black arrow: image after compensation. GT: greater tubercle; LT: lesser tubercle. Blue arrows: the projected and reflected US beams.

W.T. Wu, K.V. Chang, Y. C . Hsu, P.S . Hsu, V. Ricci, L. Ozcakar,  
Artifacts in Musculoskeletal Ultrasonography: From Physics to Clinics, *Diagnostics (Basel)*, 10 (9), (2020).

# Система уравнений для вязкоупругой среды

$$\frac{\partial v_x}{\partial t} = \frac{1}{\rho} \left( \frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{xy}}{\partial y} + \frac{\partial \sigma_{xz}}{\partial z} \right)$$

$$\frac{\partial v_y}{\partial t} = \frac{1}{\rho} \left( \frac{\partial \sigma_{xy}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} + \frac{\partial \sigma_{yz}}{\partial z} \right)$$

$$\frac{\partial v_z}{\partial t} = \frac{1}{\rho} \left( \frac{\partial \sigma_{xz}}{\partial x} + \frac{\partial \sigma_{yz}}{\partial y} + \frac{\partial \sigma_{zz}}{\partial z} \right)$$

$$\frac{\partial \sigma_{xx}}{\partial t} = c_{11} \frac{\partial v_x}{\partial x} + c_{12} \frac{\partial v_y}{\partial y} + c_{13} \frac{\partial v_z}{\partial z} + c_{14} \left( \frac{\partial v_z}{\partial y} + \frac{\partial v_y}{\partial z} \right) + c_{15} \left( \frac{\partial v_z}{\partial x} + \frac{\partial v_x}{\partial z} \right) + c_{16} \left( \frac{\partial v_y}{\partial x} + \frac{\partial v_x}{\partial y} \right) - \frac{\sigma_{xx}}{\tau_0}$$

$$\frac{\partial \sigma_{yy}}{\partial t} = c_{12} \frac{\partial v_x}{\partial x} + c_{22} \frac{\partial v_y}{\partial y} + c_{23} \frac{\partial v_z}{\partial z} + c_{24} \left( \frac{\partial v_z}{\partial y} + \frac{\partial v_y}{\partial z} \right) + c_{25} \left( \frac{\partial v_z}{\partial x} + \frac{\partial v_x}{\partial z} \right) + c_{26} \left( \frac{\partial v_y}{\partial x} + \frac{\partial v_x}{\partial y} \right) - \frac{\sigma_{yy}}{\tau_0}$$

$$\frac{\partial \sigma_{zz}}{\partial t} = c_{13} \frac{\partial v_x}{\partial x} + c_{23} \frac{\partial v_y}{\partial y} + c_{33} \frac{\partial v_z}{\partial z} + c_{34} \left( \frac{\partial v_z}{\partial y} + \frac{\partial v_y}{\partial z} \right) + c_{35} \left( \frac{\partial v_z}{\partial x} + \frac{\partial v_x}{\partial z} \right) + c_{36} \left( \frac{\partial v_y}{\partial x} + \frac{\partial v_x}{\partial y} \right) - \frac{\sigma_{zz}}{\tau_0}$$

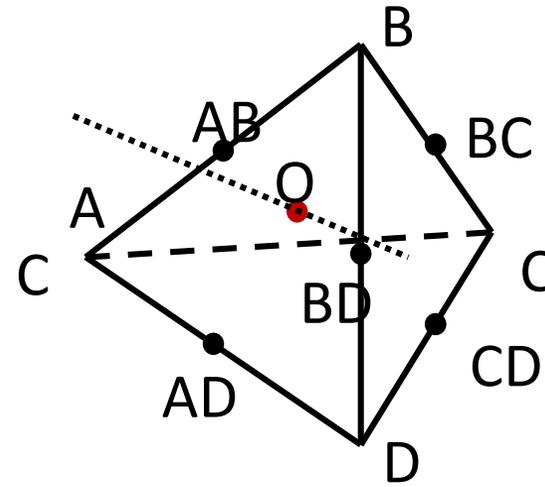
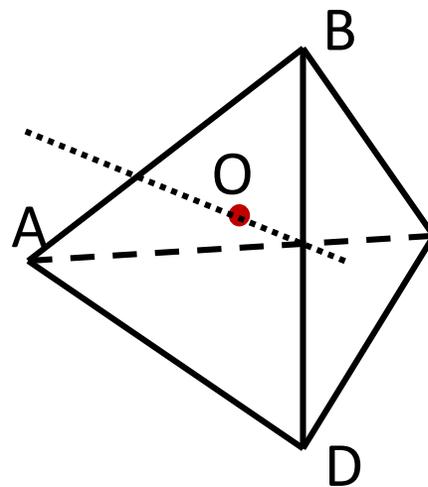
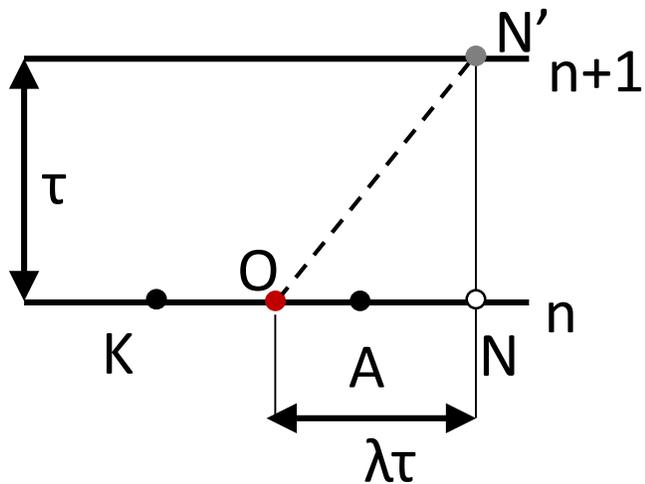
$$\frac{\partial \sigma_{yz}}{\partial t} = c_{14} \frac{\partial v_x}{\partial x} + c_{24} \frac{\partial v_y}{\partial y} + c_{34} \frac{\partial v_z}{\partial z} + c_{44} \left( \frac{\partial v_z}{\partial y} + \frac{\partial v_y}{\partial z} \right) + c_{45} \left( \frac{\partial v_z}{\partial x} + \frac{\partial v_x}{\partial z} \right) + c_{46} \left( \frac{\partial v_y}{\partial x} + \frac{\partial v_x}{\partial y} \right) - \frac{\sigma_{yz}}{\tau_0}$$

$$\frac{\partial \sigma_{xz}}{\partial t} = c_{15} \frac{\partial v_x}{\partial x} + c_{25} \frac{\partial v_y}{\partial y} + c_{35} \frac{\partial v_z}{\partial z} + c_{45} \left( \frac{\partial v_z}{\partial y} + \frac{\partial v_y}{\partial z} \right) + c_{55} \left( \frac{\partial v_z}{\partial x} + \frac{\partial v_x}{\partial z} \right) + c_{56} \left( \frac{\partial v_y}{\partial x} + \frac{\partial v_x}{\partial y} \right) - \frac{\sigma_{xz}}{\tau_0}$$

$$\frac{\partial \sigma_{xy}}{\partial t} = c_{16} \frac{\partial v_x}{\partial x} + c_{26} \frac{\partial v_y}{\partial y} + c_{36} \frac{\partial v_z}{\partial z} + c_{46} \left( \frac{\partial v_z}{\partial y} + \frac{\partial v_y}{\partial z} \right) + c_{56} \left( \frac{\partial v_z}{\partial x} + \frac{\partial v_x}{\partial z} \right) + c_{66} \left( \frac{\partial v_y}{\partial x} + \frac{\partial v_x}{\partial y} \right) - \frac{\sigma_{xy}}{\tau_0}$$

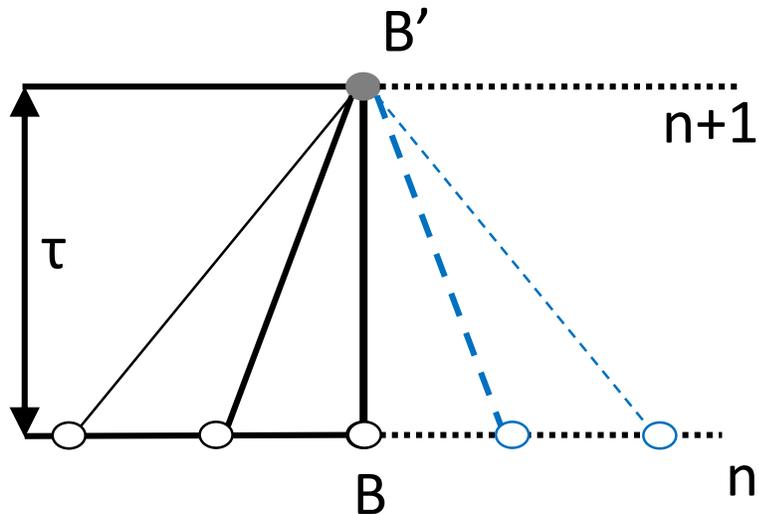
# Сеточно-характеристический метод

- Полная система уравнений МДТТ
- Расщепление по направлениям
- Переход к инвариантам Римана и характеристикам
- Неструктурированные сетки, интерполяция в тетраэдре для переноса значений на новый временной слой
- Получение компонент вектора скорости и тензора напряжений из инвариантов Римана



