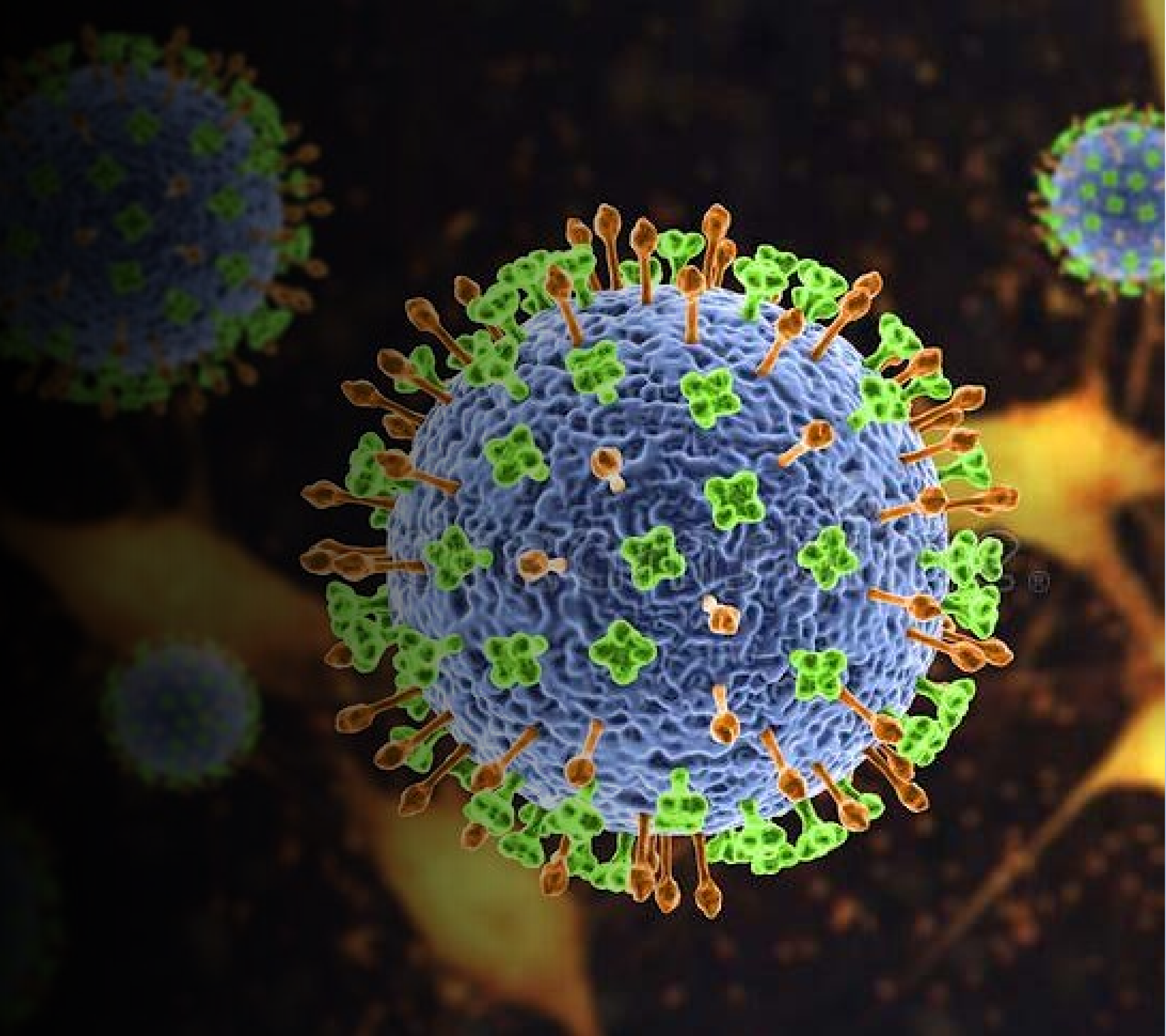
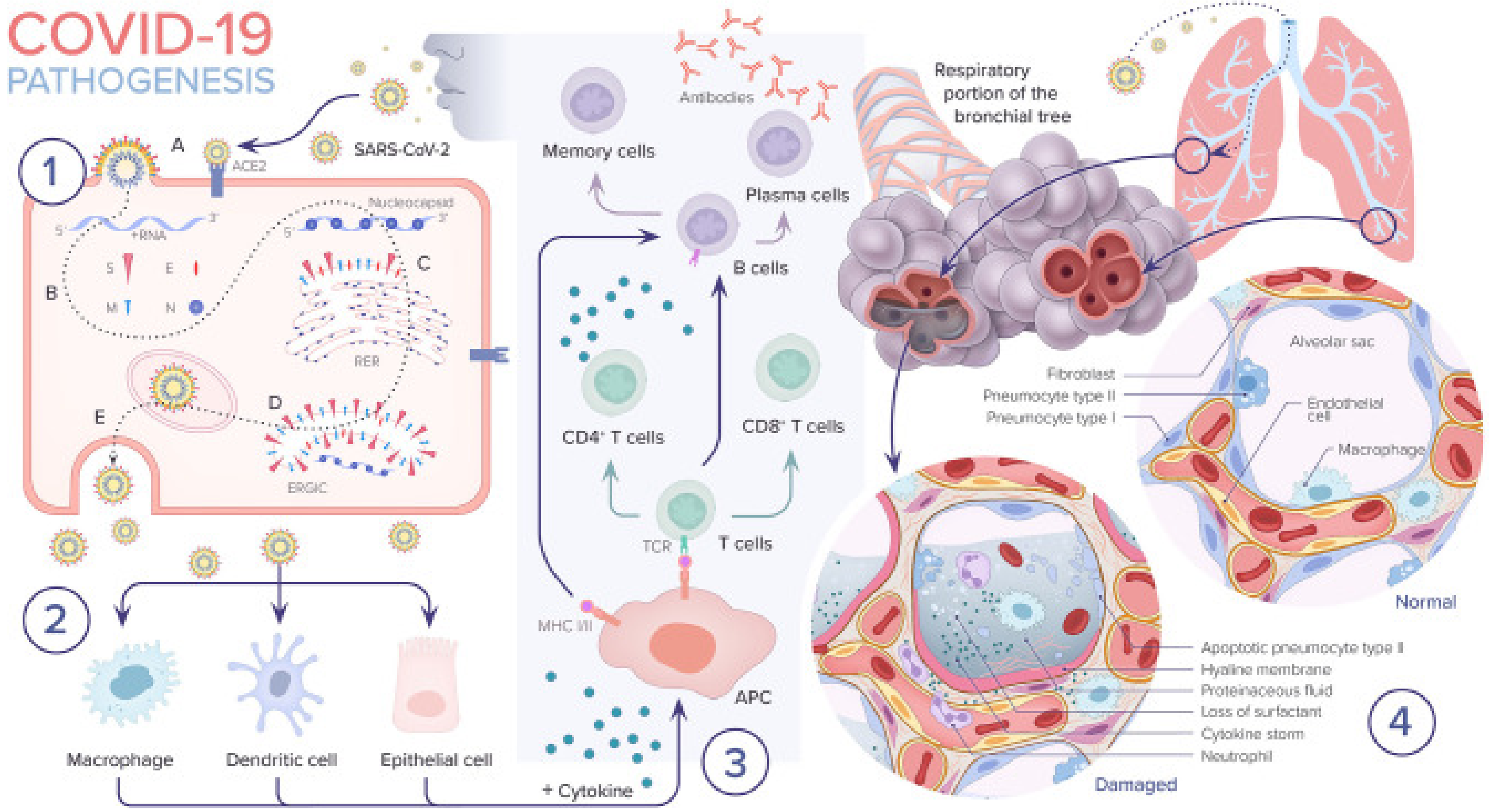
Multiscale Mathematical Modelling of Viral Infection

