A POROELASTIC APPROACH TO THE HYDROCEPHALUS MODELING

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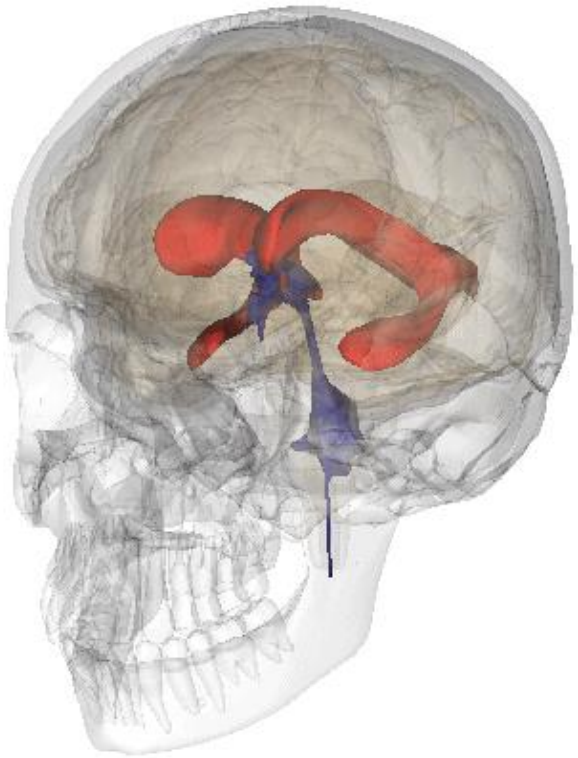
Cerebrospinal fluid

Cerebrospinal fluid (CSF, liquor) — is the biological liquid around the brain and spinal cord. In the brain cerebrospinal fluid is concentrated in the four ventricles located in the center of the skull.

The total volume of cerebrospinal fluid is approximately 150 ml and it is produced at a rate of 450 ml per day (thus replacing itself three times a day).



CSF functions:

- CSF removes waste products from cerebral metabolism and supplies nutrients to nervous system tissue;
- CSF protects the brain and spinal cord from trauma;
- CSF is necessary to maintain “constant intracranial pressure”

Images are generated by Life Science Databases(LSDB). - from Anatomography website maintained by Life Science Databases(LSDB)

