Numerical modeling of blood flow: left ventricular assist device case

Alexander Timofeev[1], Yuri Vassilevski[2,3,4], Sergey Simakov[3,4]

[1] Lomonosov Moscow State University
[2] Institute of Numerical Mathematics RAS
[3] Sechenov University
[4] Moscow Institute of Physics and Technology

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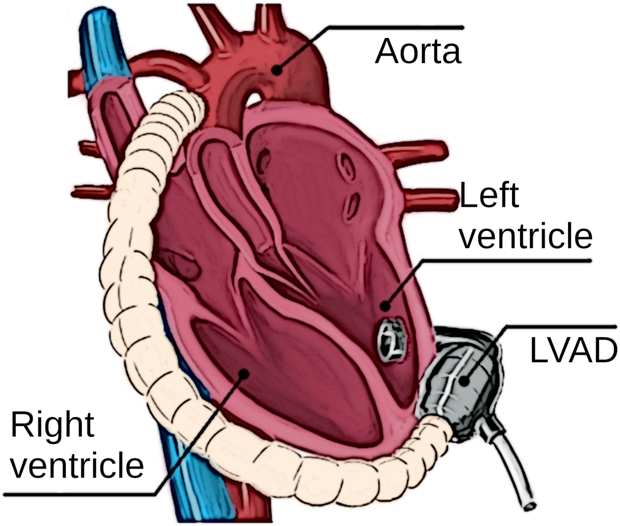
Introductory facts

Heart failure: the heart is unable to pump sufficiently to provide the body with enough blood flow.

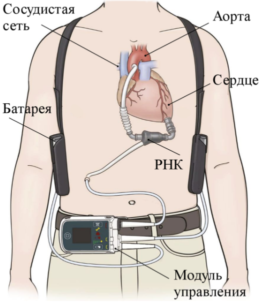
- 8 million in Russia suffer from a heart failure
- 2.4 million of them have an acute heart failure
- More than 1 million hospitalizations due to an acute heart failure per year in USA
- 450 000 with cardiac failure die in USA annually
- Cardiac transplantation is a gold-standard therapy in an acute heart failure
- Cardiac transplantations per year:
  2000 in USA
  100 in Russia
- Ventricular Assist Device implantations per year:
  $>$ 2500 in USA
  $<$ 20 in Russia

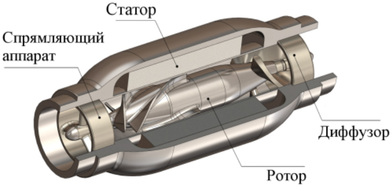
Basic anatomy of the heart



Sputnik Ventricular Assist Device

- Developed in Russia
- 3 devices: Sputnik 1, Sputnik 2, Pediatric Sputnik
- Data is provided by the development team



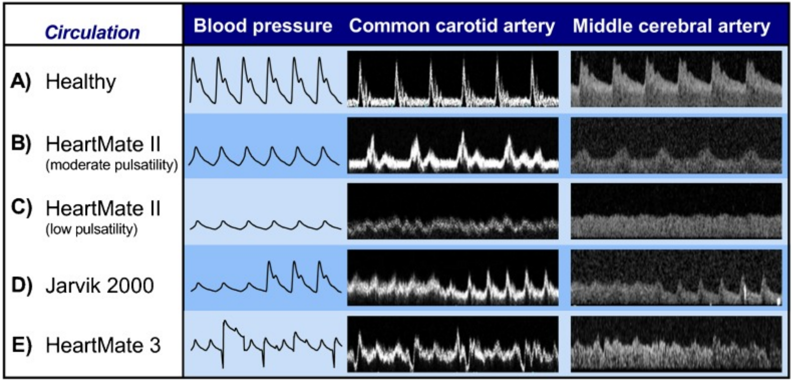


Структурно-параметрическая идентификация имплантируемых роторных насосов крови в аппаратах вспомогательного кровообращения, Петухов Д. С.