Ryumina Kristina Andreevna
Volpert Vitaly Aizikovich
Mozokhina Anastasia Sergeevna

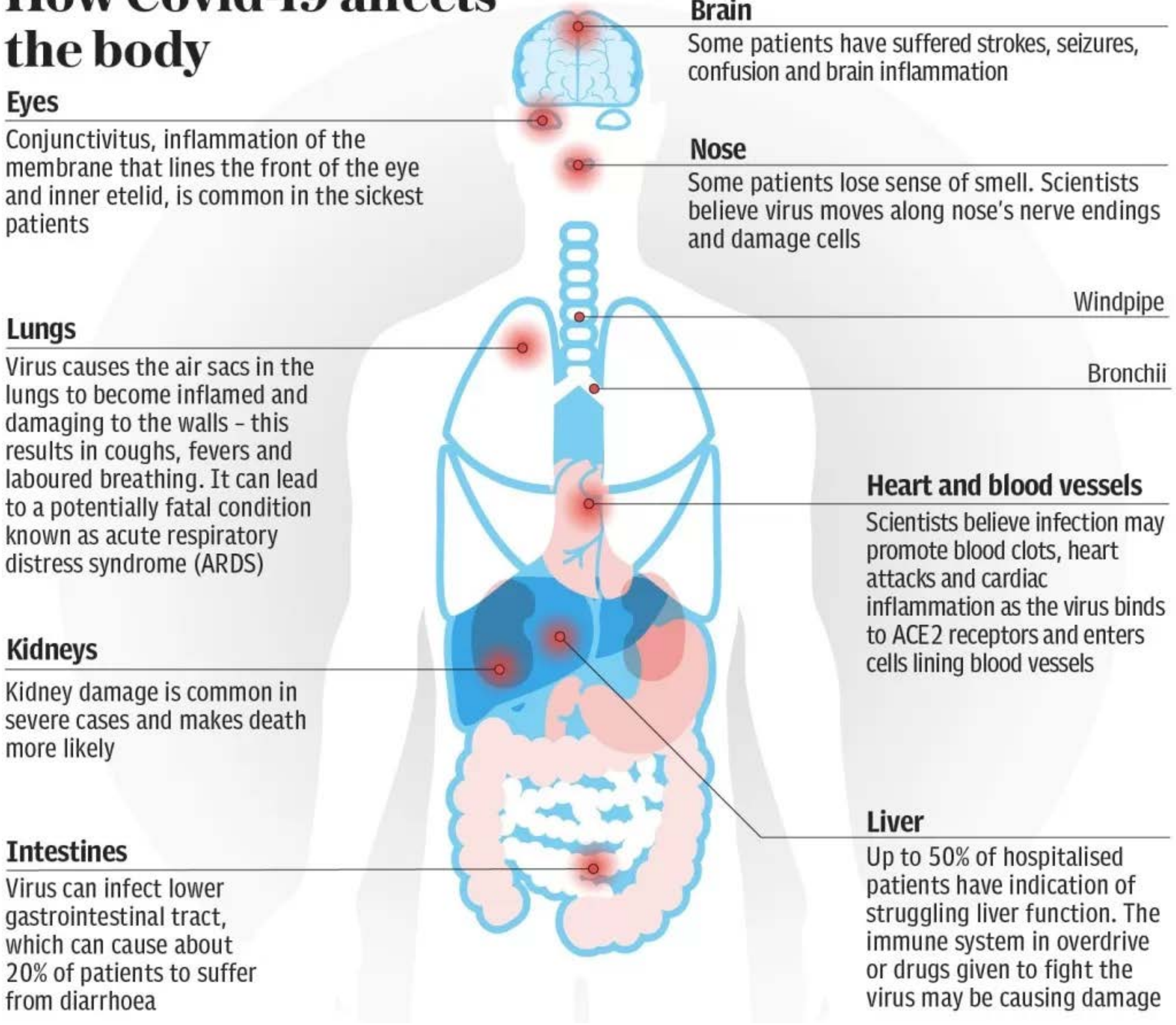


COVID-19 pathogenesis.

- 1. SARS-CoV-2 enters the epithelial cell
- 2. SARS-CoV-2 infection induces inflammatory factors
- 3. Antigen presentation of SARS-CoV-2 stimulates humoral and cellular immunity resulting
- 4. In severe COVID-19 cases, the virus reaches the lower respiratory tract and infects type II pneumocytes leading



How Covid-19 affects the body



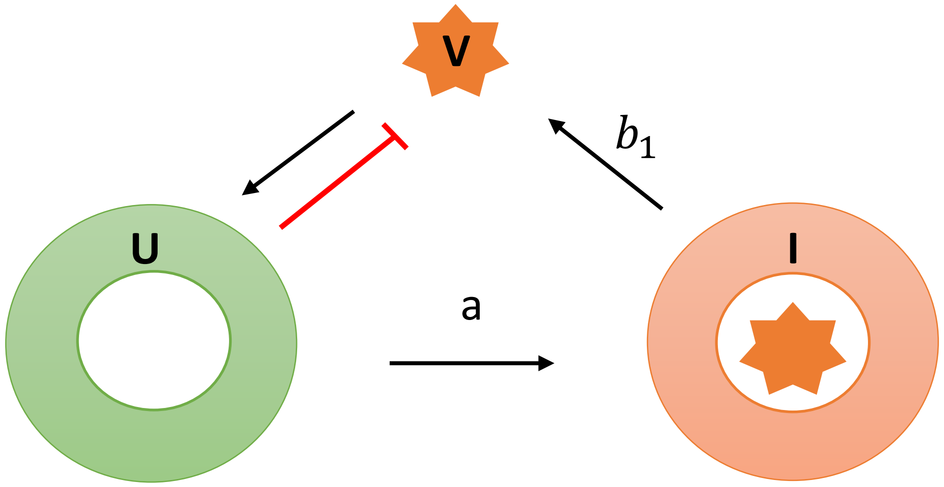
Circulating virus in the organism

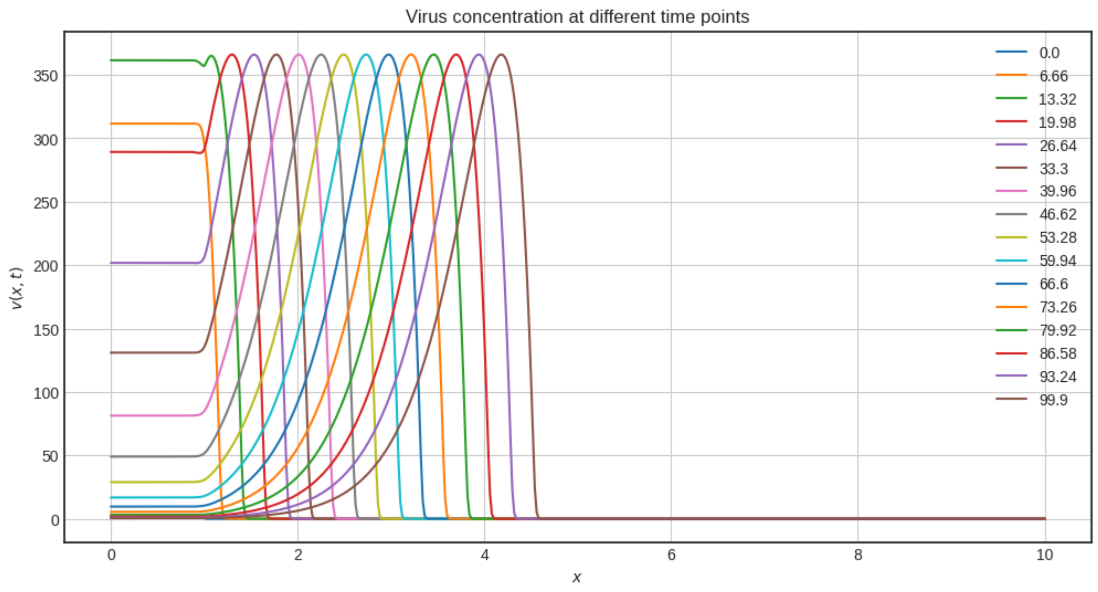
$$\frac{\partial U}{\partial t} = -aUV, \quad (1)$$

