The Week of Applied Mathematics and Mathematical Modelling

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Ricker model parameter estimation using neural networks

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Moran – Ricker model with one year time lag

$$\begin{cases} x_{n+1} = a x_n \exp(-x_n - \rho \cdot y_n) \\ y_{n+1} = x_n \end{cases} \qquad \textbf{(1)}$$

x_n is a population size at time moment n,
n is reproductive season number,
a is reproductive potential,

The parameter $\boldsymbol{\rho}$ characterizes the previous generation "participation" in the density-dependent regulation of the population birth rate.

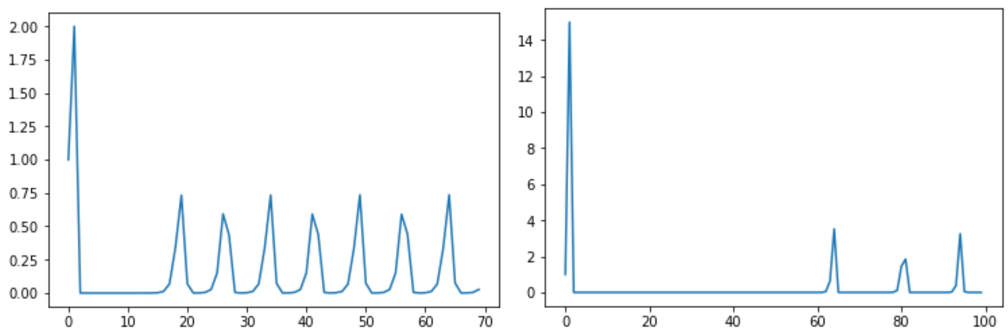
If $\boldsymbol{\rho}<1$ then the previous generation "participation" in the density-dependent regulation of the population birth rate does not exceed the "participation" of the current generation. Thus, the resources used by a population are significantly recovered between two successive periods of reproduction.

At $\rho>1$ the previous generation "participation" in the density-dependent regulation of the population birth rate is more than the "participation" of the current generation. The higher the value of parameter ρ (ρ>1), the fewer resources are available for the next generation.

Neural network for parameter identification

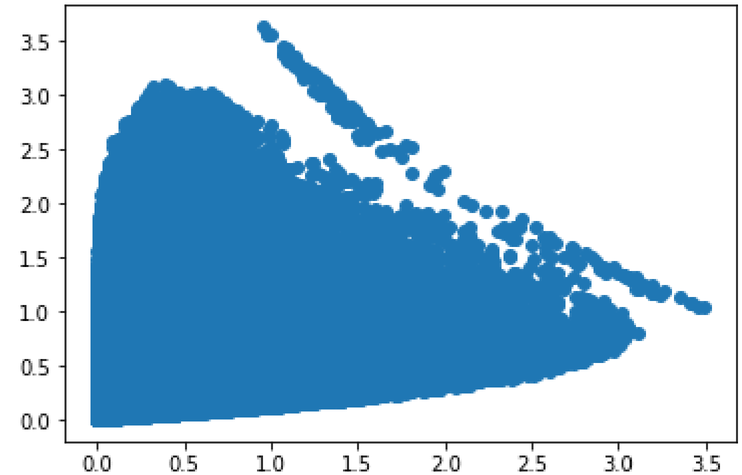
Input Data

Values of a, ρ, x_0, y_0 and the corresponding model trajectory consisting of 38 points (in according with used real data)



Examples of trajectories of system (1). Due to the exponential factor, the population size very quickly decreases to almost zero. Then arises oscillations of population number.

Independently of the range of initial conditions, after reaching the oscillatory mode, the possible range of population size is significantly narrowed. This fact was also taken into account when we prepared the data.