Motivation

The interaction of the VAD with a human circulatory system is examined in order to archieve the following goals with an impeller speed control:

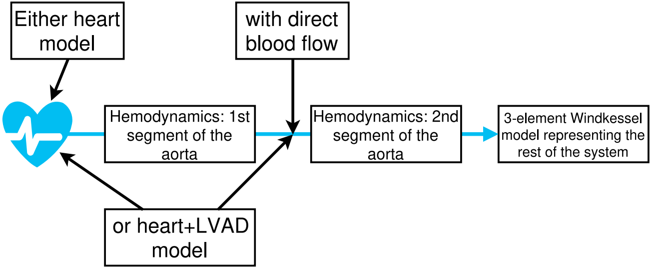
- Recovery of the cardiac output (in accordance with an exertion)
- Aortic valve opening
- Blood flow pulsation

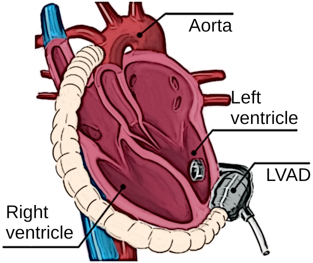


Castagna, Francesco, et al. "The unique blood pressures and pulsatility of LVAD patients: current challenges and future opportunities."Current hypertension reports 19.10 (2017): 85.

Model

1D hemodynamic model in vessels requires boundary conditions computed using 0D junction point models.





1D hemodynamics

1D flow in a separate vessel is a viscous incompressible fluid flow through an elastic tube.

- Mass and momentum balance equations

$$\partial S_k/\partial t + \partial(S_k u_k)/\partial x = \phi_k \quad (1)$$

$$\partial u_k/\partial t + \partial\left(u_k^2/2 + p_k/\rho\right)/\partial x = f_{fr}\left(S_k, u_k, S_k^0\right) + \psi_k \quad (2)$$

t – time, x – coordinate along the vessel, S – cross-section area of the vessel, u and p – the averaged over the cross-section linear velocity and blood pressure, ρ – blood density

- Numerical solution

Explicit two-step hybrid scheme (2nd order in space, 1st order in time)
Сеточно-характеристические численные методы, Магомедов К.М., Холодов А.С.

1D hemodynamics

- Elastic properties of vessel

$$p_k(S_k) - p_{*k} = \rho c_k^2 f(S_k) \qquad (3)$$

$$f(S_k) = \begin{cases} \exp\left(S_k/S_k^0 - 1\right) - 1, & S_k > S_k^0 \\ \ln\left(S_k/S_k^0\right), & S_k \leqslant S_k^0 \end{cases} \qquad (4)$$

- Initial conditions

$$S_k(0, x) = S_k^0, u_k(0, x) = Q_0/S_k^0 \qquad (5)$$

t – time, x – coordinate along the vessel, S – cross-section area of the vessel, u and p – the averaged over the cross-section linear velocity and blood pressure, ρ – blood density

0D model of vessels bifurcation

- Mass conservation

$$\sum_{k=k_1,k_2,\ldots,k_M} \varepsilon_k S_k\left(t,\tilde{x}_k\right) u_k\left(t,\tilde{x}_k\right) = 0 \tag{6}$$

- Bernoulli integral conservation

$$p_i\left(S_i\left(t,\tilde{x}_i\right)\right) + \frac{\rho u_i^2\left(t,\tilde{x}_i\right)}{2} = p_j\left(S_j\left(t,\tilde{x}_j\right)\right) + \frac{\rho u_j^2\left(t,\tilde{x}_j\right)}{2}, \tag{7}$$

- Discretized compatibility conditions along characteristics of hyperbolic equations

$$u(t,x_0) = \alpha(t,x_0)S(t,x_0) + \beta(t,x_0) \tag{8}$$

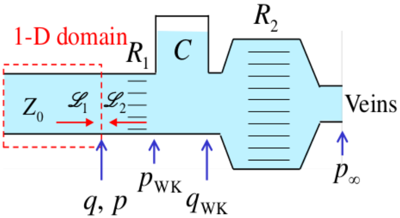
3-element Windkessel node

- Discretized compatibility conditions along characteristics of hyperbolic equations

$$u(t, x_0) = \alpha(t, x_0) S(t, x_0) + \beta(t, x_0) \quad (9)$$

- 3-element Windkessel model

$$\frac{dq}{dt} = \frac{1}{R_1}\left(\frac{dp}{dt} - \frac{dp_{WK}}{dt}\right)$$
$$\frac{dp_{WK}}{dt} = \frac{q}{C}\left(1 + \frac{R_1}{R_2}\right) - \frac{p - p_\infty}{R_2 C} \quad (10)$$



Characteristic outflow boundary conditions for simulations of one-dimensional hemodynamics, Bo-Wen Lin

0d heart model

- The dynamics of the volume of a heart chamber

$$I_k \frac{d^2 V_k}{dt^2} + R_k \frac{dV_k}{dt} + E_k(t)\left(V_k - V_k^0\right) + P_k^0 = P_k, k = lv, la, \tag{11}$$

  k – index of a chamber, V^0 – reference volume of the chamber, P^0 – reference pressure in the chamber, I – inertia parameter, R – hydraulic resistance of the compartment.