Cerebrospinal fluid circulation

Lateral ventricle's choroid plexuses

CSF

Lateral ventricles

Through interventricular foramina

Third ventricle's choroid plexuses

CSF

Third ventricle

Through cerebral aqueduct

Fourth ventricle's choroid plexuses

CSF

Fourth ventricle

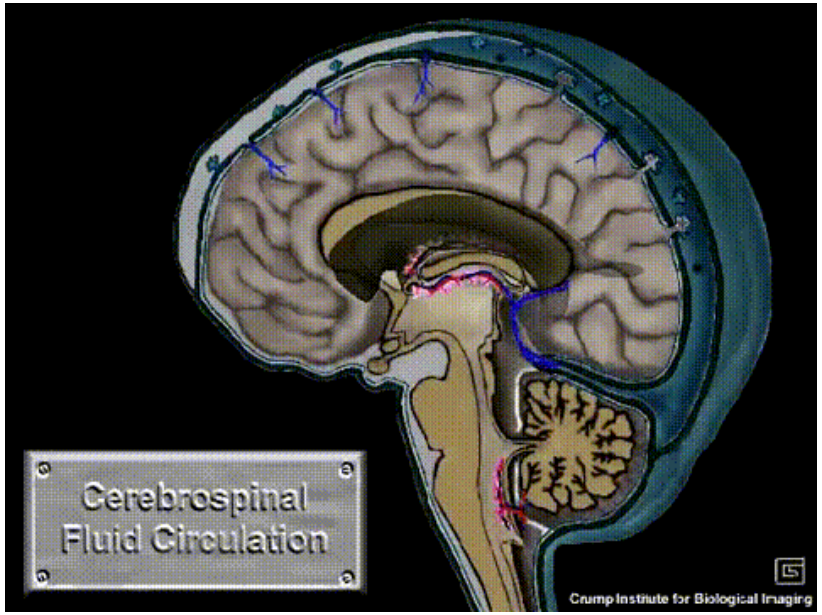
Through lateral and median apertures

Subarachnoid space

Arachnoid villi of dural venous sinuses

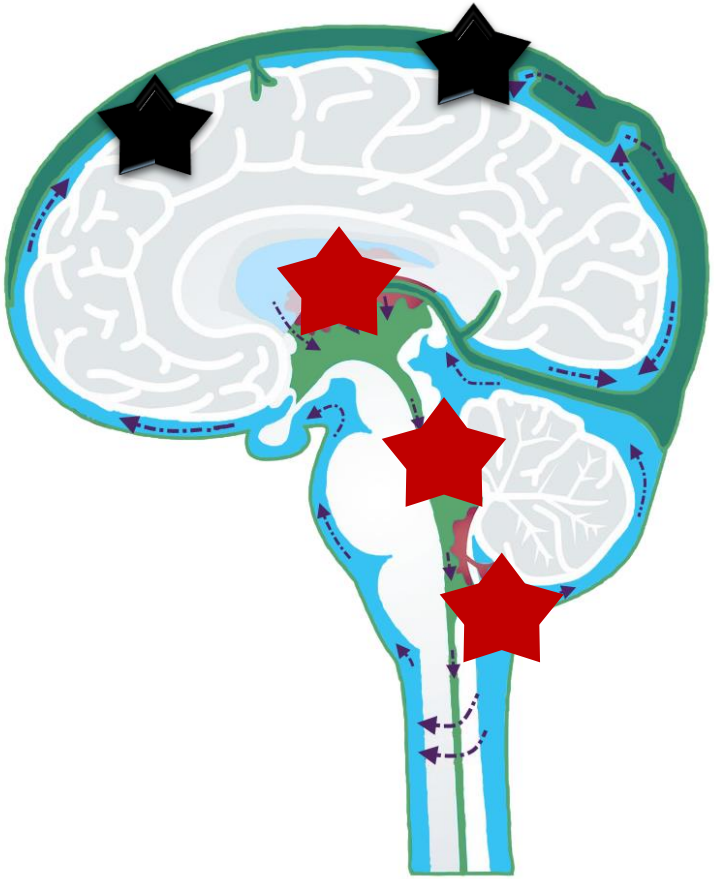
Venous blood

Heart



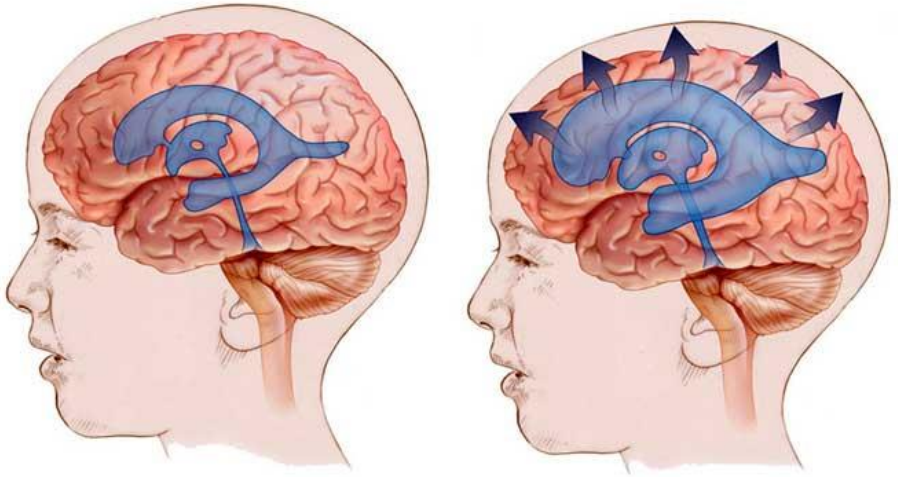
Hydrocephalus

It is important to maintain a continuous equilibrium between the secretion, circulation and resorption of the CSF. In the case of imbalance, the fluid accumulates in the system causing a condition called **hydrocephalus**



Disruption of liquor production or its movement

Disruption of absorption into the circulatory system



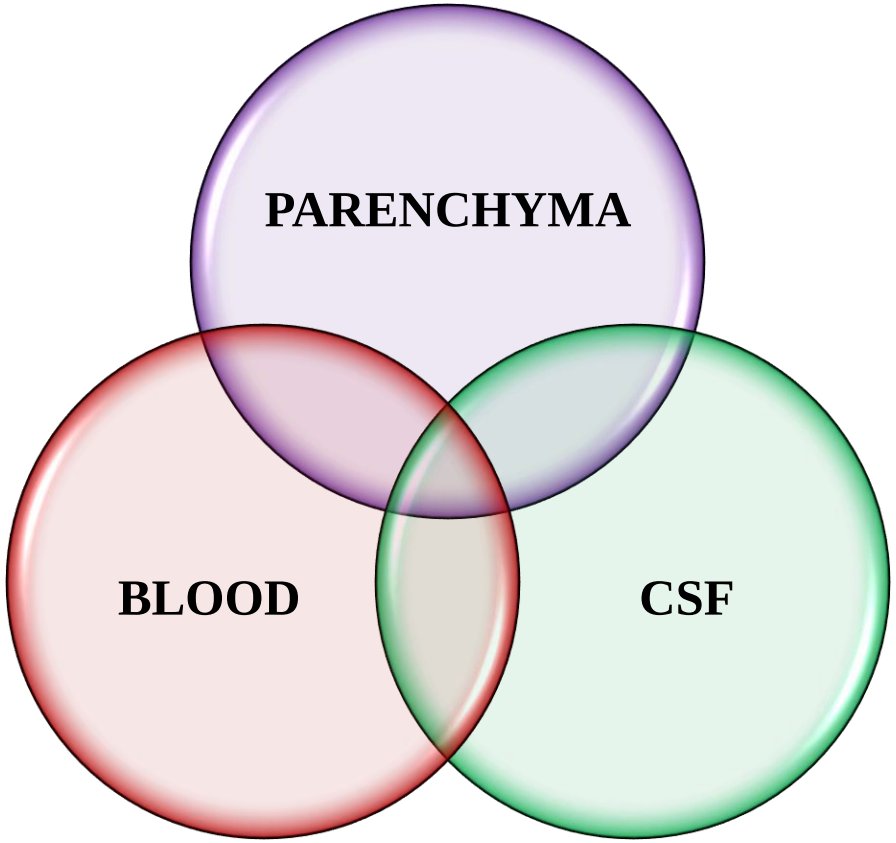
Hydrocephalus

The Monro Kellie Doctrine

The volume of the skull is a constant which contains:

- Brain tissue (parenchyma) –80 — 85% intracranial space,
- Blood – 6 — 8% intracranial space,
- CSF – 5 — 15% intracranial space.