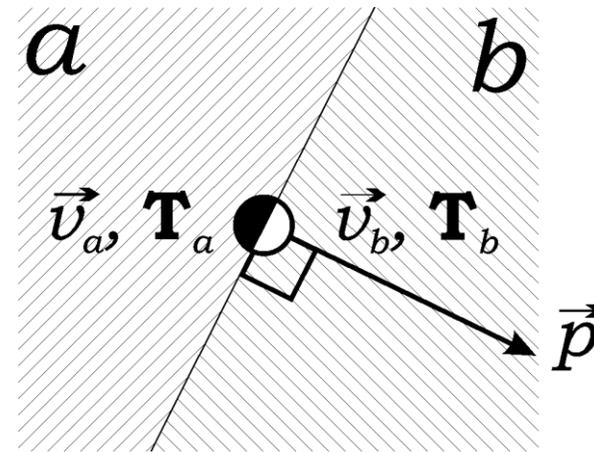
# Граничные и контактные условия

Внешние поверхности



- Заданная внешняя сила  
 $\mathbf{T}\vec{p} = \vec{f}$
- Заданная скорость границы  
 $\vec{v} = \vec{V}$
- Поглощающие  
(неотражающие) условия

Поверхности раздела сред



A – реальный узел  
B – виртуальный узел

- Полное сцепление  
 $\vec{v}_a = \vec{v}_b = \vec{V}, \quad \vec{f}_a = -\vec{f}_b$
- Свободное скольжение
- Скольжение с трением
- Разрушаемый контакт

# Оценка поглощенной энергии

$$\frac{\partial \vec{u}}{\partial t} + \mathbf{A}_x \frac{\partial \vec{u}}{\partial x} + \mathbf{A}_y \frac{\partial \vec{u}}{\partial y} + \mathbf{A}_z \frac{\partial \vec{u}}{\partial z} + \vec{f} = 0$$

$$\vec{f} = \left\{ 0, 0, 0, -\frac{\sigma_{xx}}{\tau_0}, -\frac{\sigma_{yy}}{\tau_0}, -\frac{\sigma_{zz}}{\tau_0}, -\frac{\sigma_{yz}}{\tau_0}, -\frac{\sigma_{xz}}{\tau_0}, -\frac{\sigma_{xy}}{\tau_0} \right\}^T$$

$$\vec{u}^{n+1} = F(\mathbf{A}_x, \mathbf{A}_y, \mathbf{A}_z, \vec{f}) \vec{u}^n \longrightarrow \vec{u}^{n+1} = F_r(\vec{f}) F_a(\mathbf{A}_a) F_b(\mathbf{A}_b) F_c(\mathbf{A}_c) \vec{u}^n$$

$$\vec{u}^c = F_c(\mathbf{A}_c) \vec{u}^n,$$

$$\vec{u}^{bc} = F_b(\mathbf{A}_b) \vec{u}^c,$$

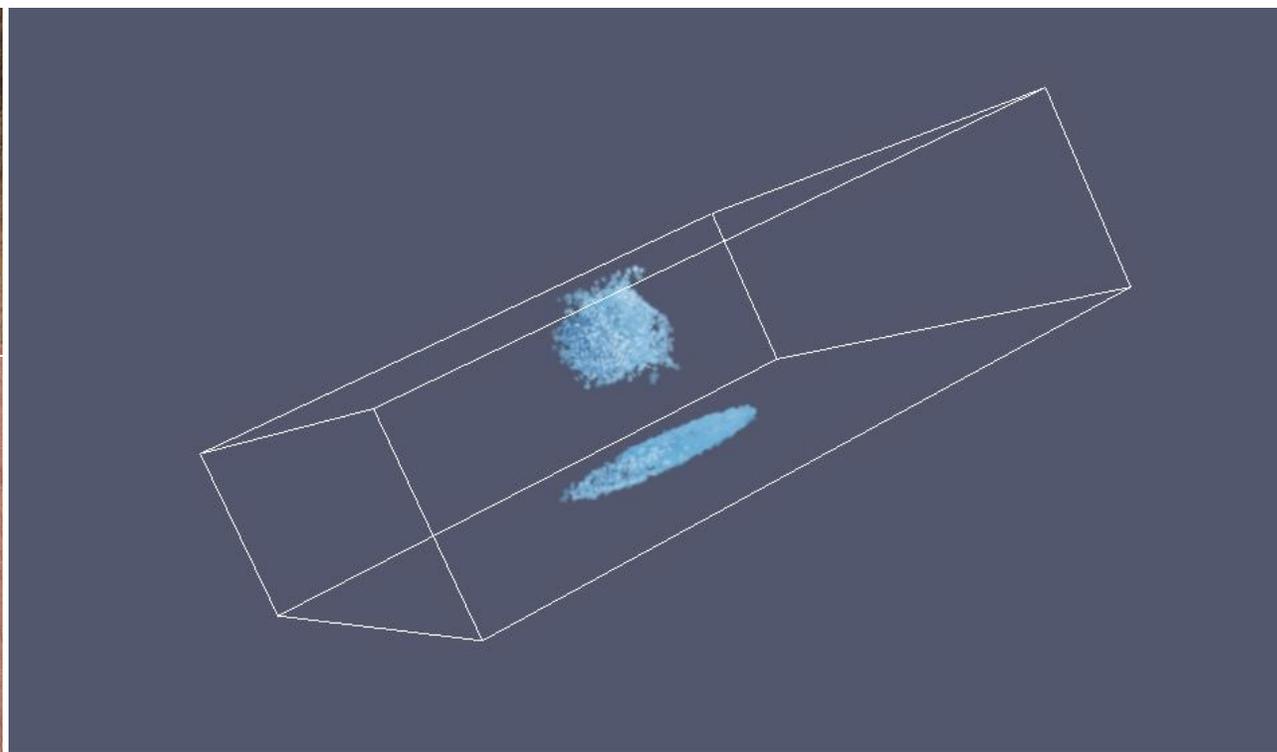
$$\vec{u}^{abc} = F_a(\mathbf{A}_a) \vec{u}^{bc}.$$

$$\vec{u}^{n+1} = F_r(\vec{f}) \vec{u}^{abc}$$

$$\begin{aligned} \delta E = & \frac{(\sigma_{xx}^{abc})^2 - (\sigma_{xx}^{n+1})^2}{2Y} + \frac{(\sigma_{yy}^{abc})^2 - (\sigma_{yy}^{n+1})^2}{2Y} + \frac{(\sigma_{zz}^{abc})^2 - (\sigma_{zz}^{n+1})^2}{2Y} \\ & + \frac{(\sigma_{xy}^{abc})^2 - (\sigma_{xy}^{n+1})^2}{2\mu} + \frac{(\sigma_{xz}^{abc})^2 - (\sigma_{xz}^{n+1})^2}{2\mu} + \frac{(\sigma_{yz}^{abc})^2 - (\sigma_{yz}^{n+1})^2}{2\mu} \end{aligned}$$

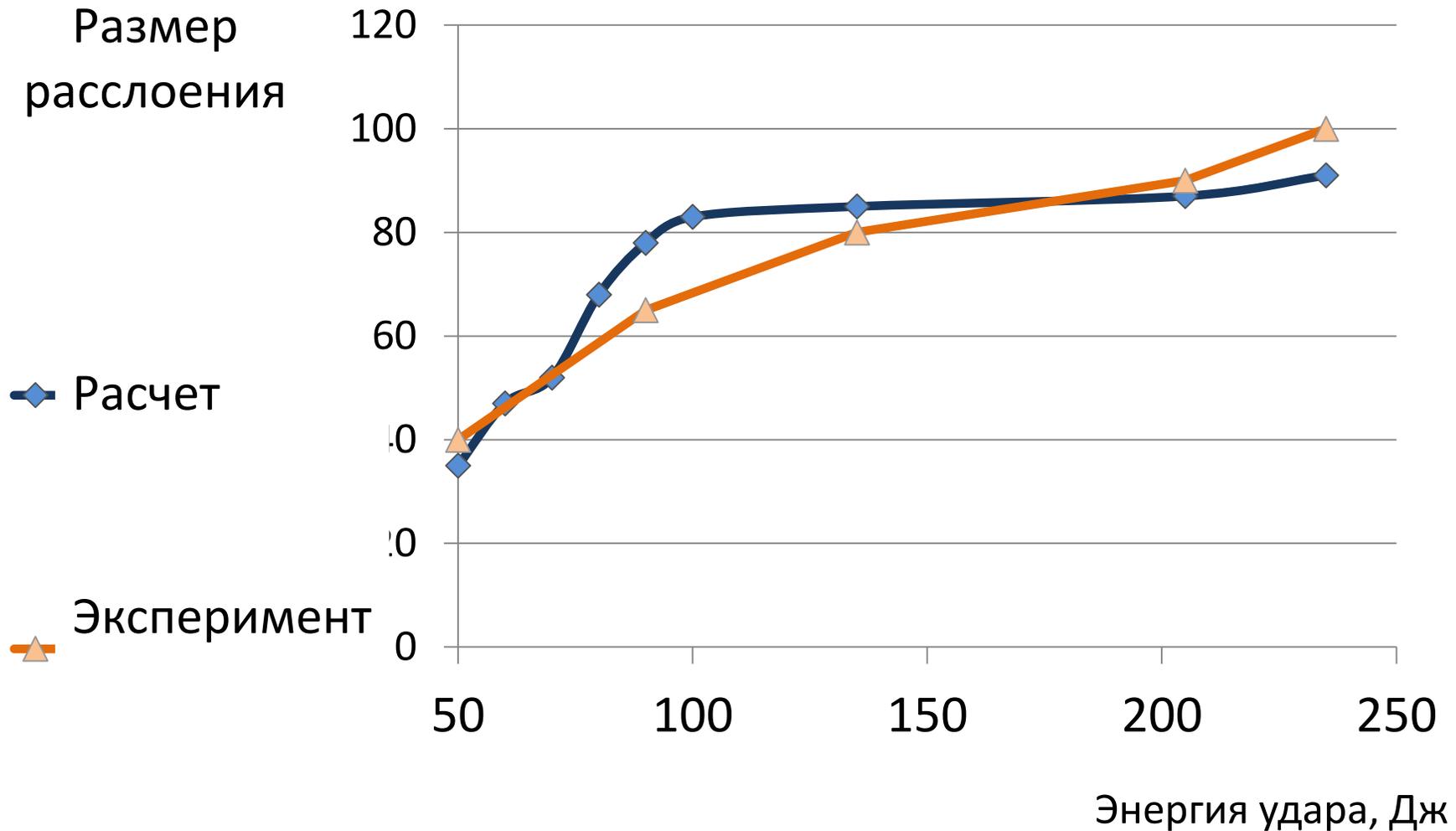
# Верификация: короткий лазерный импульс

New method of the polymeric material properties experimental investigation under powerful energy flux impact / B.A. Demidov, V.P. Efremov, Yu.G. Kalinin et al. // Proceedings of XXX International Conference on Interaction of Intense Energy Fluxes with Matter. – Journal of Physics: Conference Series (JPCS), 2015.