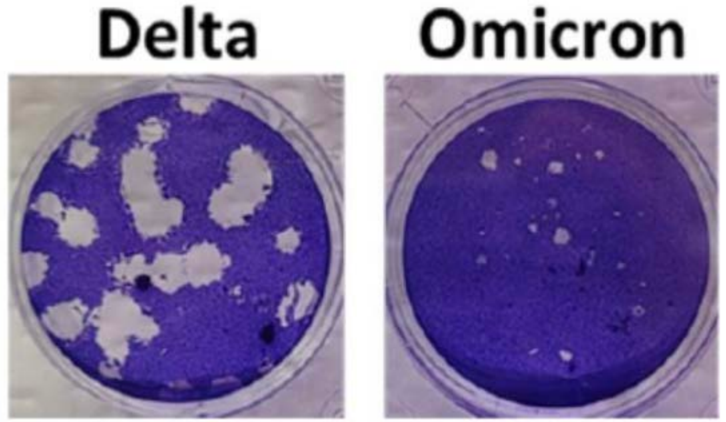
$$\frac{\partial I}{\partial t} = aUV - \beta I, \quad (2)$$

$$\frac{\partial V}{\partial t} = D\frac{\partial^2 V}{\partial x^2} + b_1 I_\tau - \sigma_1 V \quad (3)$$

U	Concentration of uninfected cells
I	Concentration of infected cells
V	Concentration of virus in tissue



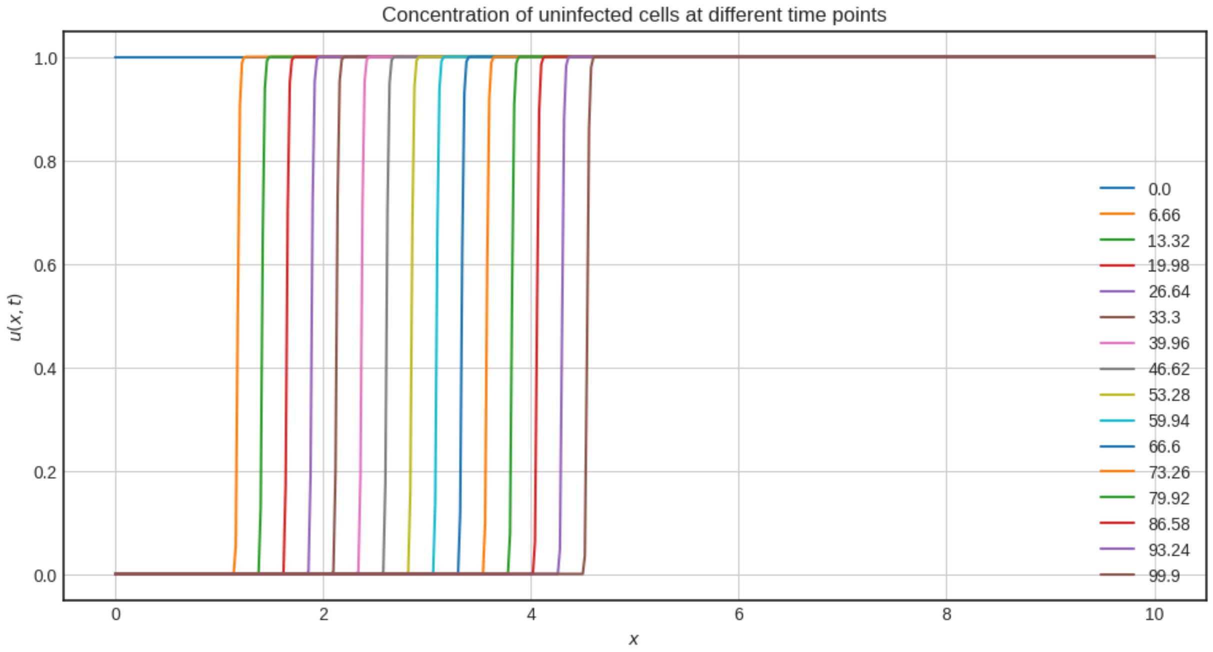


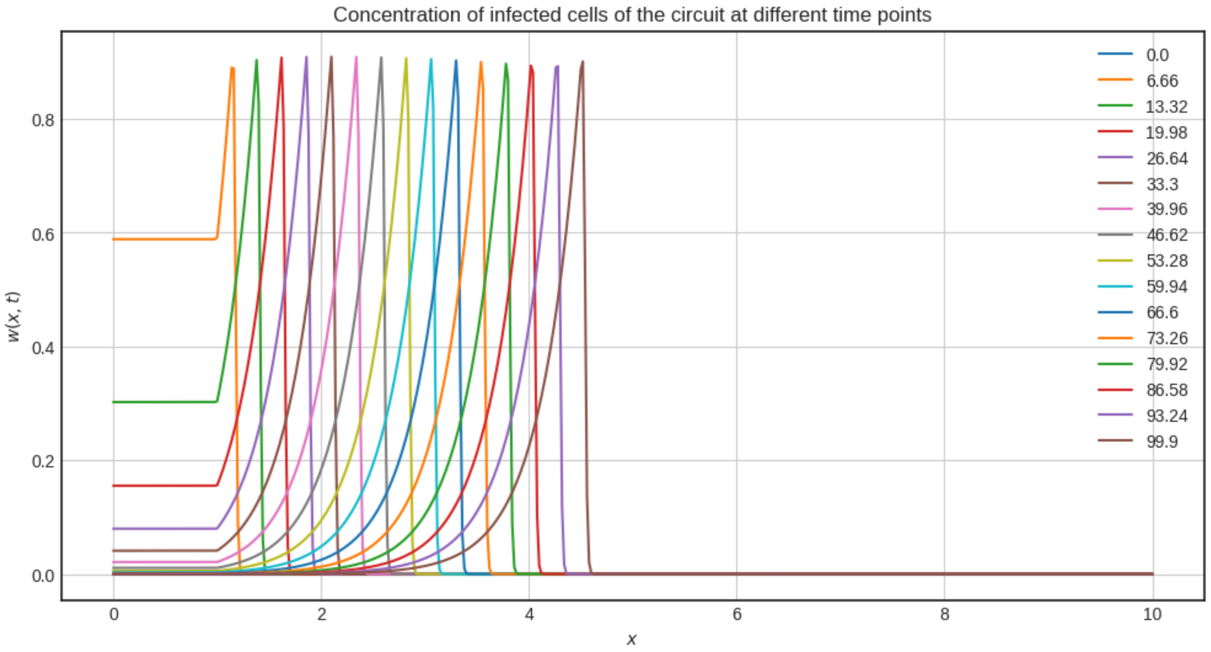


$$V_x \approx bcu_0/(\beta\sigma)$$

$$c = \min_{\mu > \mu_0} F(\mu) \equiv \frac{\sqrt{D}\mu}{\sqrt{\mu + \sigma - abu_0 e^{-\mu\tau}/(\mu + \beta)}}$$