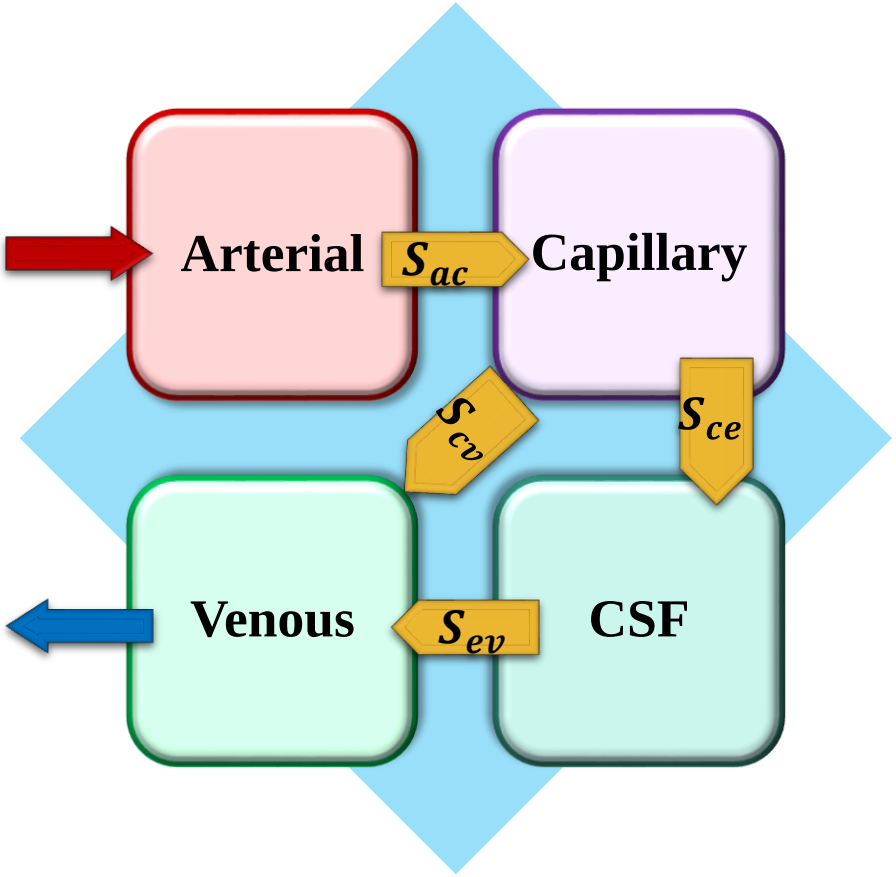
An increase in the volume of one component is compensated for by a decrease in the volume of one or more of the other components



Background on mathematical modeling of intracranial dynamics

- Hakim et al., (1971, 1976) the first authors to use the methods of continuum mechanics to study hydrocephalus. They suggested that the brain parenchyma can be seen as an "open-cell sponge made of viscoelastic material."
- Nagashima, Tamaki et al., (1986, 1987) formalized this notion of Hakim et al. in terms of poroelasticity theory and the finite element method.
- Pena et al. (1999) improved Nagashima's model. The time-dependent evolution of ventricular expansion was presented in the model.
- Most studies using poroelasticity theory are based on simplified geometries, where the brain is assumed to be either spherical or cylindrical: (Kaczmarek, et al., 1997), (G Tenti et al., 1999), (Levine, 1999, 2000, 2008), (Smillie et al., 2005), (Sobey & Wirth 2006), (Tully & Ventikos 2009).
- Extension of poroelasticity models to include detailed mechanisms of transport between cerebral blood and extracellular fluid / CSF: (Tully & Ventikos 2011), (Vardakis et al., 2013), (Guo, Vardakis et al., 2018).

Mathematical modeling of hydrocephalus*



4 networks :

- Arterial blood (a)
- Arteriole/capillary blood (c)
- Venous blood (v)
- CSF (e)

The fluid flow direction. For example, from the arterial network to the capillary.

It is assumed that direct fluid transfer from the arterial network to the CSF or venous network does not occur.

∗ Tully, Ventikos: Cerebral water transport using multiple-network poroelastic theory: application to normal pressure hydrocephalus // J. Fluid Mech. (2011), 667

Final system

$$\mu\Delta\,\boldsymbol{u} + (\mu+\lambda)\nabla(div\,\boldsymbol{u}) - (\alpha_a\nabla p^a + \alpha_c\nabla p^c + \alpha_e\nabla p^e + \alpha_v\nabla p^v) = 0 \quad (1)$$

$$-\frac{k^a}{\mu^a}\Delta p^a + |\dot{s}_{a\to c}| = 0 \quad (2)$$

$$-\frac{k^c}{\mu^c}\Delta p^c - |\dot{s}_{a\to c}| + |\dot{s}_{c\to e}| + |\dot{s}_{c\to v}| = 0, \quad (3)$$

$$-\frac{k^e}{\mu^e}\Delta p^e - |\dot{s}_{c\to e}| + |\dot{s}_{e\to v}| = 0, \quad (4)$$

$$-\frac{k^v}{\mu^v}\Delta p^v - |\dot{s}_{c\to v}| - |\dot{s}_{e\to v}| = 0 \quad (5)$$

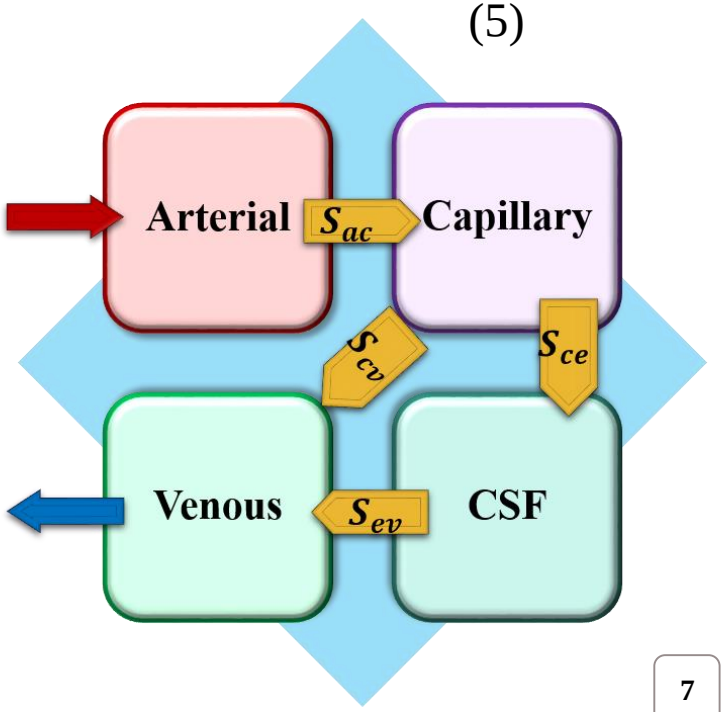
$\boldsymbol{u}$ – brain tissue displacement, p – pore pressure fluid