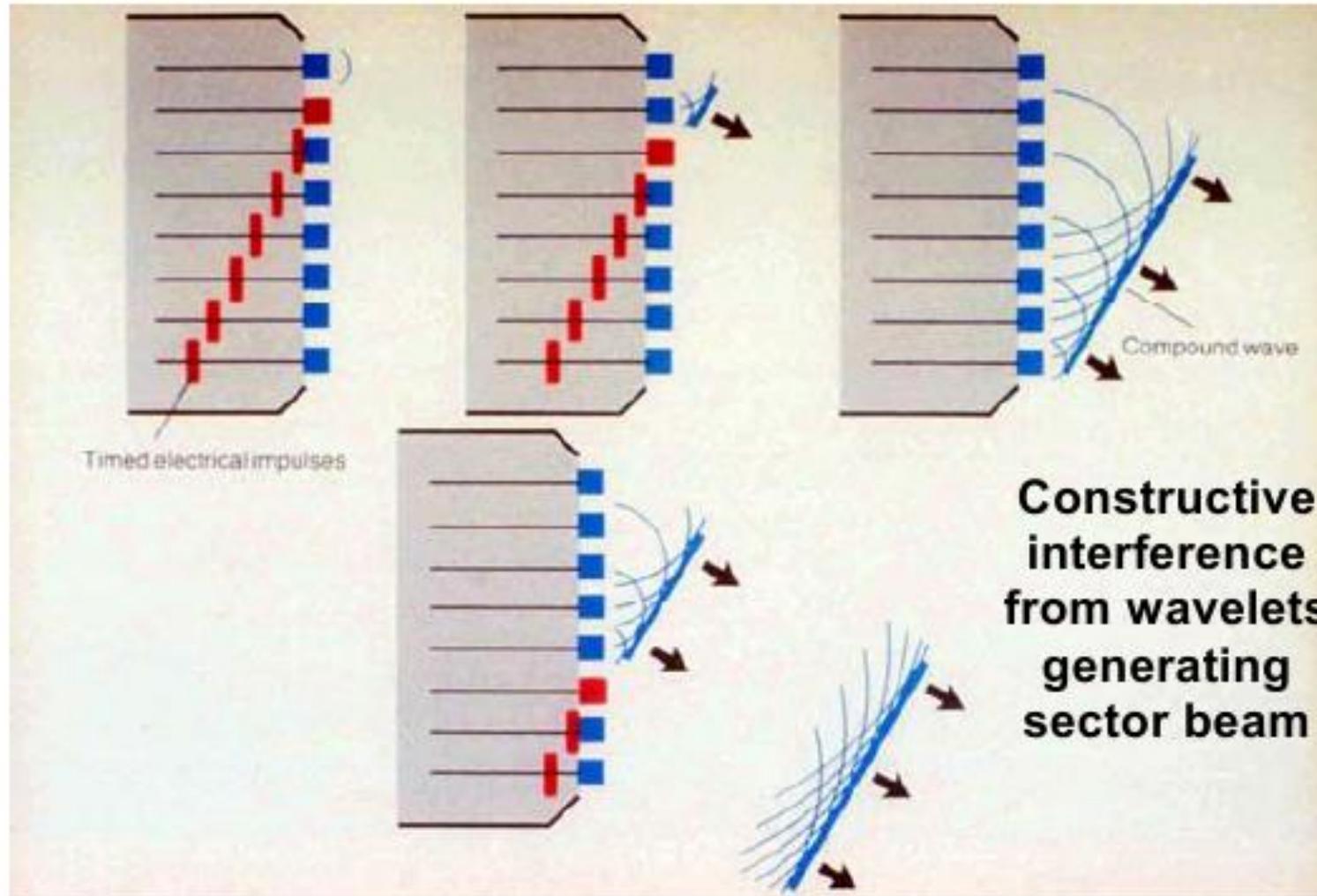
# Верификация: авиационный композит

## сравнение с экспериментом ЦАГИ

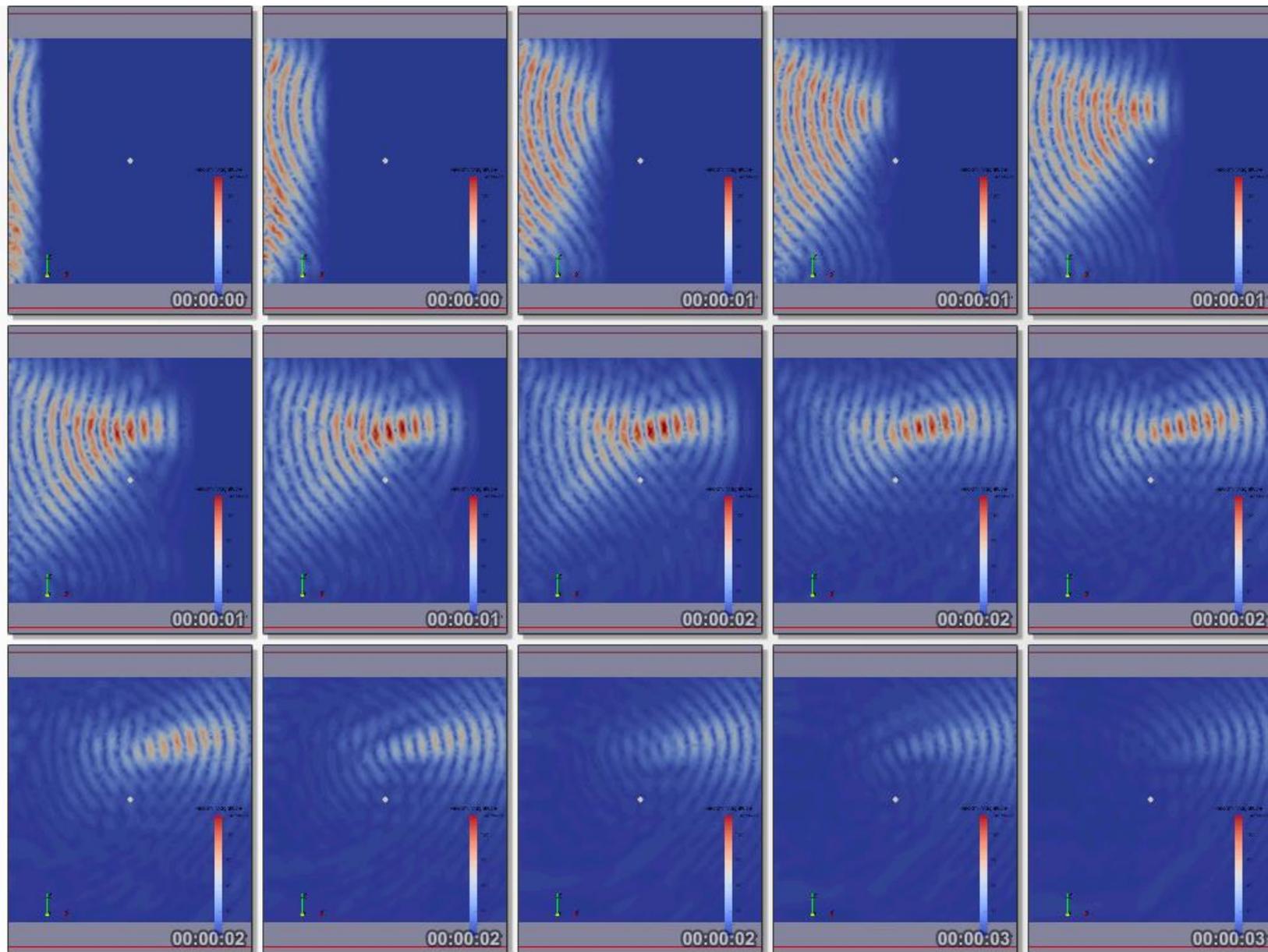


# Фазированная решетка: общий принцип

Each waveform merges to form a compound wave, generating a sector beam.



# Фазированная решетка: расчет



# Формирование В-скана: постановка задачи

15x15x15 мм

Линейная фазовая решетка из 20 элементов.

Фокусное расстояние 10 мм.

Максимальная амплитуда сигнала 800 Па.