Concentration of Uninfected and Infected





Circulating virus in the organism

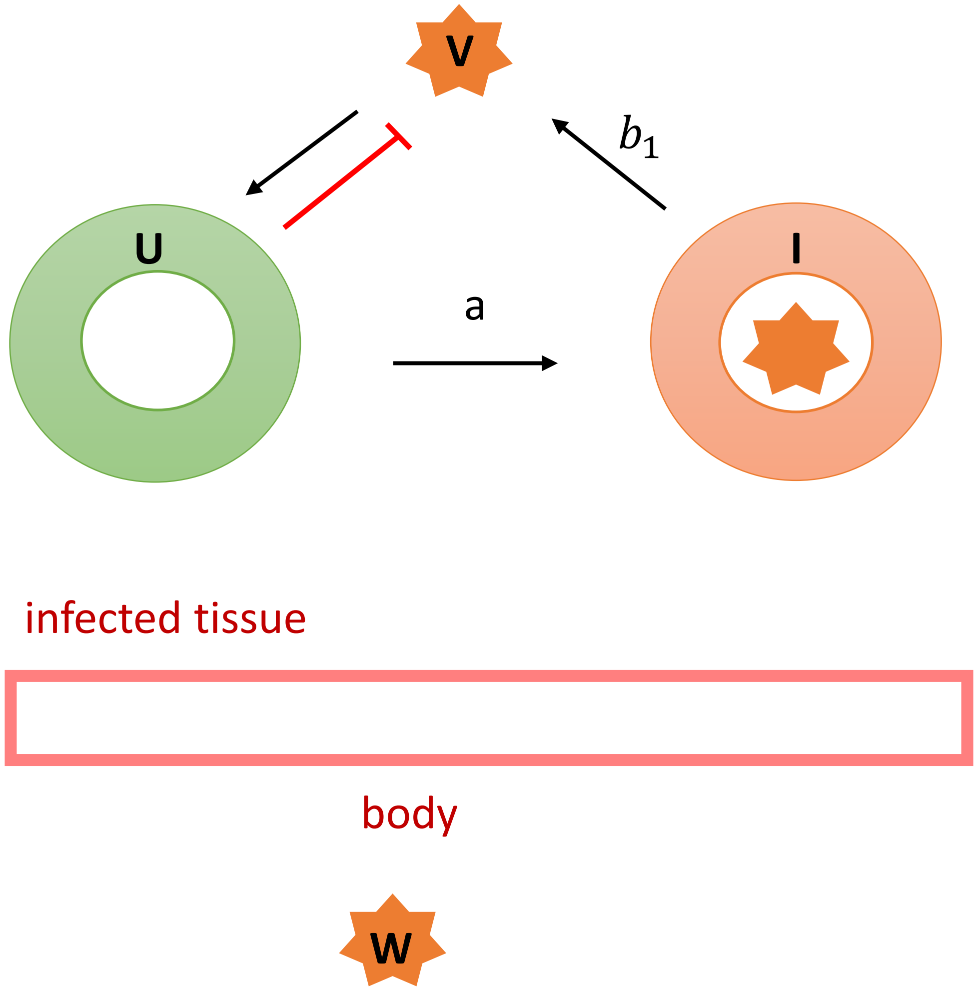
$$\frac{\partial U}{\partial t} = -aUV, \quad (1)$$

$$\frac{\partial I}{\partial t} = aUV - \beta(T)I, \quad (2)$$

$$\frac{\partial V}{\partial t} = D\frac{\partial^2 V}{\partial x^2} + b_1 I_\tau - (k_2 + \sigma_1)V, \quad (3)$$

$$\frac{dW}{dt} = k_3 J(V) - \sigma_2 W, \quad (4)$$

U	Concentration of uninfected cells
I	Concentration of infected cells
V	Concentration of virus in tissue
W	Concentration of virus in body



$$J(V) = \int_{-\infty}^{\infty} V(x,t)\,dx$$

Circulating virus in the organism

$$\frac{\partial U}{\partial t} = -aUV, \quad (1)$$

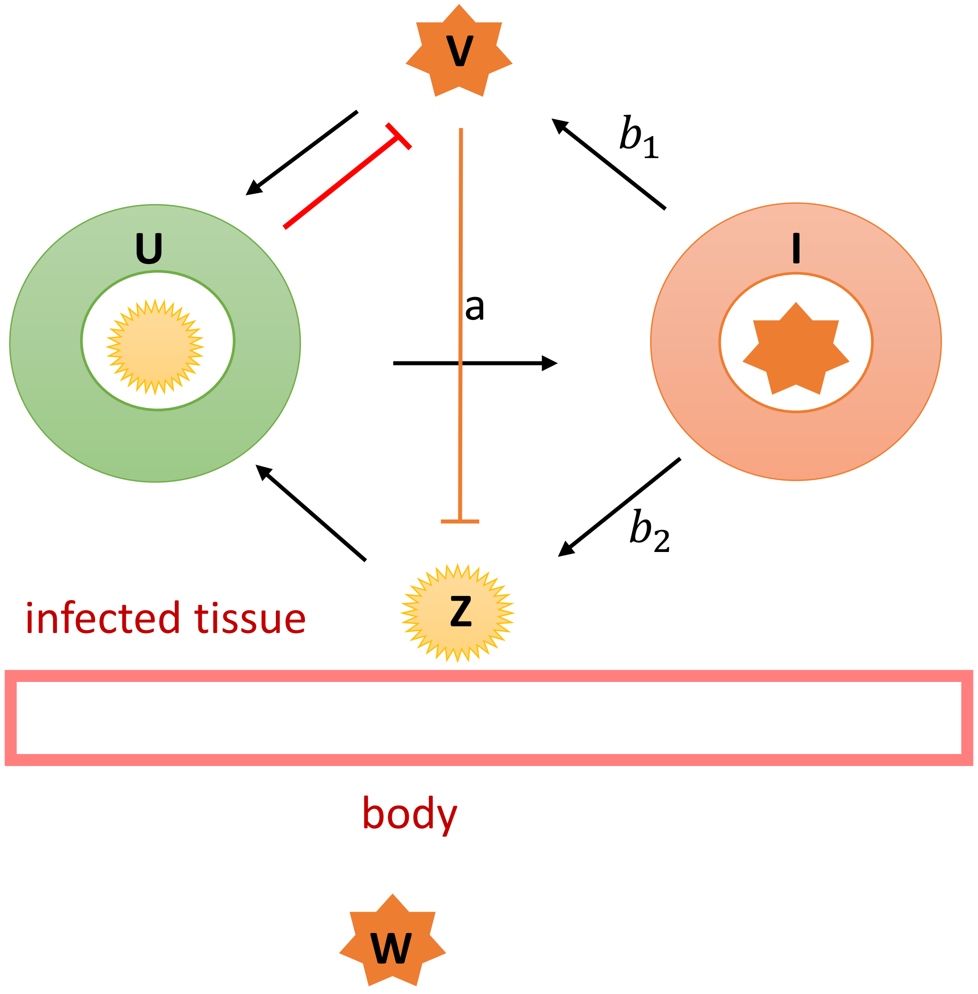
$$\frac{\partial I}{\partial t} = aUV - \beta(T)I, \quad (2)$$

$$\frac{\partial V}{\partial t} = D\frac{\partial^2 V}{\partial x^2} + \frac{b_1}{1 + k_1 Z} I_\tau - (k_2 + \sigma_1)V, \quad (3)$$

$$\frac{dW}{dt} = k_3 J(V) - \sigma_2(A)W, \quad (4)$$

$$\frac{dZ}{dt} = b_2 J(I)e^{-k_4 J(V)} - \sigma_3 Z \quad (5)$$

U	Concentration of uninfected cells
I	Concentration of infected cells
V	Concentration of virus in tissue
W	Concentration of virus in body
Z	Concentration of interferon



$$J(I) = \int_{-\infty}^{\infty} I(x,t)dx, \qquad J(V) = \int_{-\infty}^{\infty} V(x,t)dx$$