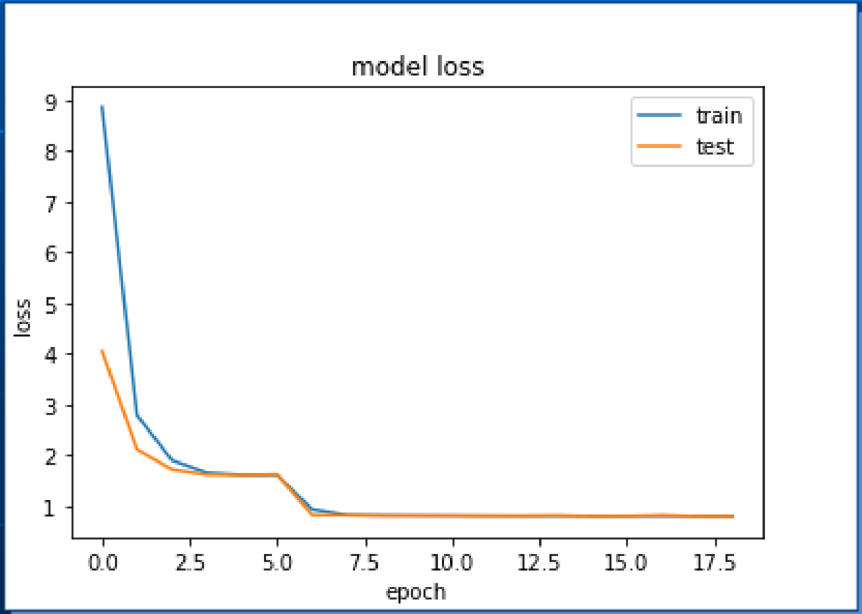
The dependence of values y_0 (ordinate axis) and the values of x_0 (abscissa axis) for system (1) after reaching the oscillation mode.

Neural network for parameter identification

We use a fully connected neural network to solve the regression problem.

Network Training

The neural network was trained on 10^6 trajectories and tested on 10^4 trajectories.




```
model = Sequential()
model.add(Dense(100, input_dim=38))
model.add(ReLU())
model.add(Dense(100))
model.add(ReLU())
model.add(Dense(100))
model.add(ReLU())
model.add(Dense(100))
model.add(ReLU())
model.add(Dense(4))
```

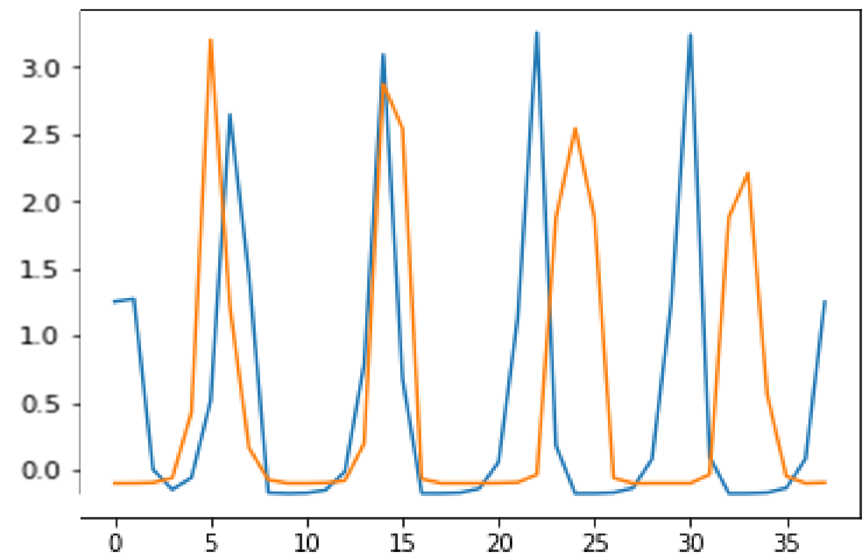
Application of the Moran–Ricker model with time lag to insect population

We are trying to use the neural network to estimate a parameters of a real insects population within the framework of the Moran Ricker model.

The time series used can be freely downloaded online from the Global Population Dynamics Database (GPDD)

To test our neural network we used data set #1525. It contains dynamic of Larch budmoth in Engadine Valleyand, Switzerland.