Для «медленного» материала параметры Ламе уменьшены на 20%.

$$\lambda = 12.0 \text{ MPa}$$

$$\mu = 5.0 \text{ MPa}$$

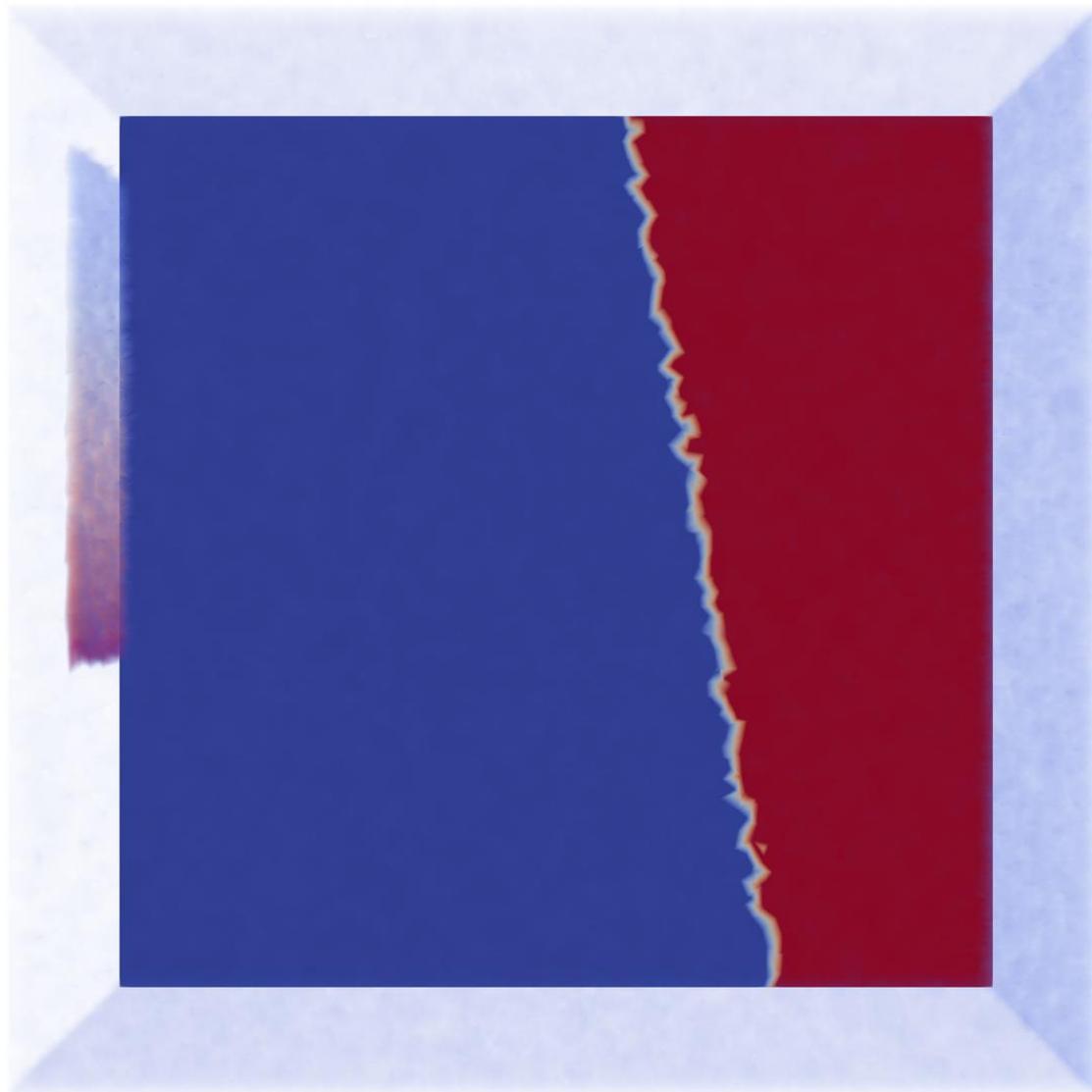
$$\rho = 0.45 \text{ g/mm}^3$$

$$\tau_0 = 1.0 \text{ s}$$

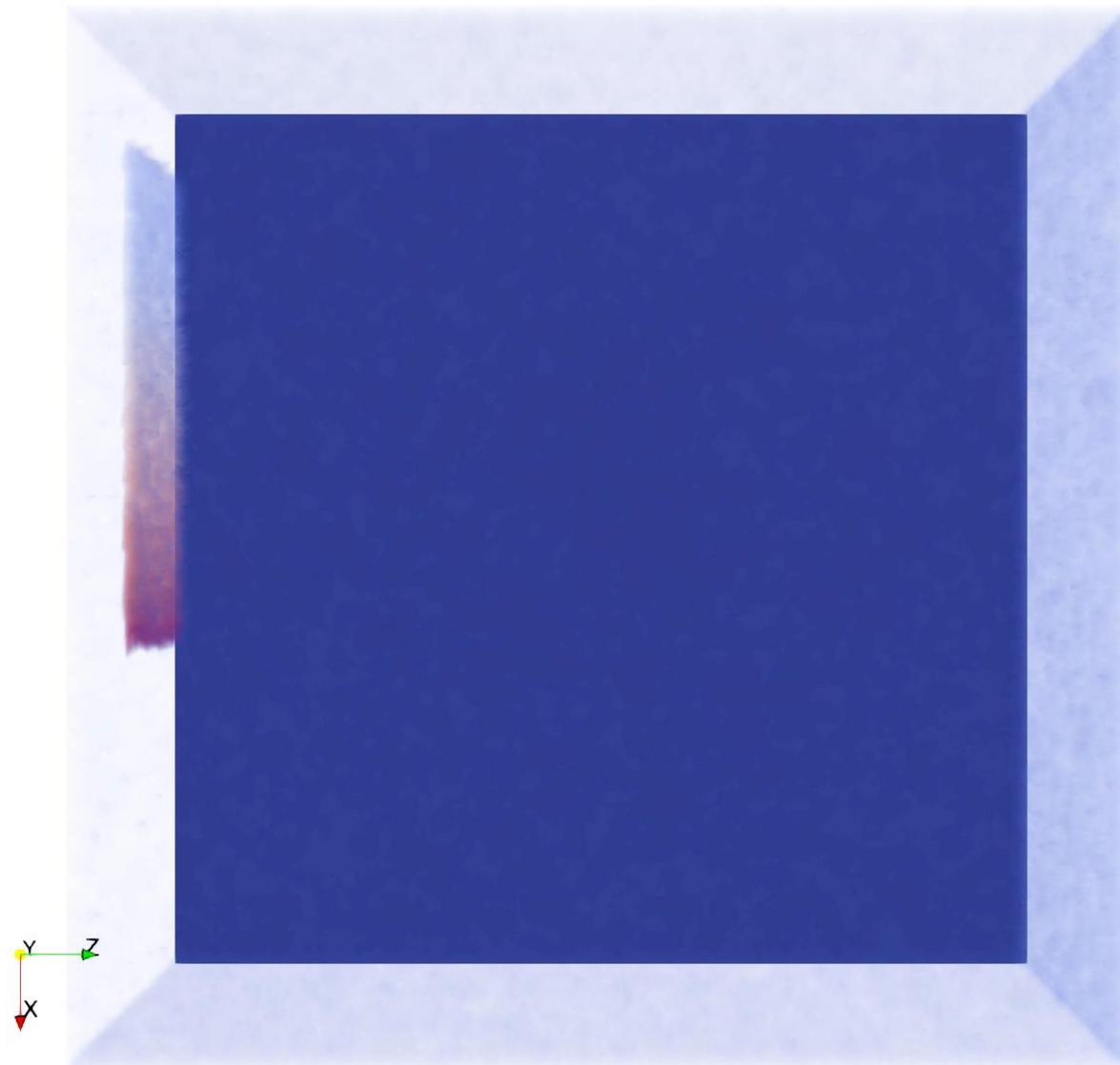
$$\sigma_{zz}^{border} = P \cdot e^{-(t-t_0)^2/2t_{pulse}^2} \cdot \cos(\omega(t-t_0)), t > t_0$$

$$\sigma_{zz}^{border} = 0, t < t_0,$$

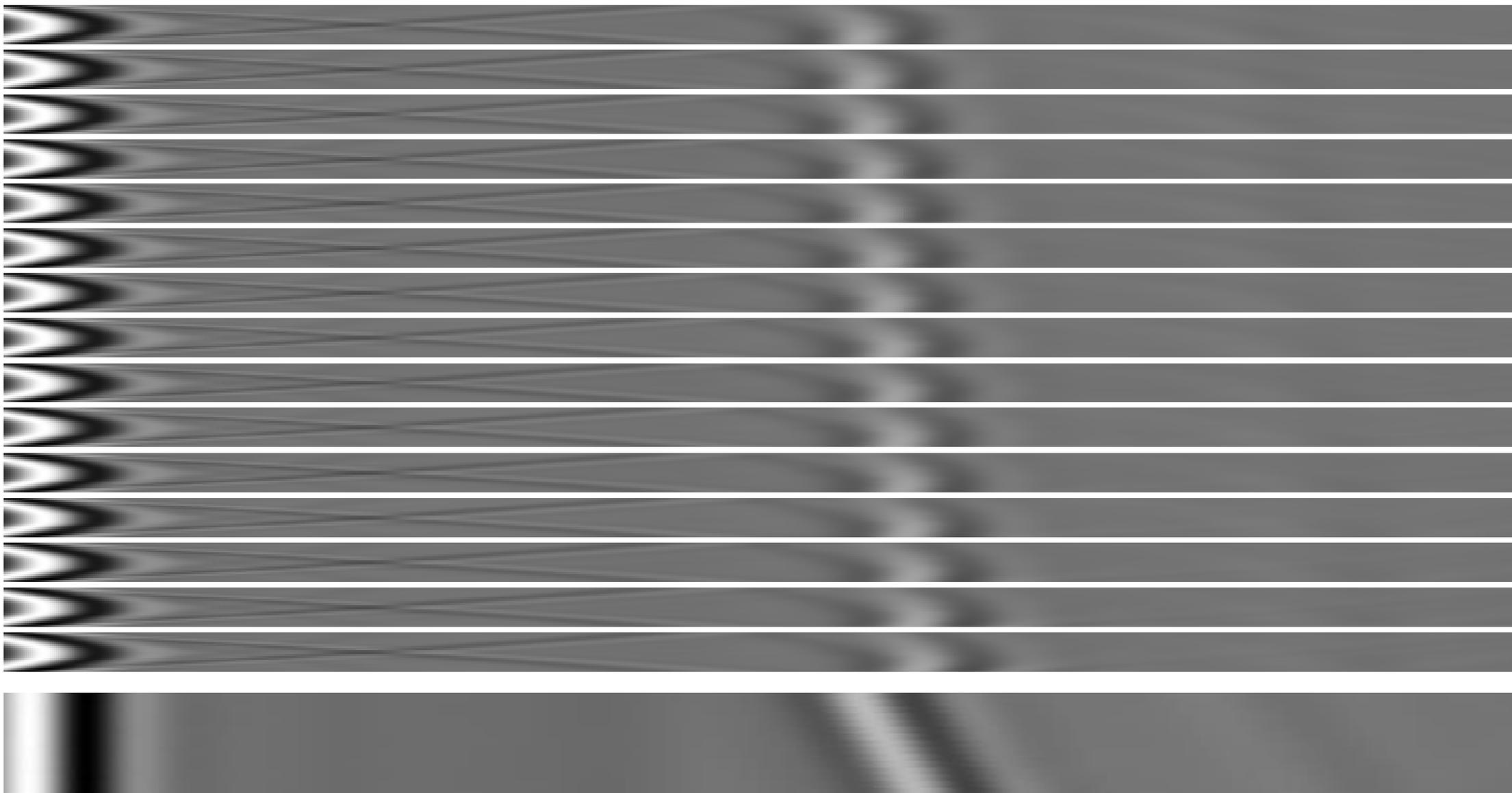
# Формирование В-скана: наклонная поверхность