Circulating virus in the organism

$$\frac{\partial U}{\partial t} = -aUV, \quad (1)$$

$$\frac{\partial I}{\partial t} = aUV - \beta(T)I, \quad (2)$$

$$\frac{\partial V}{\partial t} = D\frac{\partial^2 V}{\partial x^2} + \frac{b_1}{1 + k_1 Z} I_\tau - (k_2 + \sigma_1(A))V, \quad (3)$$

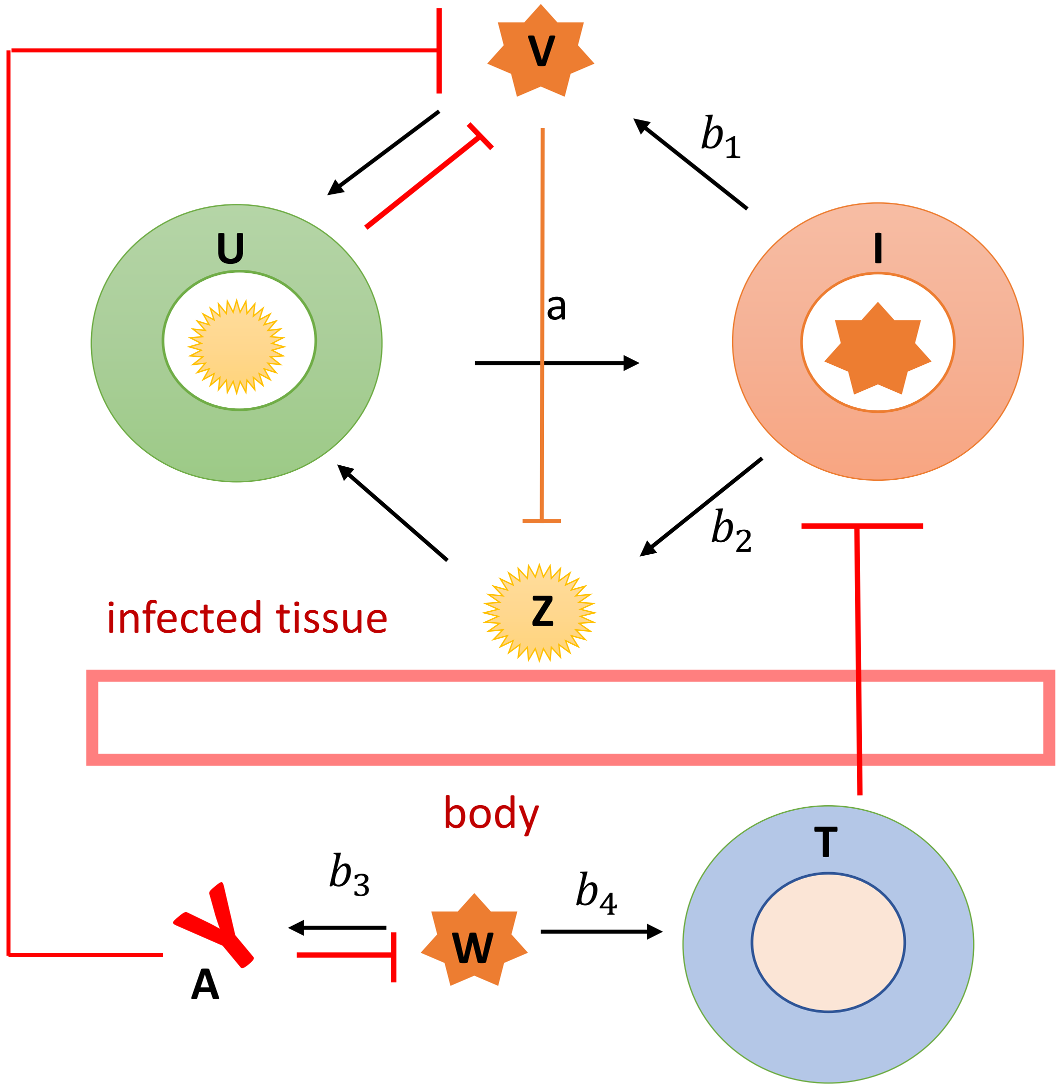
$$\frac{dW}{dt} = k_3 J(V) - \sigma_2(A)W, \quad (4)$$

$$\frac{dZ}{dt} = b_2\, J(I)e^{-k_4 J(V)} - \sigma_3 Z, \quad (5)$$

$$\frac{dT}{dt} = b_3\, W - \sigma_4 T\, J(I), \quad (6)$$

$$\frac{dA}{dt} = b_4\, W - \sigma_5 W A. \quad (7)$$

U	Concentration of uninfected cells
I	Concentration of infected cells
V	Concentration of virus in tissue
W	Concentration of virus in body
Z	Concentration of interferon
T	Concentration of cytotoxic T-lymphocytes
A	Concentration of antibodies



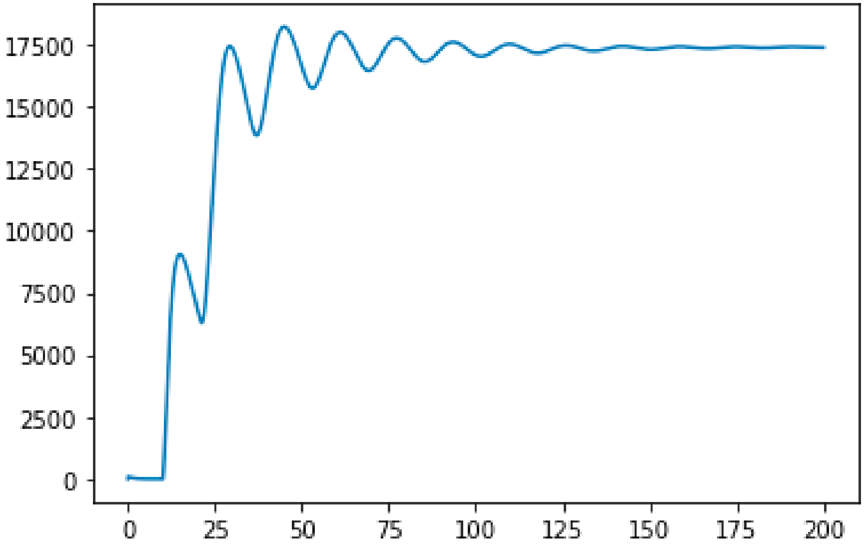
$$J(I) = \int_{-\infty}^{\infty} I(x,t)\,dx, \qquad J(V) = \int_{-\infty}^{\infty} V(x,t)\,dx$$

Analytical solution

$$J(v) = \frac{cu_0}{\beta(\theta)(k_2+\sigma_1(y))} b(J) = \frac{cu_0}{\beta(\theta)(k_2+\sigma_1(y))}(b_1 - \alpha(y)J(v)e^{-k_4 J(v)})$$