(1) – equilibrium equation
(2) – (5) – mass conservation law for pore fluids

$+|\dot{s}_{x\to y}|$ – fluid out–flux from network x to network y;
$-|\dot{s}_{y\to x}|$ – fluid in–flux into the network y from the network x.

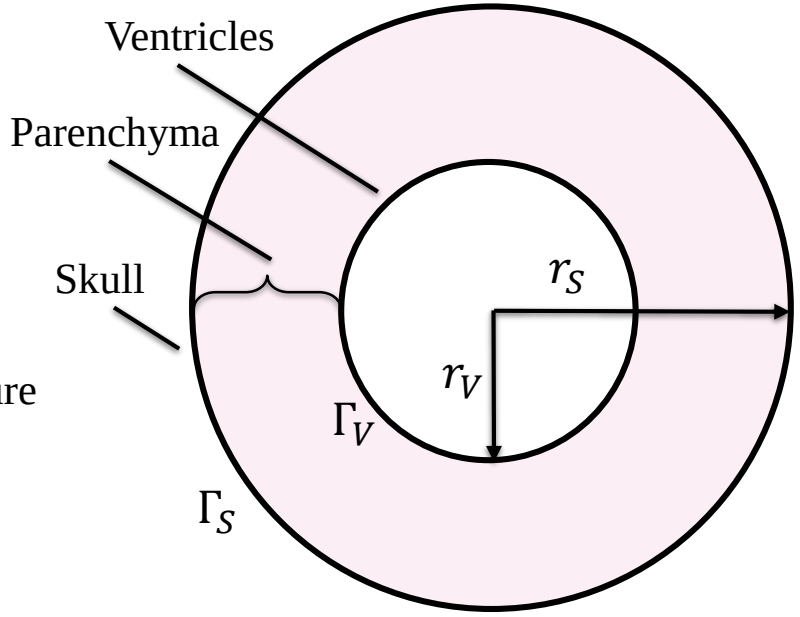
The transfer is considered to be driven by a hydrostatic pressure gradient:

$$\dot{s}_{y\to x} = -\gamma_{yx}\,[p^x - p^y]$$



Boundary conditions

At the ventricle wall Γ_V:



- The stress must be continuous across the wall:

$$2\,\mu\varepsilon(\boldsymbol{u})\cdot\boldsymbol{n}+\lambda\epsilon(\boldsymbol{u})\boldsymbol{n}=\sum_{i=a,c,e,v}\left(\alpha^i-1\right)p^i\,\boldsymbol{n}$$

- The production of CSF from the blood results in a pressure drop in the arteriole/capillary blood network:

$$-k_{c\to ventricle}\nabla p^c\boldsymbol{n}=-Q_p$$

$k_{c\to ventricle}$ - is the resistance of the flow from the capillary network to the ventricles (through the choroid plexus).

- There is no flow into or out of the arterial and venous blood networks:

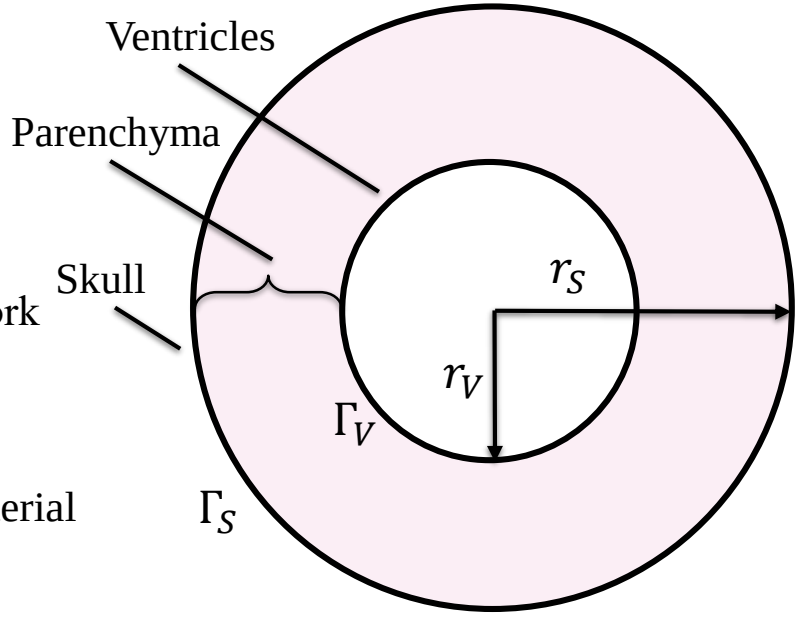
$$\nabla p^a\boldsymbol{n}=\nabla p^v\boldsymbol{n}=0$$

- The mass of fluid in the ventricles is conserved (with d being the diameter of the cerebral aqueduct):

$$Q_p=\frac{\pi\,d^4}{128\,\mu\,L}\left(p^e|_{\Gamma_V}-p^e|_{\Gamma_S}\right)-\oint_{Ventricle}\left(-\frac{k^e}{\mu^e}\nabla p^e\right)\cdot\boldsymbol{n}dS$$

Boundary conditions

At the skull Γ_S:



- The displacement is zero:

$$\boldsymbol{u} = 0$$

- There is no flow into or out of the capillary blood network at the skull :

$$\nabla p^c \boldsymbol{n} = 0$$

- The blood pressures are given by physically relevant arterial and venous blood pressures:

$$p^a = p_{arterial,}\ p^v = p_{venous,}$$

- The absorption of CSF into the venous network results in a pressure rise:

$$p^e = p^v + \mu_e R Q_0$$

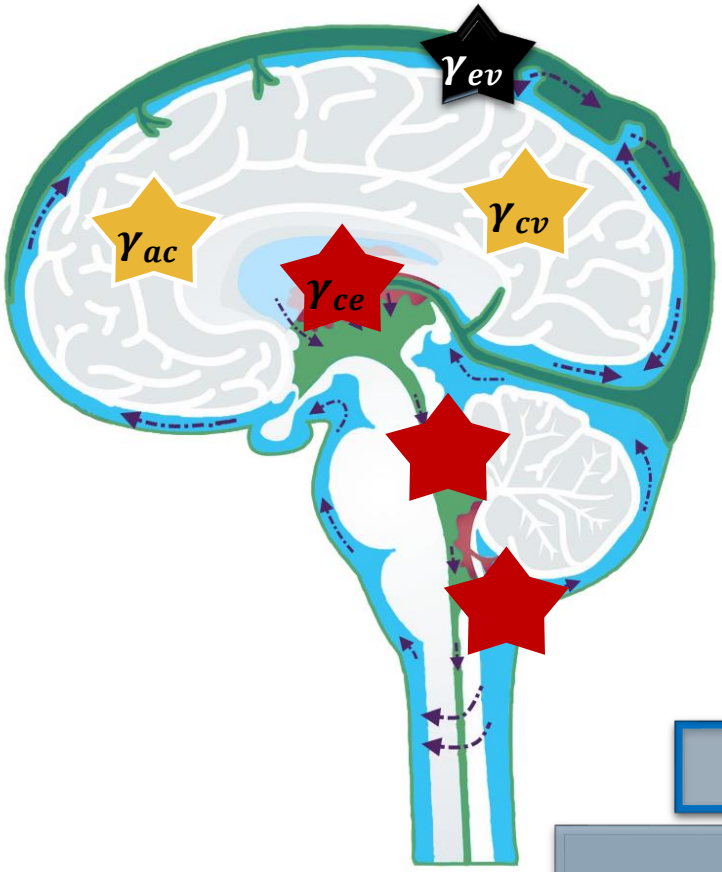
μ_e −is the viscosity of the CSF,

R − is the resistance to outflow through the arachnoid granulations,

Q_0 − is the out−flux of CSF at the skull

Hydrocephalus

Model parameters:



γ_{yx} – pressure mutual influence coefficient between network y and network x:

γ_{ac} – arterial network and a network of arteriole/capillary

γ_{cv} – network of arteriole/capillary and venous network

γ_{ce} – network of arteriole/capillary and CSF network

γ_{ev} – CSF network and venous network

Biot coefficients:

α_a - Biot coefficient of arterial network

α_c - Biot coefficient of capillary network

α_v - Biot coefficient of venous network

α_e - Biot coefficient of CSF network

Parameter space

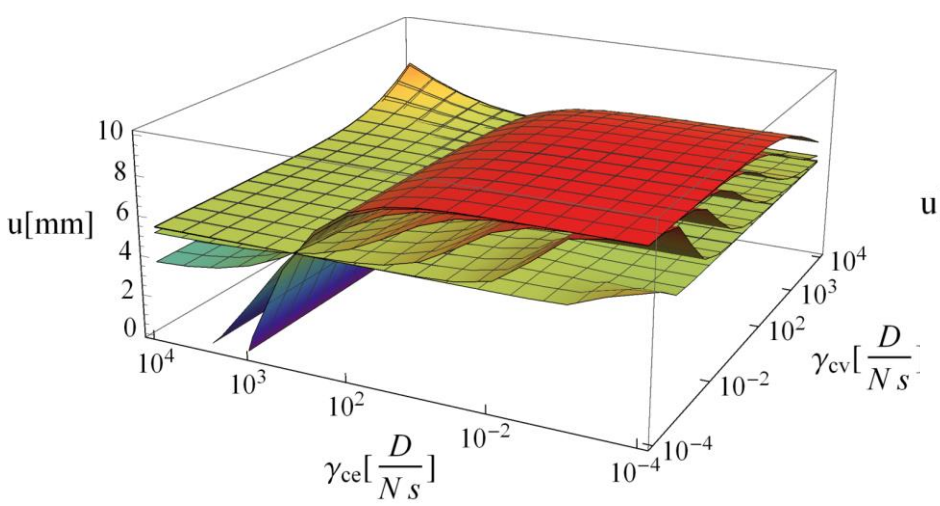
The range of $\gamma_{ac}, \gamma_{cv}, \gamma_{ce}, \gamma_{ev}$: $10^{-4}\ \frac{D}{N\,s}$ to $10^{4}\ \frac{D}{N\,s}$.