# Формирование В-скана: наклонная поверхность



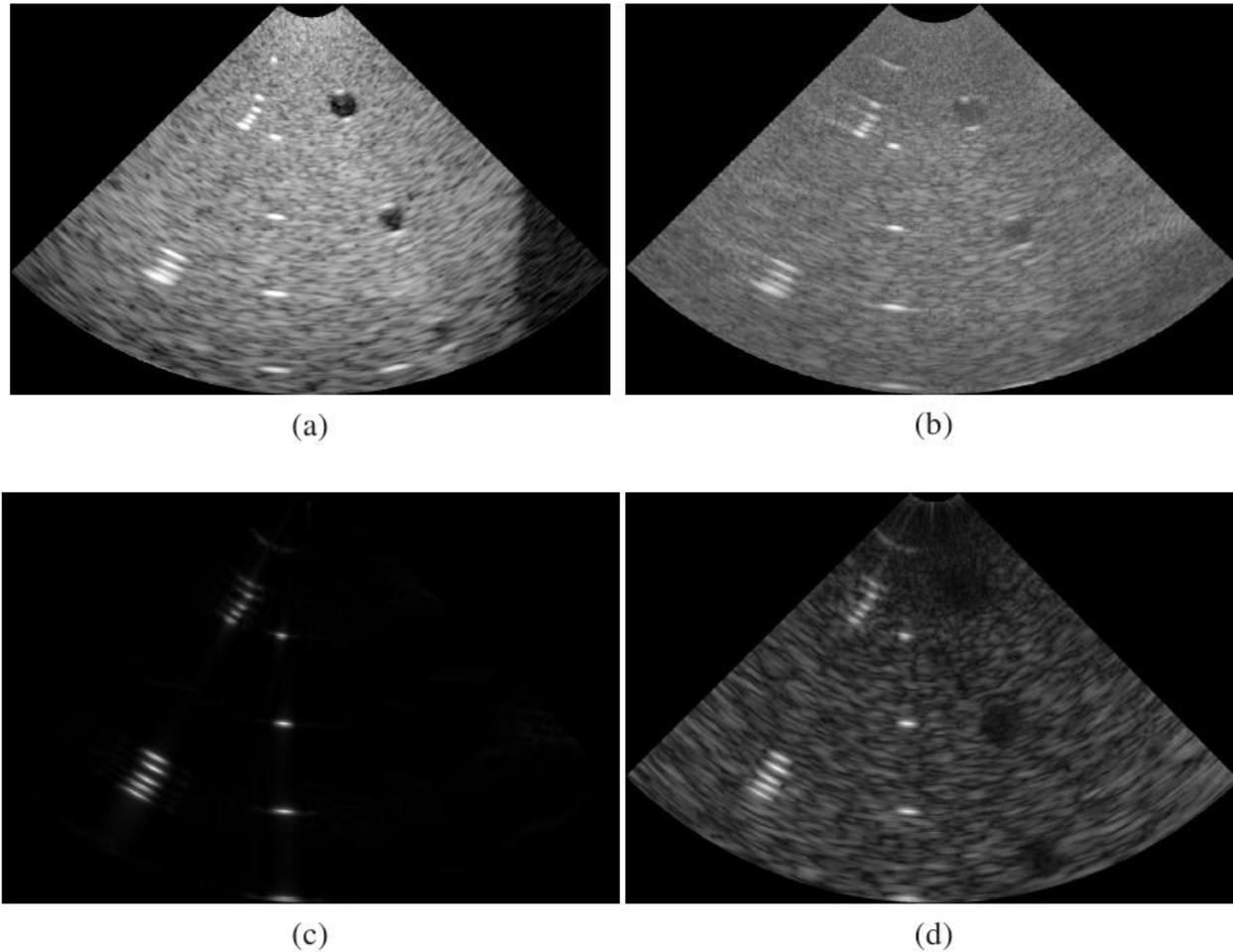
# Формирование В-скана: наклонная поверхность



# Формирование В-скана: гипоэхогенность наклонной поверхности

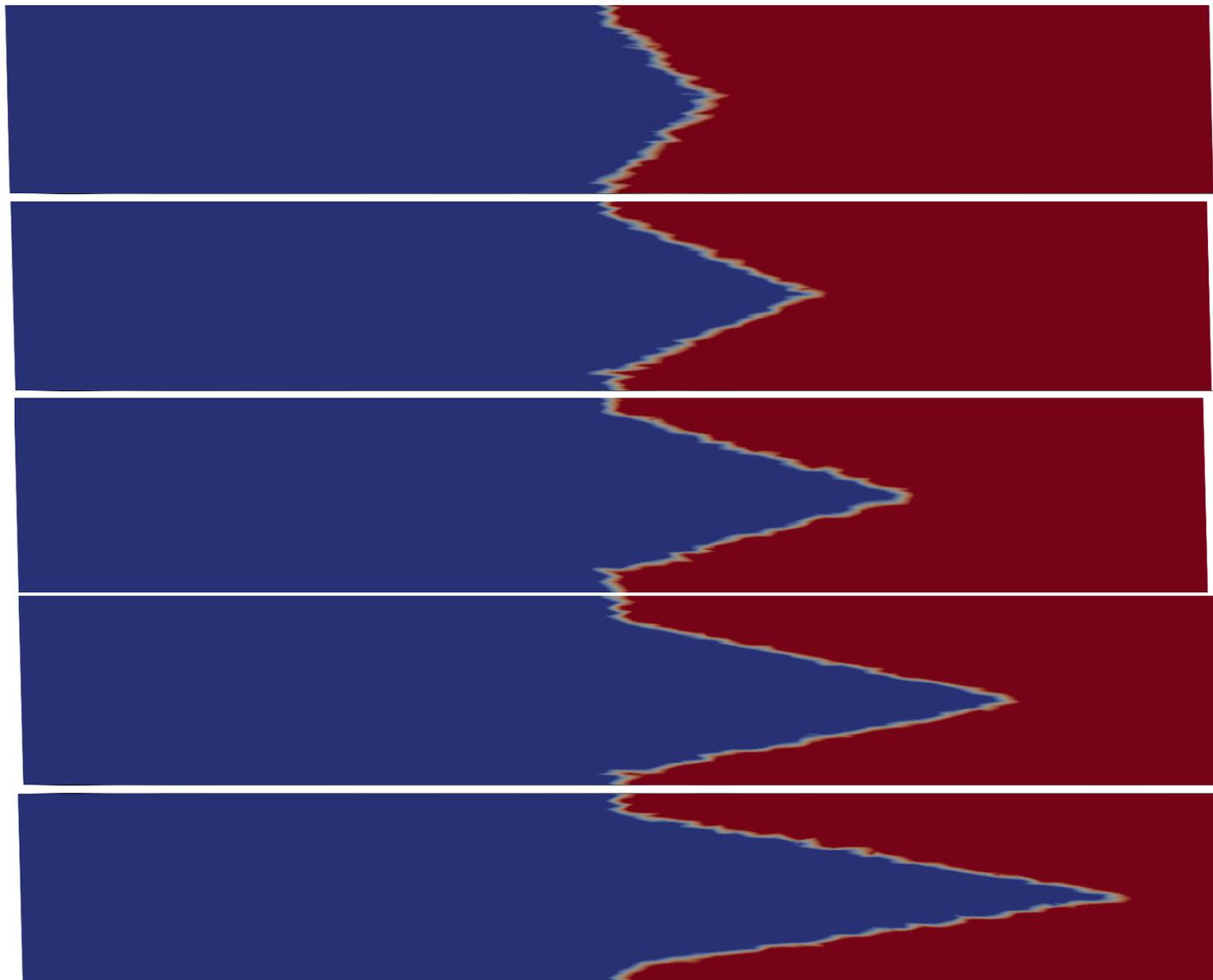


# Верификация алгоритма обработки сигнала



**Figure 8.** B-scan reconstructed on the ultrasound equipment (a), B-scan reconstructed by the numerical postprocessing of the experimental raw data (b), B-scan reconstructed by the numerical postprocessing of the simulated raw data which account bright pins only (c), B-scan reconstructed by the numerical postprocessing of the simulated raw data which account both pins and background reflectors in the gel (d).