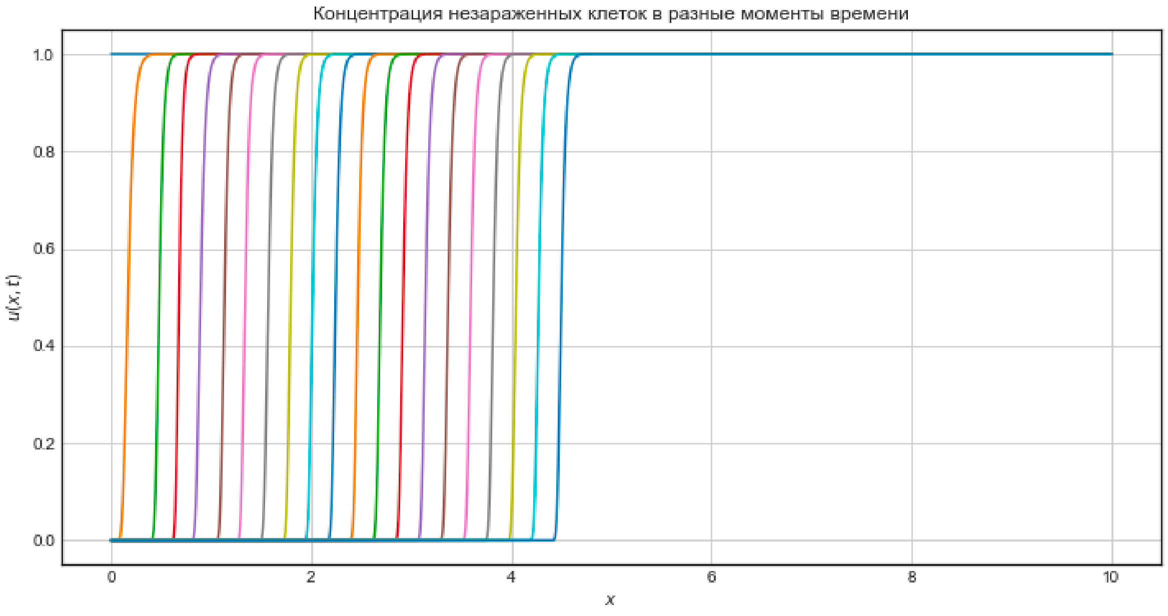
$$w = \frac{cu_0\sigma}{\beta(\theta)(k_2+\sigma_1(y))}(b_1 - \alpha(y)J(v)e^{-k_4 J(v)}), \qquad \sigma = \frac{k_3\sigma_{51}}{\sigma_{20}\sigma_{51}+\sigma_{21}b_4}$$

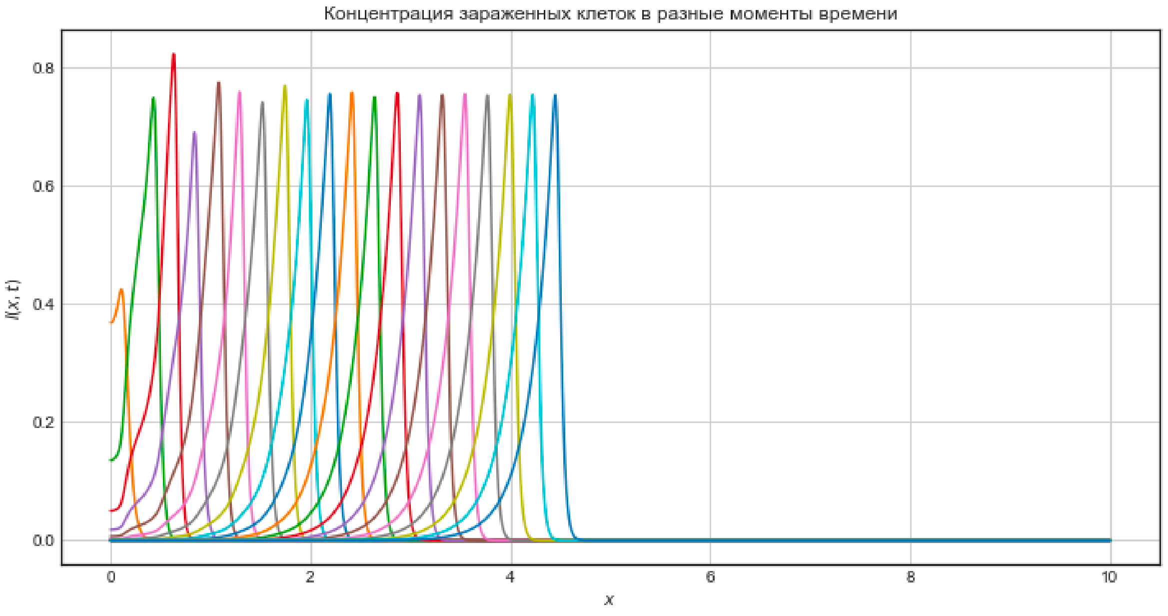
$$c^2 = \frac{D\mu^2(\mu+\beta(\theta))}{(\mu+\sigma_1(y)+k_2)(\mu+\beta(\theta)) - au_0 b(J)e^{-\mu\tau}}$$



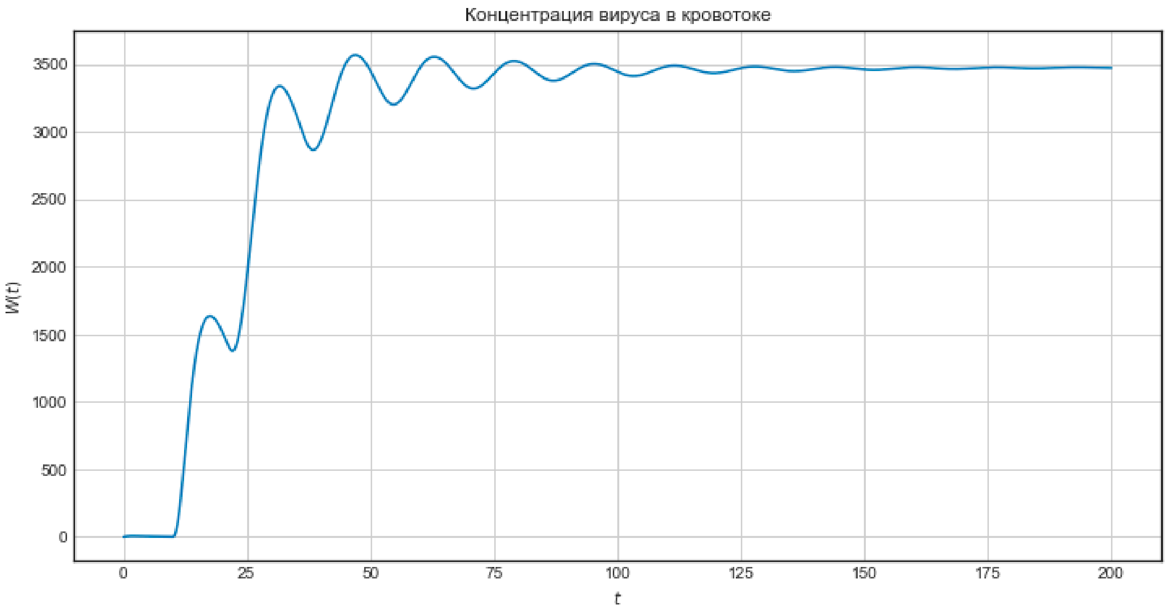
parameter	numerical	analitical
J(v)	17334.7119	17450.6305
W	34673.7332	34901.2610
wave speed	0.0225	0.0228