Each of the parameters $\gamma_{ac}, \gamma_{cv}, \gamma_{ce}, \gamma_{ev}$ independently took values from a fifteen-element set covering the range of parameters

$$\alpha_a = \alpha_c = \alpha_v = \alpha_e = 0{,}99.$$

The calculation was carried out by the explicit Runge-Kutta method with automatic selection of the integration step and method order

Results: ventricular wall displacement



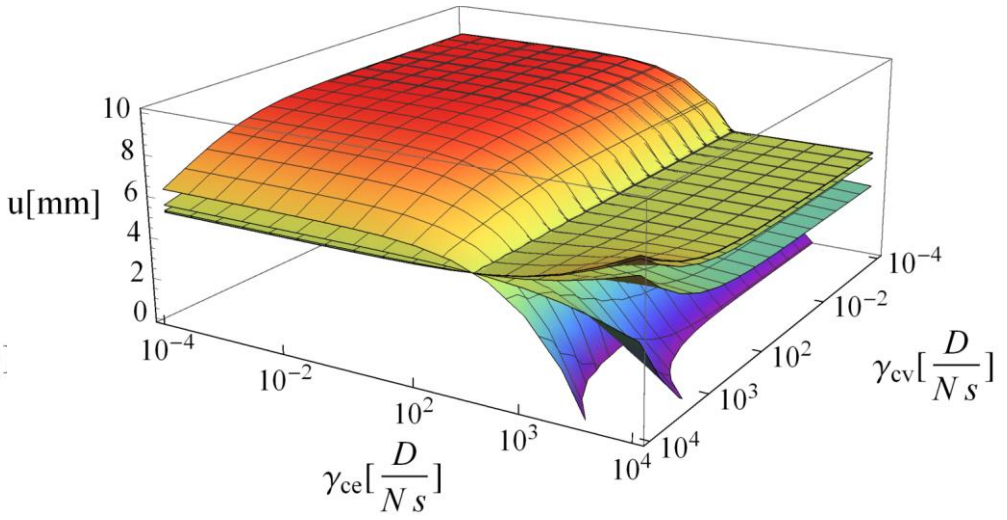
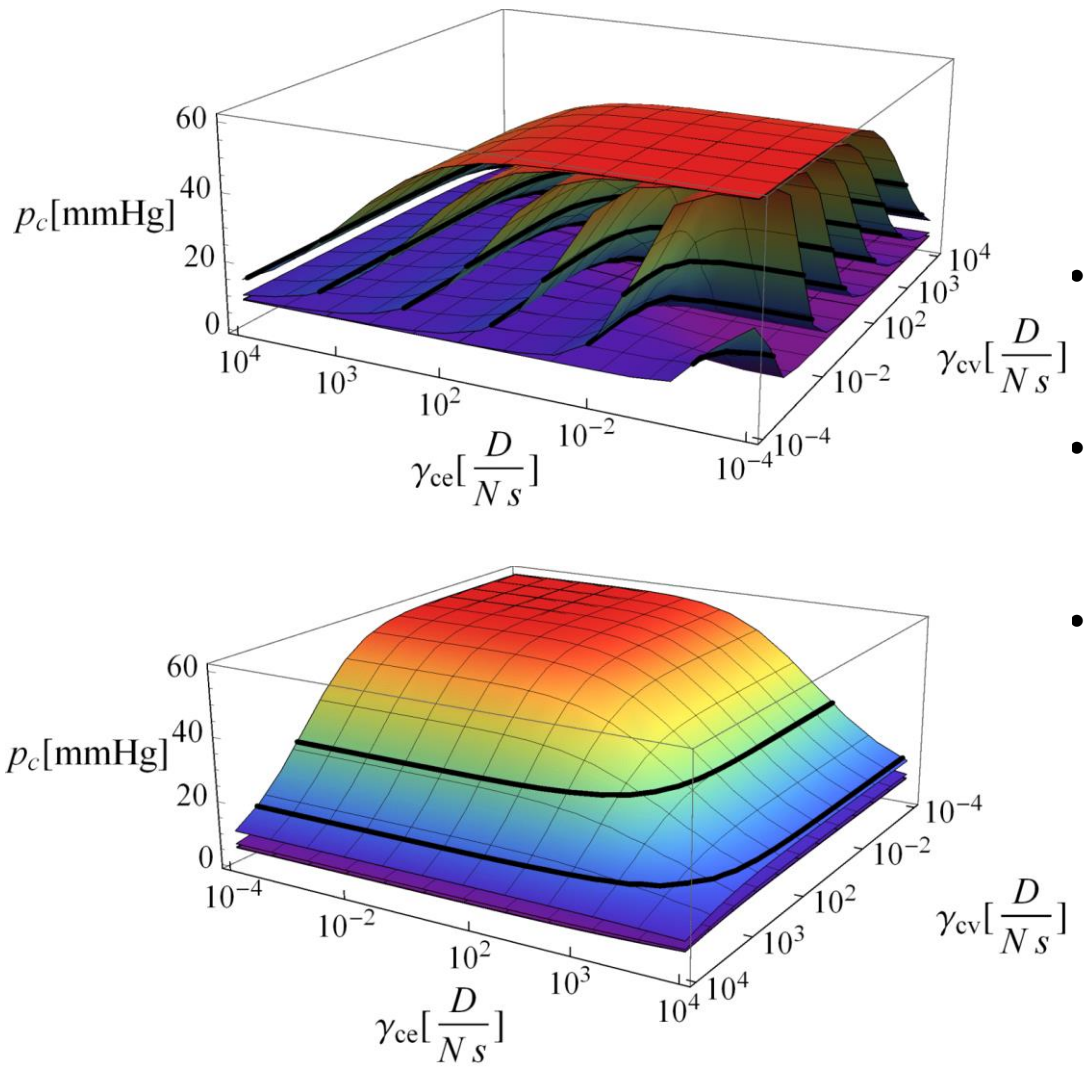


Fig.: Ventricular wall displacement dependence on $\gamma_{ac}, \gamma_{cv}, \gamma_{ce}$ with $\gamma_{ev} = 100\,\frac{D}{N\,s}$

- Decrease γ_{ce} – diminishing of the mutual influence of the capillary-cerebrospinal fluid component → transfer pulsation diminishing → CSF retention in the ventricles → increased ventricular wall displacement
- Increase γ_{ac} – increased the mutual influence of the arterial - capillary component → correct distribution of the pulse wave from arterial to capillary component → small ventricular wall displacement
- Decrease γ_{ac} – arterial-capillary flow obstruction → significant displacement of the ventricular wall and decrease capillary pressure
- Increase γ_{cv} – increased the mutual influence capillary-venous component → overload of the venous component and inverse effect on the cerebrospinal fluid component → increased ventricular wall displacement