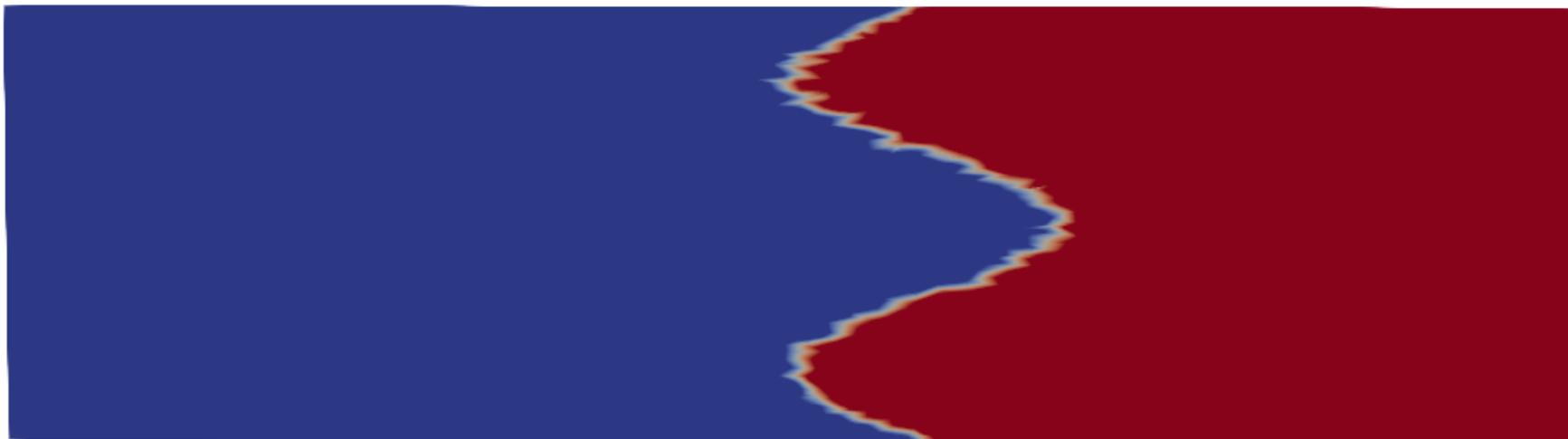
# Формирование В-скана: «уголок», геометрия в расчете



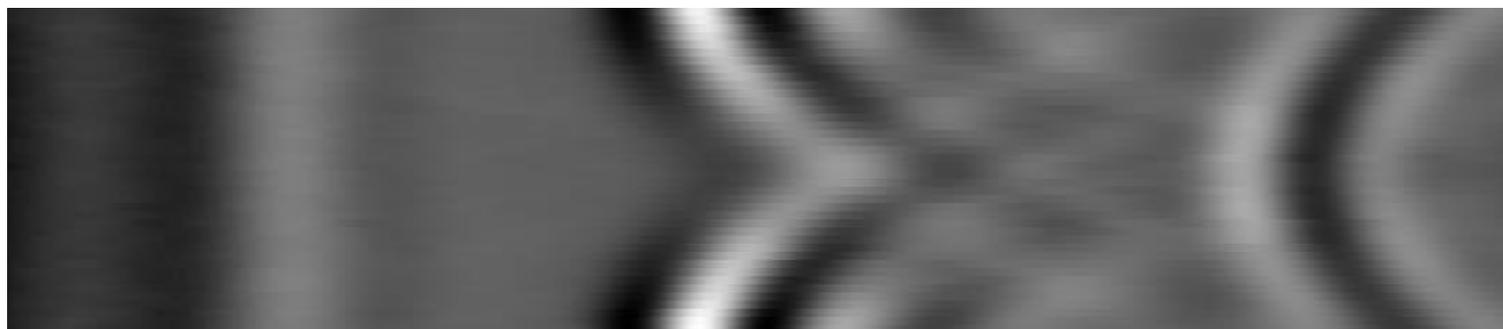
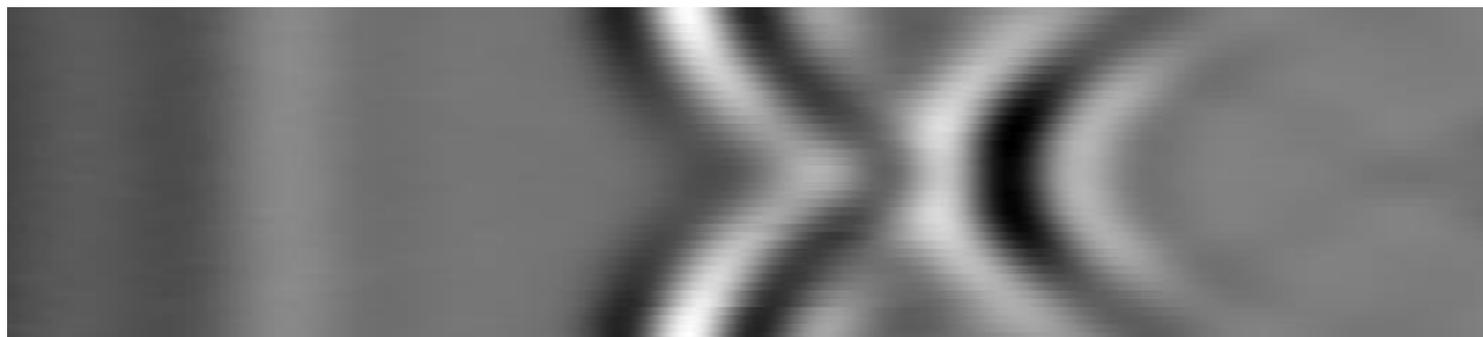
# Формирование В-скана: «уголок», расчетный В-скан



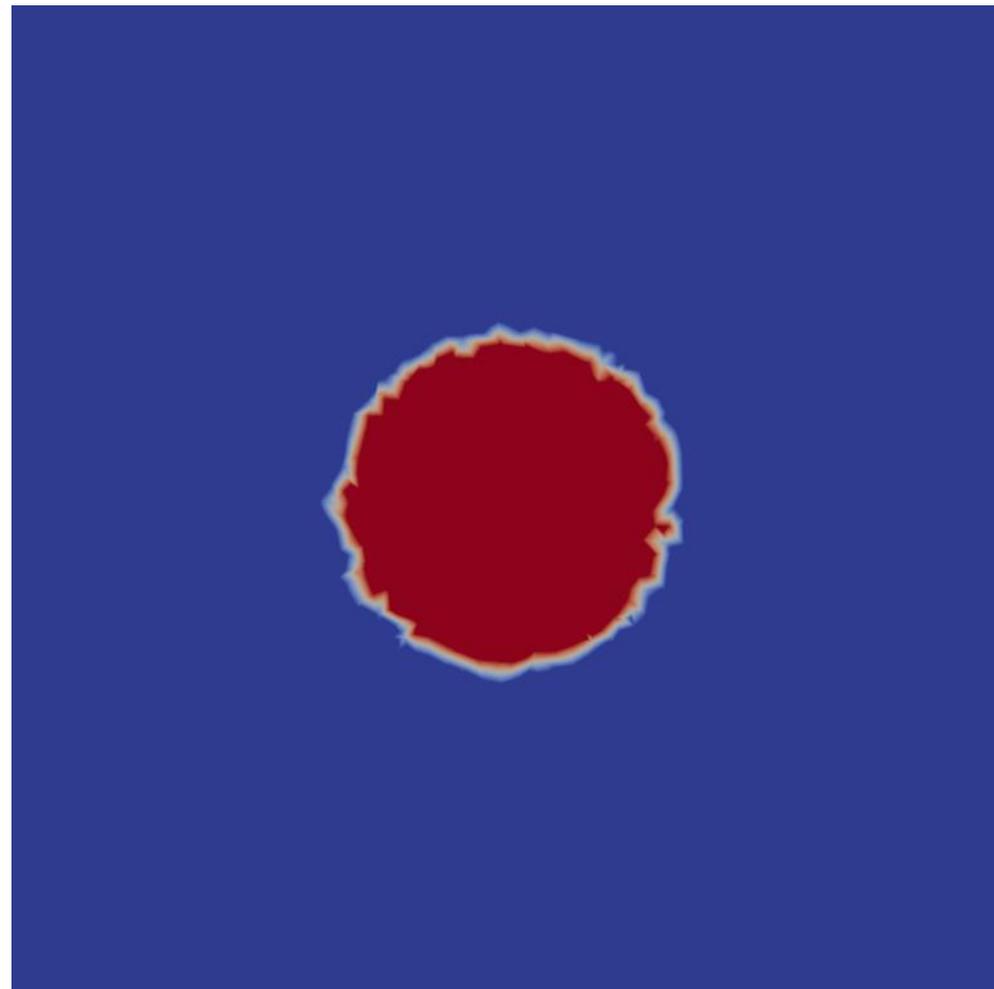
# Расчетная область: синусоидальная выемка