- Variable elastance

$$E(t) = E^d + (E^s - E^d)e(t), 0 \leqslant e(t) \leqslant 1, \tag{12}$$

  $e(t)$ is a periodic function with a period equals to the duration of the heart cycle.

  - For the left ventricle

$$e_{lv}(t) = \frac{1}{2}\begin{cases} 1 - \cos\dfrac{t}{T_{s1}}\pi, 0 \leqslant t \leqslant T_{s1}, \\ 1 - \cos\dfrac{t - T_{s2}}{T_{s1}}\pi, T_{s1} < t < T_{s2}, \\ 0, T_{s2} \leqslant t \leqslant T. \end{cases} \tag{13}$$

  - For the left atrium

$$e_{la}(t) = \frac{1}{2}\begin{cases} 0, 0 \leqslant t \leqslant T_{pb}, \\ 1 - \cos\dfrac{t - T_{pb}}{T_{pw}}2\pi, T_{pb} < t < T_{pb} + T_{pw}, \\ 0, T_{pb} + T_{pw} \leqslant t \leqslant T. \end{cases} \tag{14}$$

0d heart model

- The flow rate through the chamber is described by the mass conservation condition

$$\frac{dV_{lv}}{dt} = Q_{mi} - Q_{ao}, \qquad \frac{dV_{la}}{dt} = Q_{lpv} - Q_{mi}. \tag{15}$$

- The Poiseuille pressure drop condition for the connections between the chambers and between the chamber and the appropriate vessel

$$Q_{ao} = g_{ao}(\theta_{ao})\frac{P_{lv} - P_{sas}}{R_{ao}}, \qquad Q_{mi} = g_{mi}(\theta_{mi})\frac{P_{la} - P_{lv}}{R_{mi}}, \qquad Q_{lpv} = \frac{P_{lpv} - P_{la}}{R_{lpv}}, \tag{16}$$

$g(\theta) = \{\theta^{min} \leqslant \theta \leqslant \theta^{max}, 0 \leqslant g(\theta) \leqslant 1\}$ is a smooth monotonic valve function of the valve opening angle θ. For the closed valve it holds $g(\theta^{min}) = 0$, for the fully opened valve we have $g(\theta^{max}) = 1$.

0d heart model

- Valve function

$$
\begin{aligned}
g_{ao}\left(\theta_{ao}\right) &= \frac{\left(1-\cos\theta_{ao}\right)^2}{\left(1-\cos\theta_{ao}^{max}\right)^2}, \theta_{ao}^{min} \leqslant \theta_{ao} \leqslant \theta_{ao}^{max}, \\
g_{mi}\left(\theta_{mi}\right) &= \frac{\left(1-\cos\theta_{mi}\right)^2}{\left(1-\cos\theta_{mi}^{max}\right)^2}, \theta_{mi}^{min} \leqslant \theta_{mi} \leqslant \theta_{mi}^{max}, \\
g\left(\theta\right) &= \begin{cases} 0, \theta < \theta^{min}, \\ 1, \theta > \theta^{max}. \end{cases}
\end{aligned} \tag{17}
$$

- Newton's second law of the valve movement

$$
\begin{aligned}
\frac{d^2\theta_{ao}}{dt^2} &= -K_{ao}^f \frac{d\theta_{ao}}{dt} + \left(P_{lv} - P_{sas}\right) K_{ao}^p \cos\theta_{ao} - F_{ao}^r(\theta_{ao}), \\
\frac{d^2\theta_{mi}}{dt^2} &= -K_{mi}^f \frac{d\theta_{mi}}{dt} + \left(P_{la} - P_{lv}\right) K_{mi}^p \cos\theta_{mi} - F_{mi}^r(\theta_{mi}).
\end{aligned} \tag{18}
$$