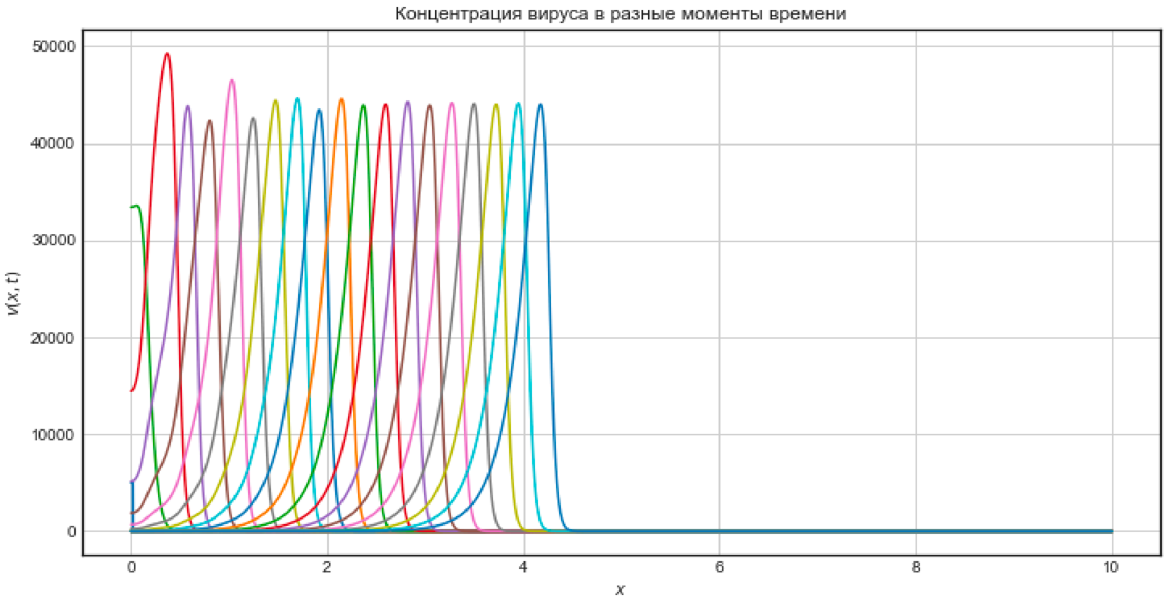
Concentration of Uninfected and Infected