# Формирование В-скана: синусоидальная выемка



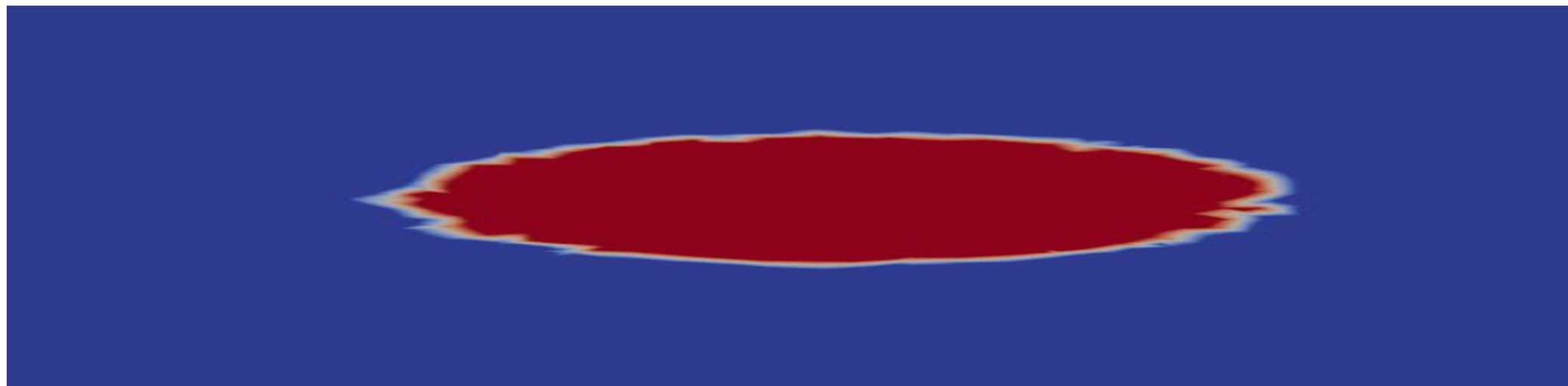
Расчётная область: цилиндр



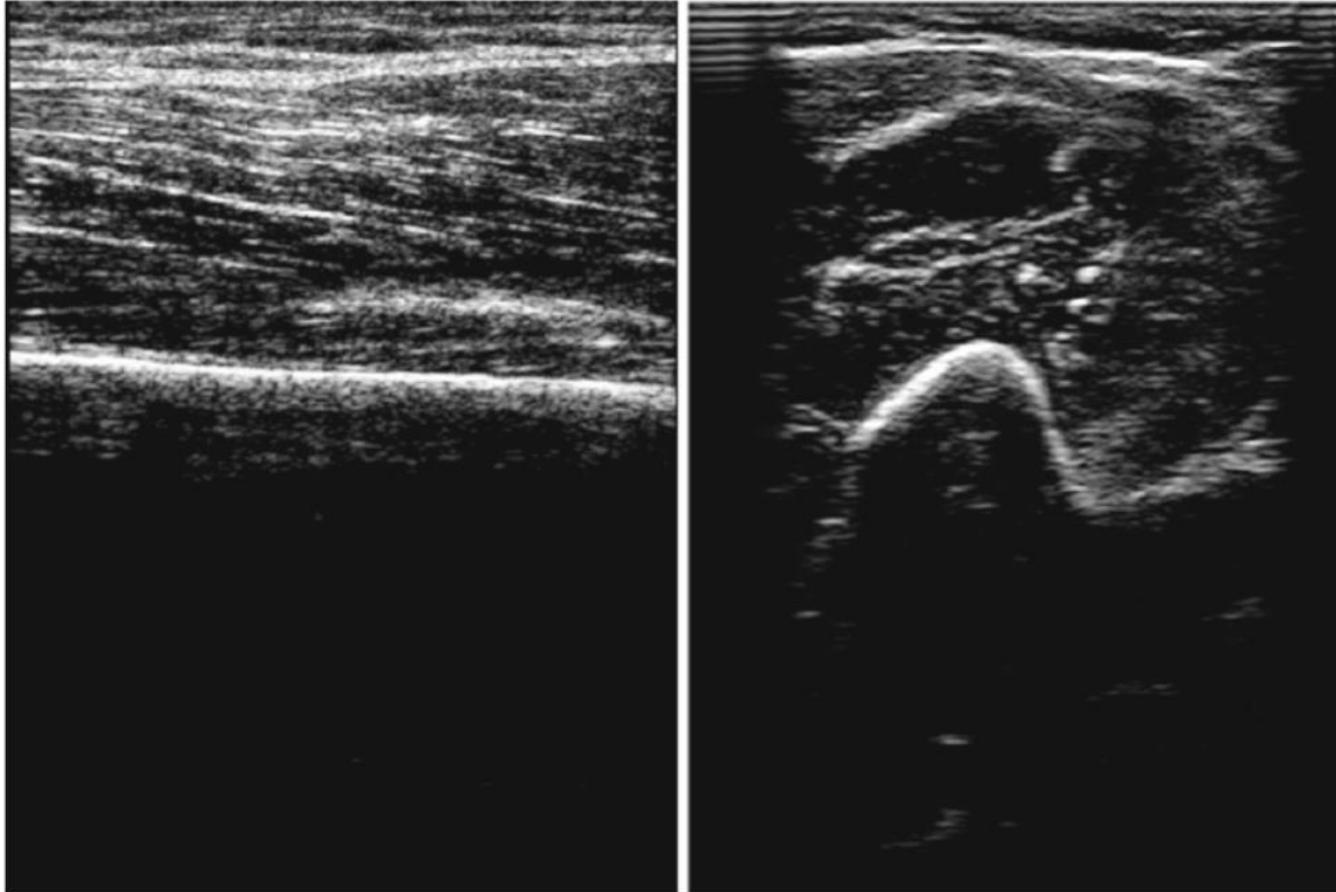
# Формирование В-скана: цилиндр



Расчетная область: цилиндр



# В-скан кости

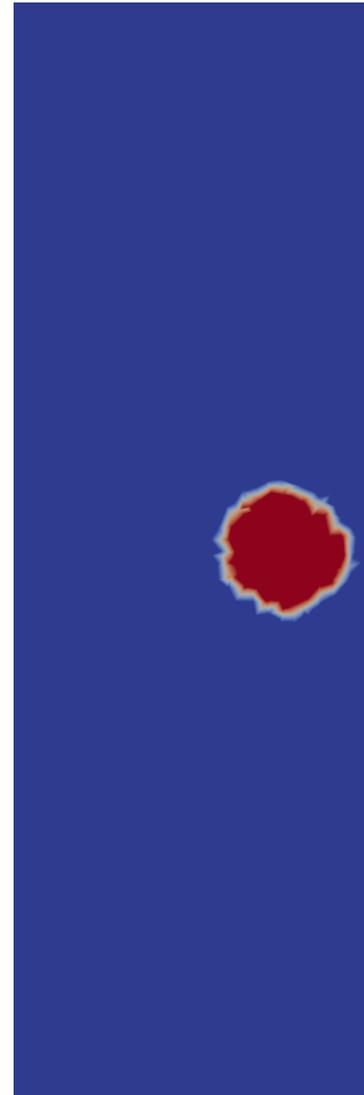


**Fig. 1** Example of a longitudinal (left) and an axial (right) view of the bone in the US image

M. Villa, G. Dardenne, M. Nasan, H. Letissier, C. Hamitouche and E. Stindel, FCN-based approach for the automatic segmentation of bone surfaces in ultrasound images, International Journal of Computer Assisted Radiology and Surgery, (2018).

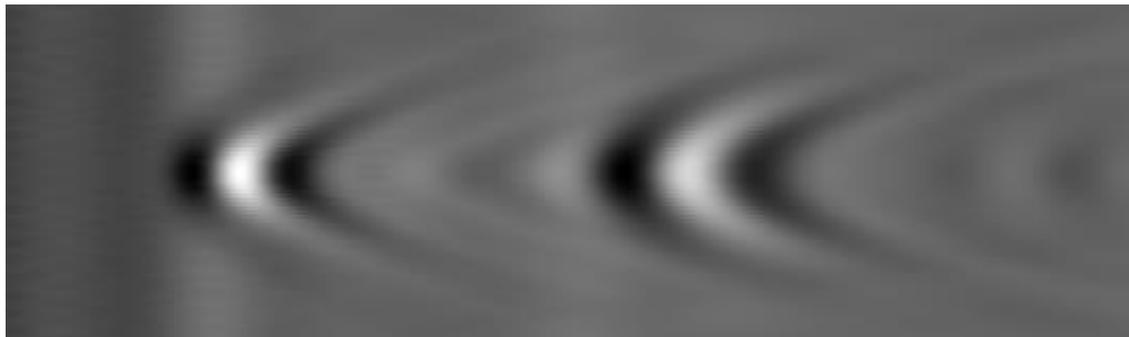
# Расчётная область: цилиндр вблизи решётки

Варьируется размер  
решётки: от  $\sim 1/6$  до  $\sim 1/2$   
размера расчётной  
области