0d heart model

- To couple the heart model with a 1D hemodynamics in the ascending aorta we need to consider the following

$$\begin{aligned} Q_{ao} &= S_{sys} u_{sys} \\ u_{sys} &= \alpha S_{sys} + \beta \\ P_{sys} &= P(S_{sys}) \end{aligned} \tag{19}$$

- In general, this leads us to the system of ordinary differential and algebraic equations

$$\begin{cases} \dot{y}_g = g(y_f, t) \\ f(y_f, t) = 0 \end{cases} \tag{20}$$

  where $y_f = (y_g, ...)$
- We use implicit Euler method to discretize it and then solve it with Newton's method.

0d pump model

$$\dot{Q}_p = a(P_A - P_{lv}) + bQ_q^2 + cQ_p\omega + d\omega^2 + \begin{cases} 0, & Q_p > e\omega \\ R_{rec}(Q_p - e\omega)^2, & Q_p \leqslant e\omega \end{cases} + \begin{cases} -R_{per}Q_p^2, & Q_p \leqslant 0 \\ R_{per}Q_p^2, & Q_p > 0 \end{cases} \quad (21)$$

Q_p – pump flow, P_A – aortic pressure in the connection node, ω – impeller speed

Boës, Stefan, et al. "Hydraulic characterization of implantable rotary blood pumps."IEEE Transactions on Biomedical Engineering 66.6 (2018): 1618-1627.

But it's not enough...

Equations additional to 0d pump model

- Need to change mass balance for the left ventricle and add more equations in order to combine 0d pump model with the heart model

$$\dot{V}_{lv} = Q_{mi} - Q_{ao} - Q_p \tag{22}$$

- Mass balance in the connection node

$$S_1 u_1 + S_2 u_2 = Q_p \tag{23}$$

- Discretized compatibility conditions for two segments of aorta

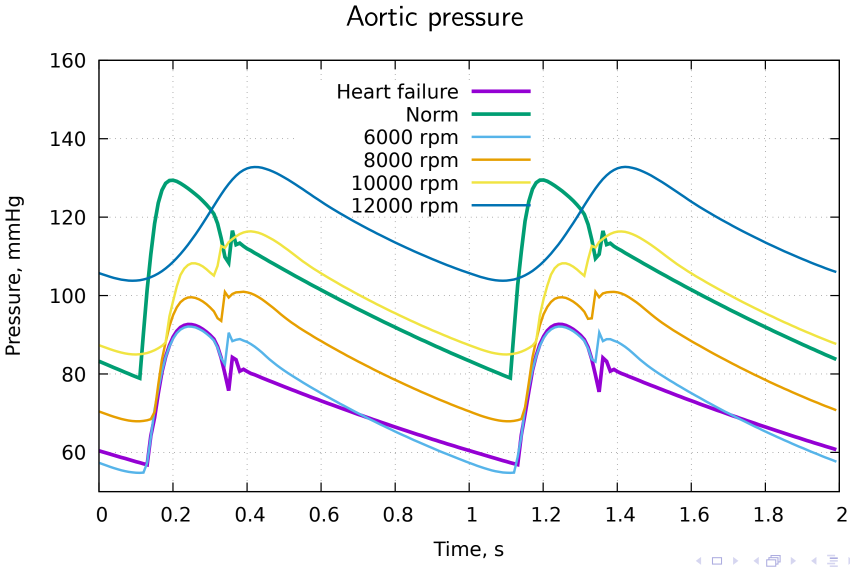
$$\begin{aligned} u_1 &= \alpha_1 S_1 + \beta_1 \\ u_2 &= \alpha_2 S_2 + \beta_2 \end{aligned} \tag{24}$$