Despite the fact that the neural network well defines the parameters of the model trajectories from the test sample application of the neural network to real populations data leads to fault.



$a=6.4$ $\rho=10.5$ $x_0=0.32$ $y_0=0.34$

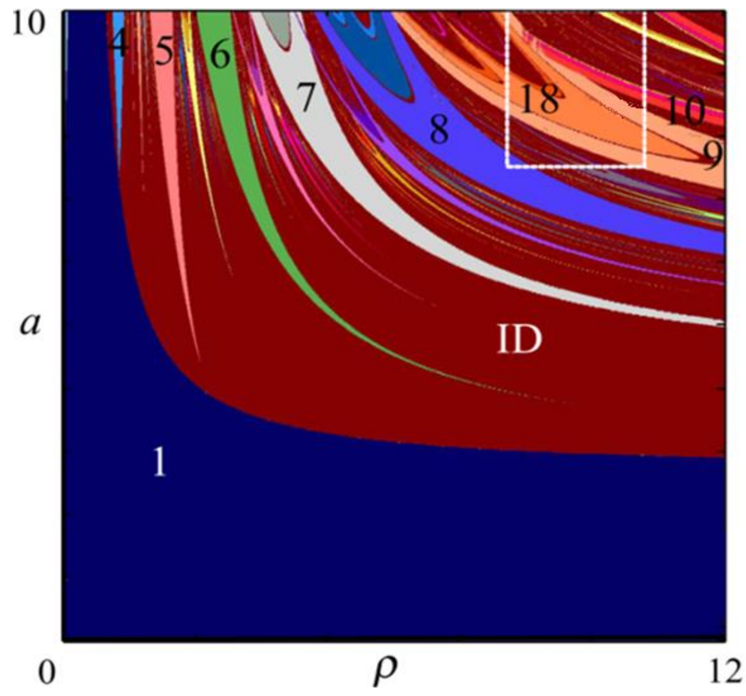
Modification of neural net

To improve the quality of the parameter estimation, we used information on the dynamic regimes of the Moran-Ricker model.

We used a map of dynamic modes to train a 1D CNN to determine the type of dynamic mode (length of cycle) of real population.

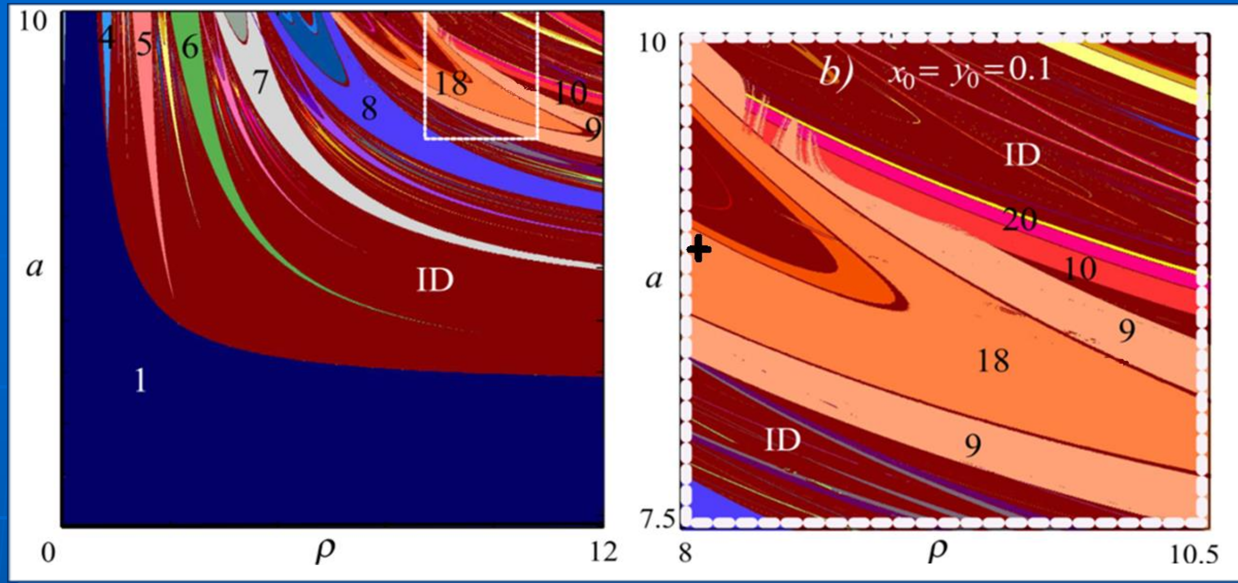
Next, we used a fully connected network, which was trained only on parameter values that tends to the determined dynamic mode.