Results: The pressure on the ventricular wall



- Decrease γ_{ce} – capillary flow obstruction → capillary pressure increase
- Decrease γ_{ac} – arterial-capillary flow obstruction → capillary pressure increase
- Increase γ_{cv} – corresponds to increased capillary outflow → capillary pressure decrease

Fig.: Capillary pressure dependence on $\gamma_{ac}, \gamma_{cv}, \gamma_{ce}$ with $\gamma_{ev} = 100\,\frac{D}{N\,s}$

Results: The range of physiological norm parameters

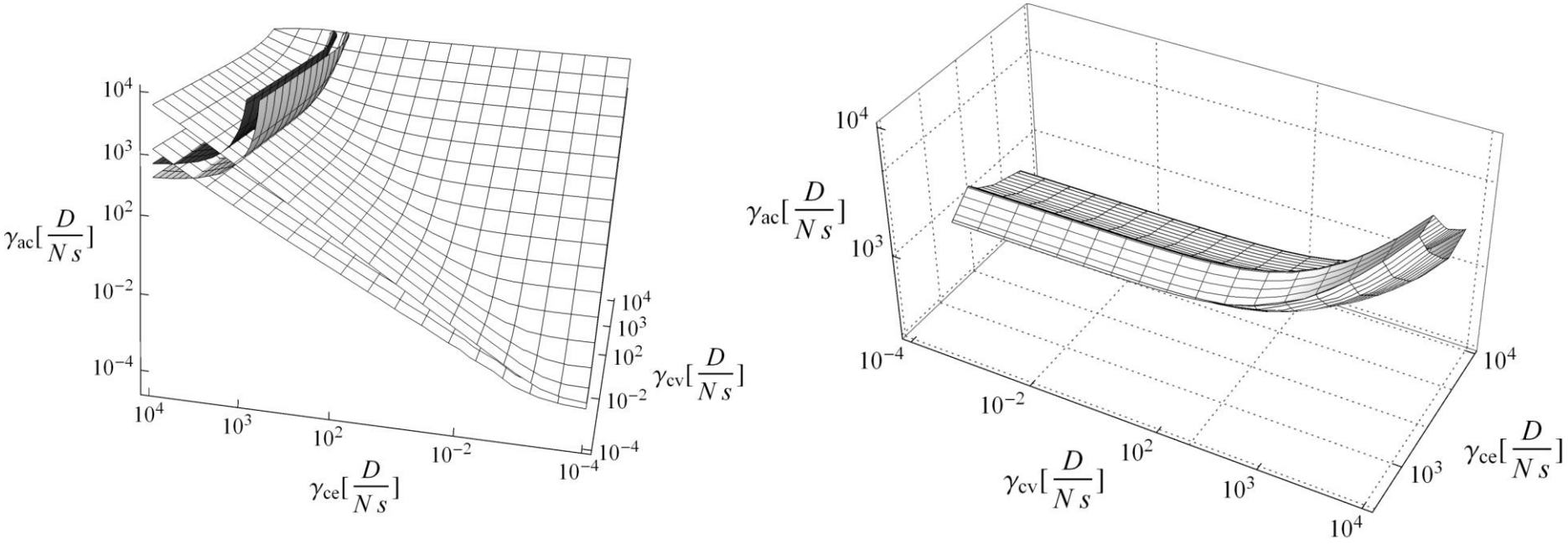


Fig.: The range of physiological norm parameters in the parameter space $\gamma_{ce}, \gamma_{cv}, \gamma_{ac}$

In accordance with the physiological norm, the walls of the ventricles are displaced by no more than 2 mm. The range of physiological norm for capillary pressure is from 15 to 30 mmHg.

Results: Numerical simulation

The governing poroelastic equations were solved with the finite element open package FreeFem++. For finite element modeling we used triangular elements with second order approximation.

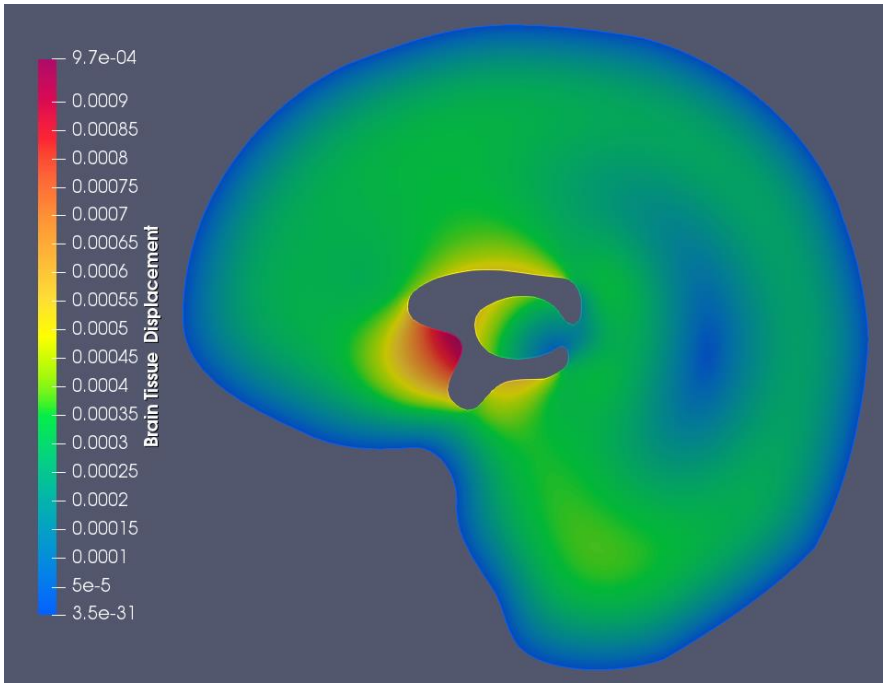


Fig.: Brain tissue displacement [mm] in

$\gamma_{ce} = 2.1 * 10^3 \frac{D}{N\,s}$ $\gamma_{cv} = 1.06 * 10^3 \frac{D}{N\,s}$