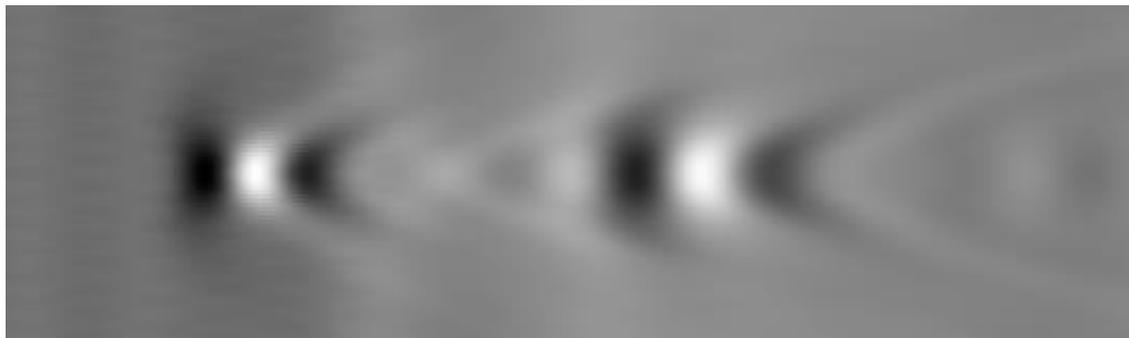


# Формирование В-сканов

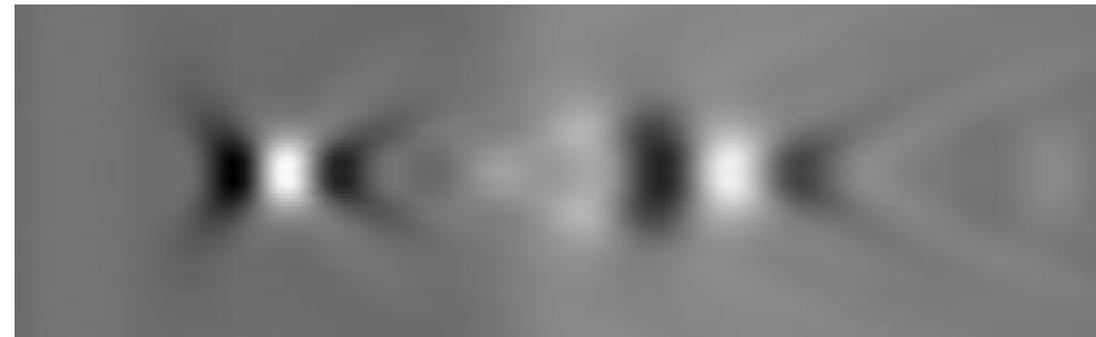
1/6 расчётной  
области



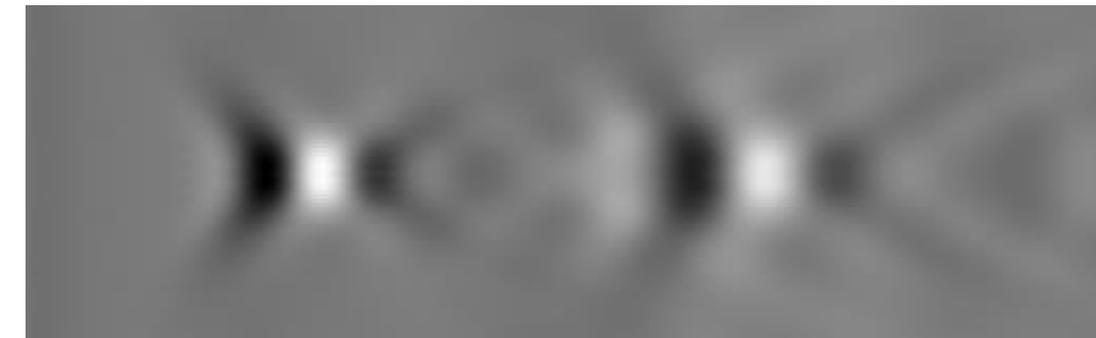
1/3 расчётной  
области



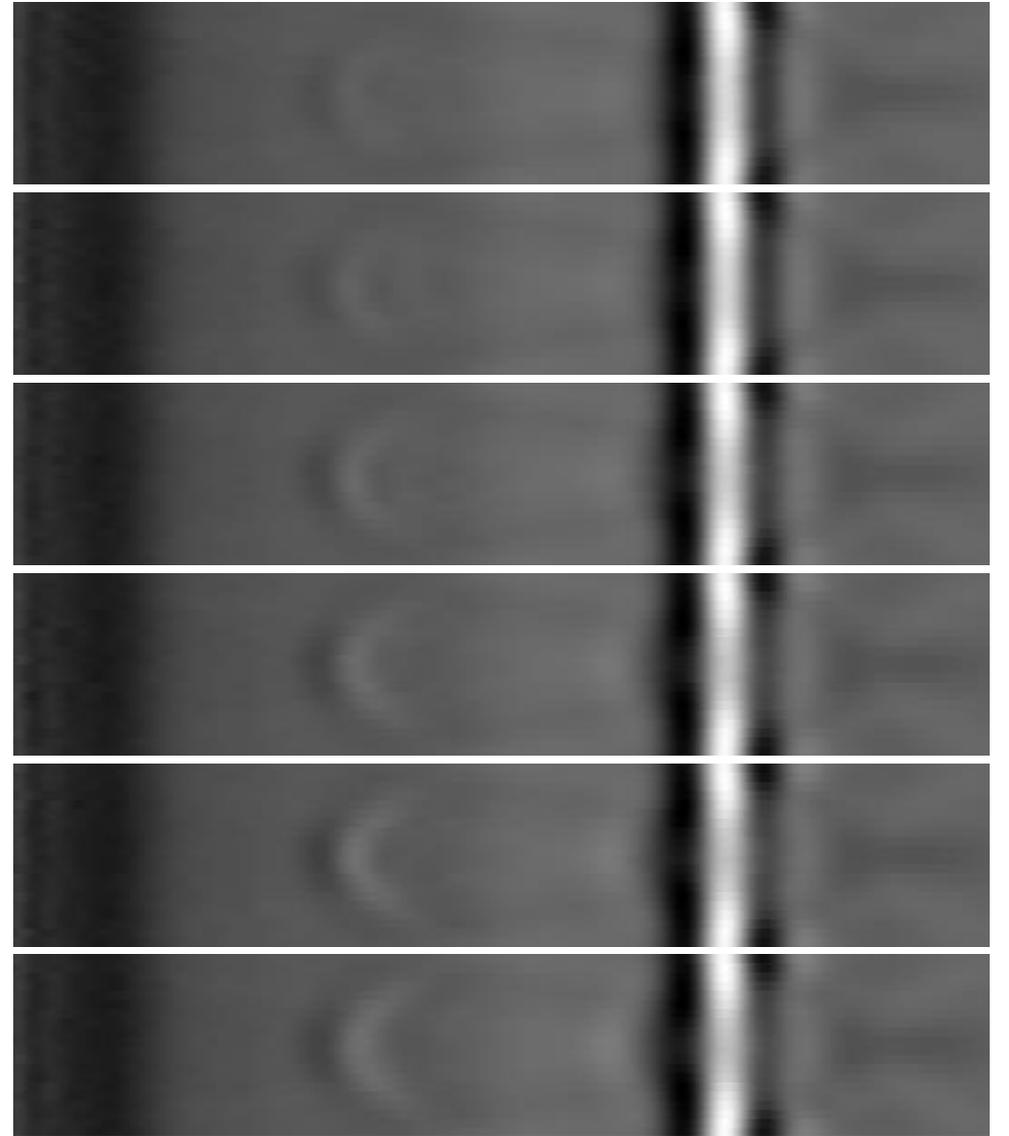
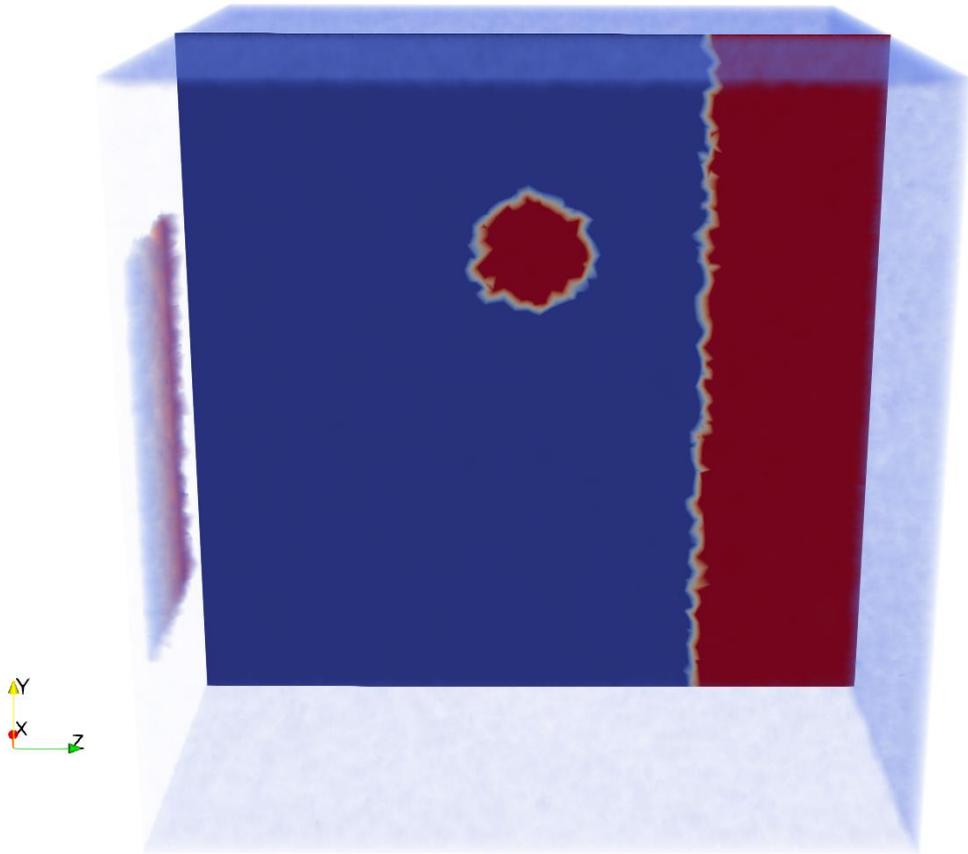
1/4 расчётной  
области



1/2 расчётной  
области



# Формирование В-скана: шарик вне плоскости сканирования



# Заключение

- Система уравнений для трехмерной **вязкоупругой среды** с учетом поглощения энергии.
- Численное моделирование при помощи **сеточно-характеристического метода** на неструктурированных сетках.
- Алгоритм формирования В-скана, аналогичный используемому в медицинских приборах и верифицированный на качественном уровне .
- Комплекс программ на основе СХМ верифицирован на аналитическом решении модельных задач и экспериментам по ультразвуку авиационных композитов.
- Алгоритм построения В-скана, соответствующий реальному устройству.
  
- Получены картины распределения напряжений и мгновенных скоростей при формировании В-скана. Каждый луч – отдельный расчет, влияние предыдущих лучей на последующие не учитывалось.
- Получен вид В-сканов для различной формы поверхности раздела двух сред.
- Воспроизведены **артефакты**, наблюдающиеся на В-сканах в эксперименте:
  - гипоэхогенный отклик от наклонной поверхности;
  - фантомный объект.