- Bernoulli integral conservation

$$p_1(S_1) + \frac{\rho u_1^2}{2} = p_2(S_2) + \frac{\rho u_2^2}{2} \tag{25}$$

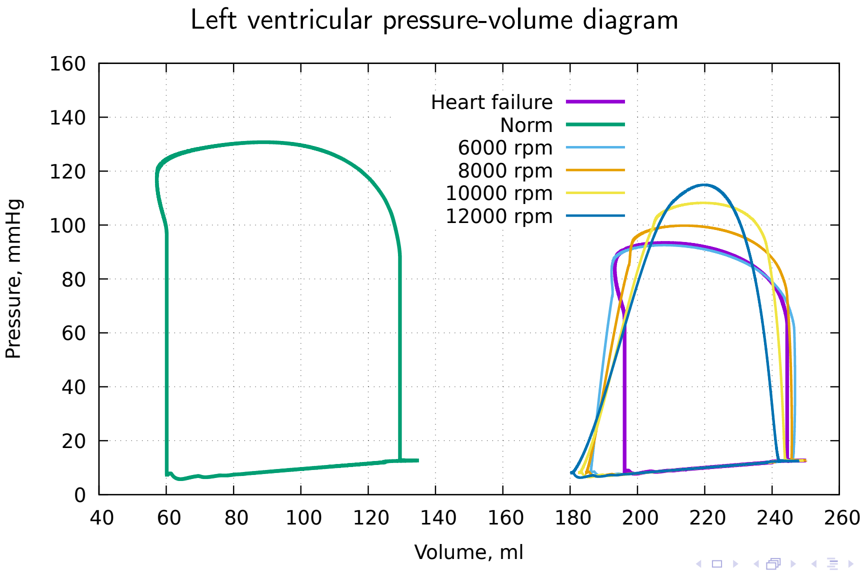
$$p_1(S_1) + \frac{\rho u_1^2}{2} = P_A + \frac{\rho (Q_p/S_p)^2}{2} \tag{26}$$

Results



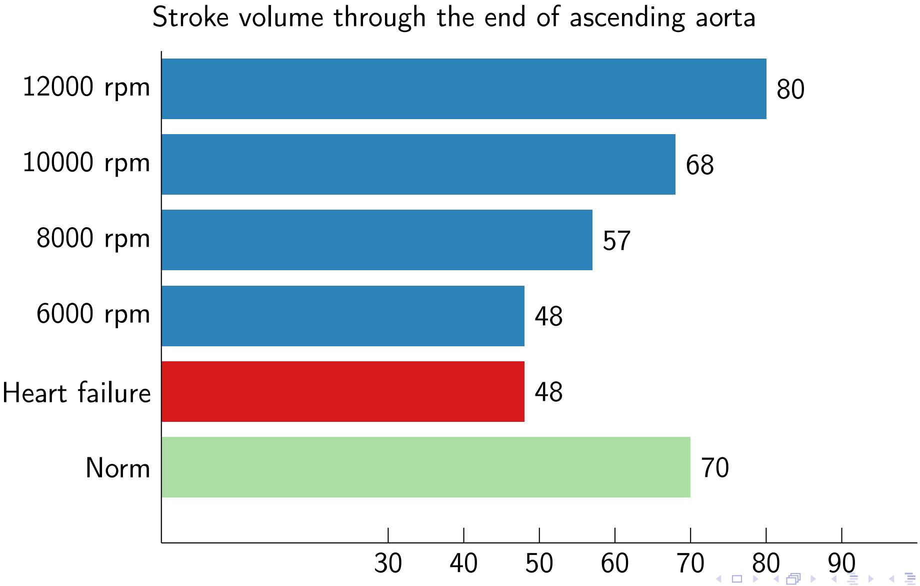
Aortic pressure
Heart failure
Norm
6000 rpm
8000 rpm
10000 rpm
12000 rpm
Pressure, mmHg
Time, s
160
140
120
100
80
60
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2

Results



Left ventricular pressure-volume diagram
Heart failure
Norm
6000 rpm
8000 rpm
10000 rpm
12000 rpm
Pressure, mmHg
Volume, ml
0
20
40
60
80
100
120
140
160
40
60
80
100
120
140
160
180
200
220
240
260

Results



Stroke volume through the end of ascending aorta
12000 rpm
10000 rpm
8000 rpm
6000 rpm
Heart failure
Norm
80
68
57
48
48
70
30
40
50
60
70
80
90

Conclusion

- New model including 0D heart model and the model of LVAD for 1D hemodynamics is proposed and implemented
- A versatile C++ software package for 1D blood flow modeling is being developed
- **TODO:** Model of the heart chamber requires some changes
- **TODO:** To experiment with the complete close-loop model of circulatory system
- **TODO:** To solve the LVAD control problem in accordance with the requirements to the next-generation pumps.

Thank you for your attention!