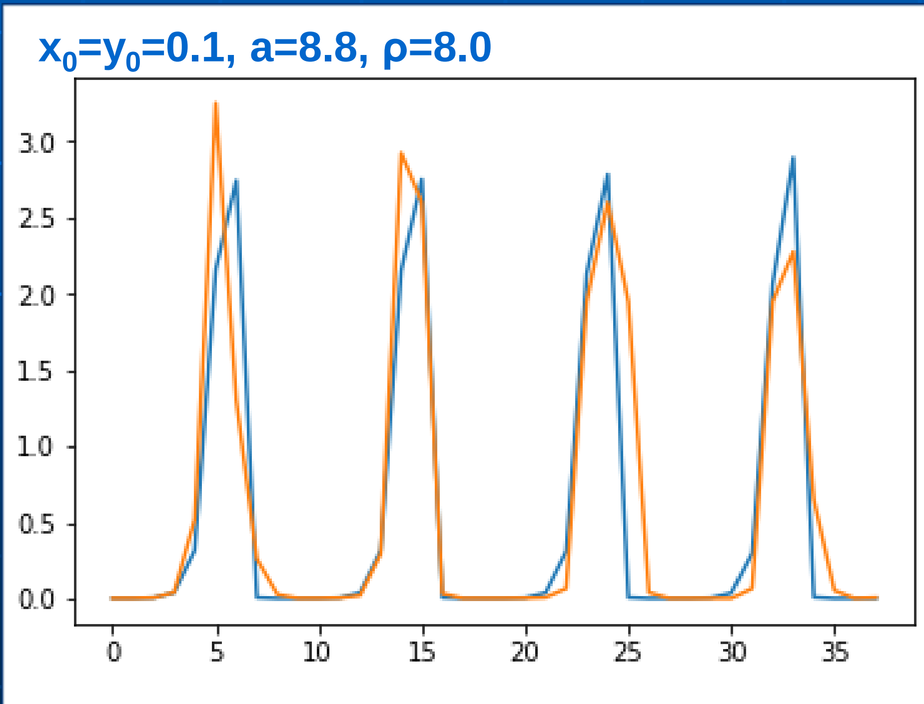
Dynamic modes of the model (1)



The figures correspond to the period of observed cycles. ID is irregular dynamics.

The estimation of the dynamic mode for Larch budmoth population showed that it oscillates with period 18. The values of parameters (a, ρ) that corresponds to this dynamic mode forms Arnold’s tongue in quasi-periodic dynamics area.





The model parameter estimations show that parameter ρ is greater than 1. The previous generation “participation” in the density-dependent regulation of the population birth rate is significantly greater than the ”participation” of the current generation.

The point estimation is placed near the border that bounded area of a cycles with period 18. A small fluctuation in the values of the parameters may lead to a change in the dynamic mode.

The conclusion

- The neural networks was applied to biological parameter estimation of real insects population within framework of Moran–Ricker model with time lag.
- The use of the qualitative properties of mathematical models in the construction of training samples for neural networks can significantly improve the quality of parameters reconstruction.
- The obtained point estimates corresponding to real population dynamics are located in a quasi-periodic fluctuations area that adjoins other dynamic modes. A variation of value demographic parameters (e.g., as a result of evolution or global climate change) can lead to a dynamic mode change.