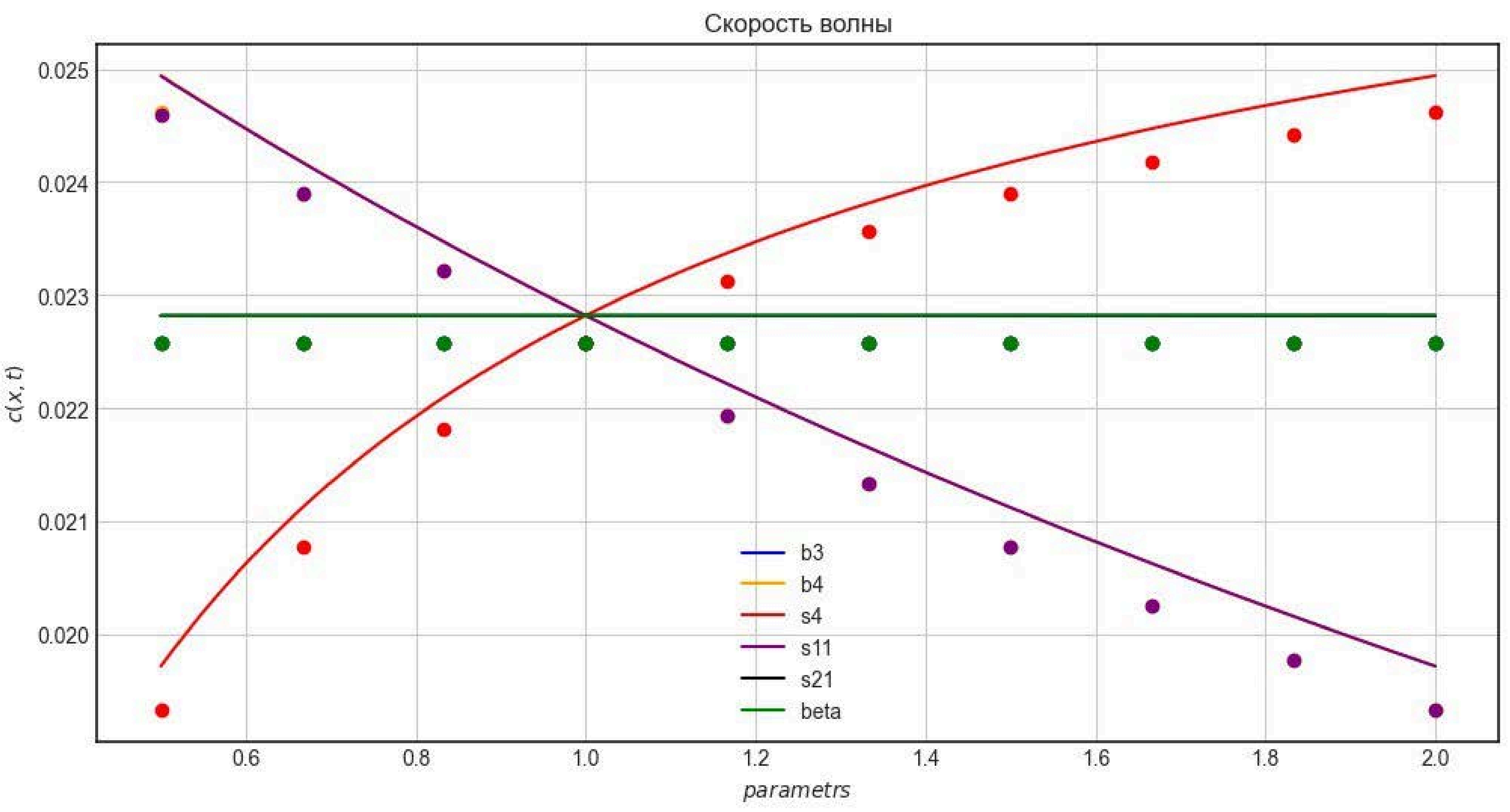


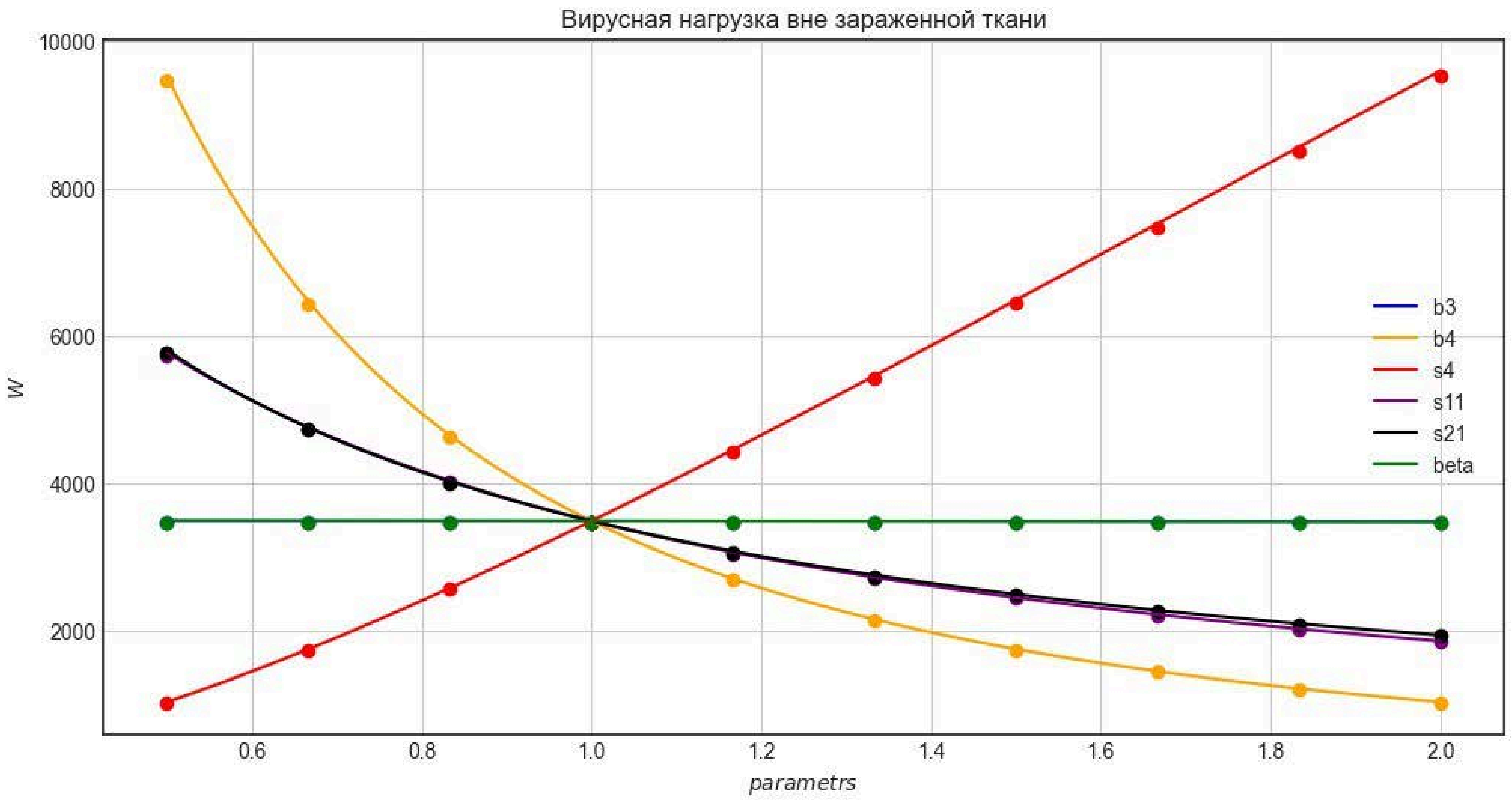


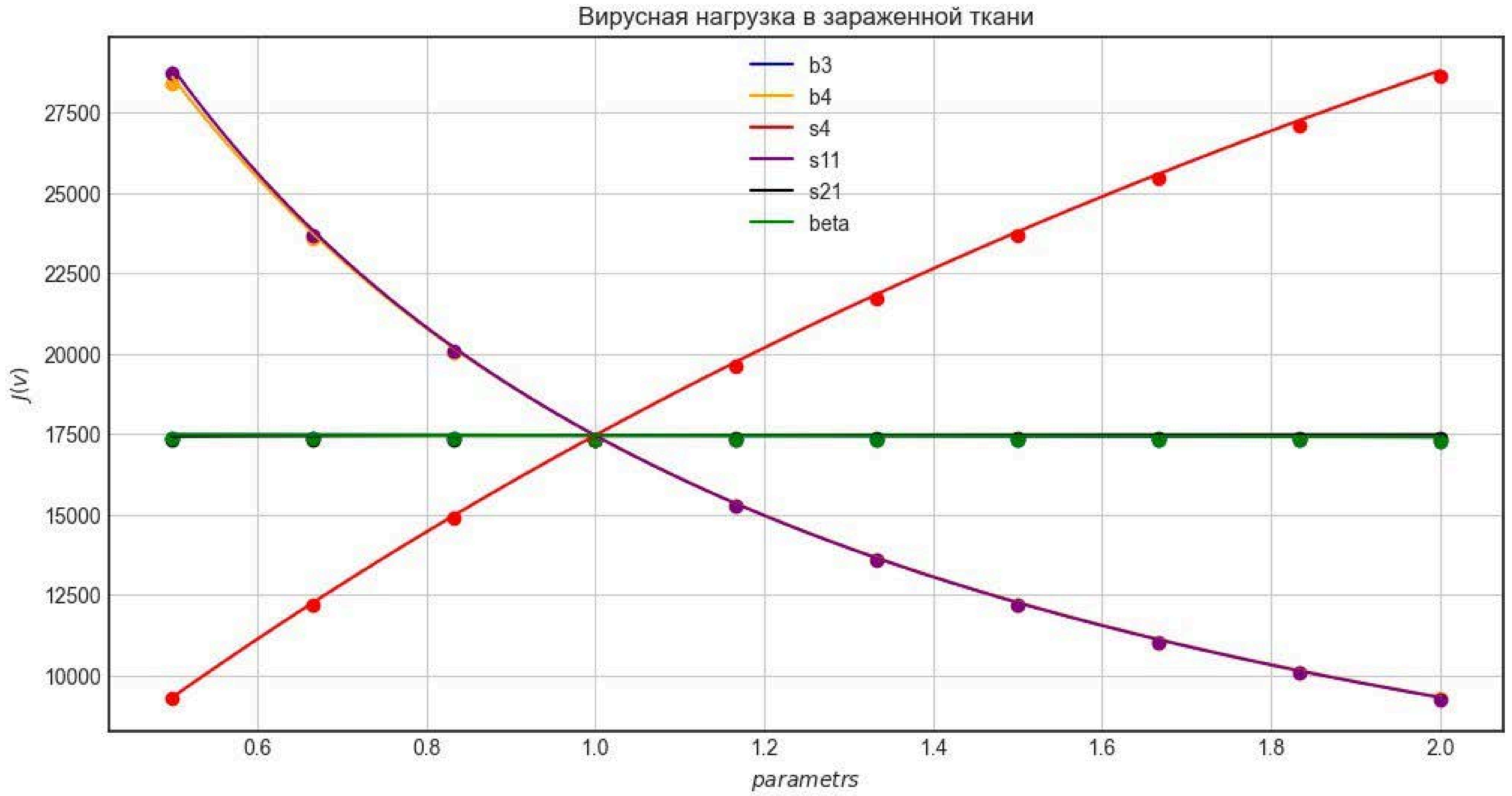
Concentration of Virus






Скорость волны
0.025
0.024
0.023
0.022
0.021
0.020
c(x, t)
0.6
0.8
1.0
1.2
1.4
1.6
1.8
2.0
parametrs
b3
b4
s4
s11
s21
beta


Вирусная нагрузка вне зараженной ткани
10000
8000
6000
4000
2000
W
0.6
0.8
1.0
1.2
1.4
1.6
1.8
2.0
parametrs
b3
b4
s4
s11
s21
beta



Вирусная нагрузка в зараженной ткани
b3
b4
s4
s11
s21
beta
27500
25000
22500
20000
17500
15000
12500
10000
J(v)
0.6
0.8
1.0
1.2
1.4
1.6
1.8
2.0
parametrs

Conclusion

- Numerical and analytical calculations of the model taking into account circulating virus were performed.

- The dependence of the wave velocity and viral load on the parameters of the initial and adaptive immune response was obtained.

- The influence of the intensity of the initial and adaptive immune response on the rate of virus spread and total viral load was evaluated.

Thank you for your attention