$\gamma_{ac} = 1.06 * 10^3 \frac{D}{N\,s}$ $\gamma_{ev} = 4.5 * 10^3 \frac{D}{N\,s}$

(range of physiological norm parameters)

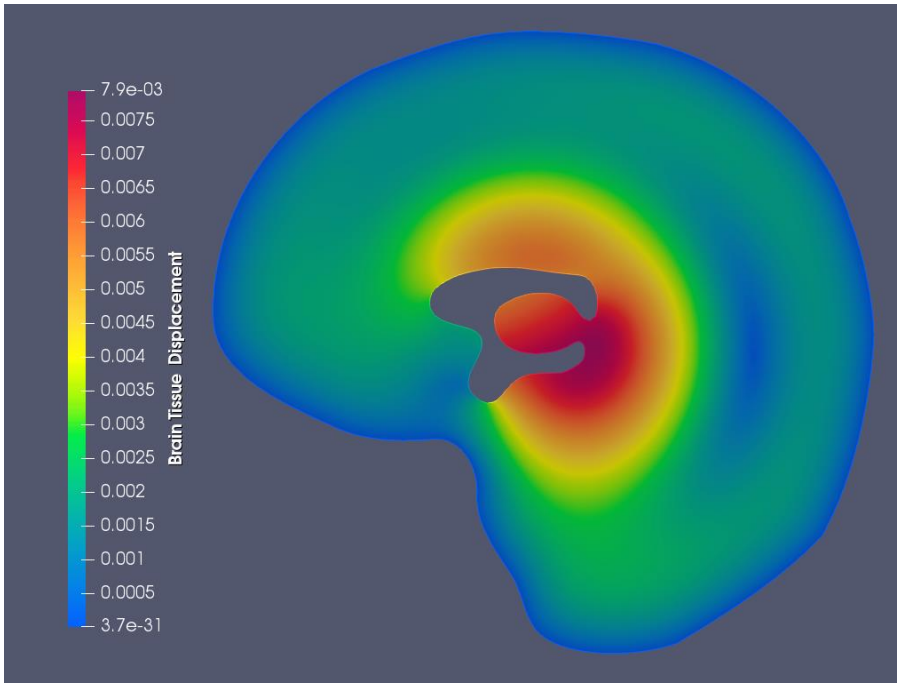


Fig.: Brain tissue displacement [mm] in

$\gamma_{ce} = 1 * 10^4 \frac{D}{N\,s}$ $\gamma_{cv} = 1 * 10^4 \frac{D}{N\,s}$

$\gamma_{ac} = 1.06 * 10^4 \frac{D}{N\,s}$ $\gamma_{ev} = 4.5 * 10^4 \frac{D}{N\,s}$

Results: Numerical simulation

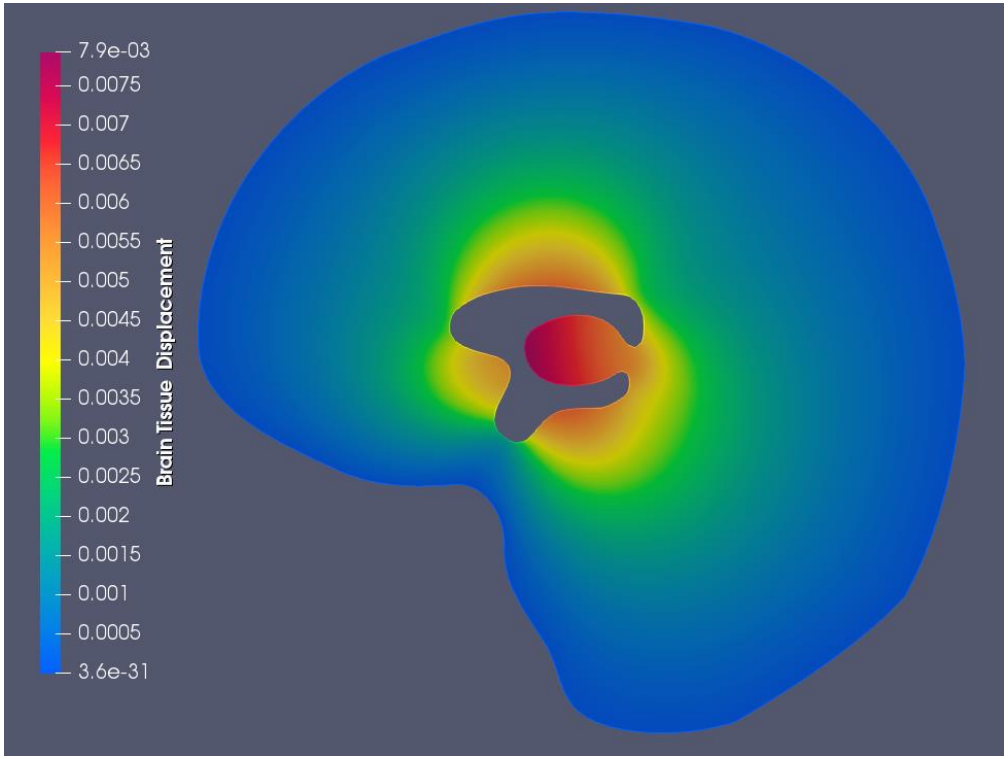


Fig.: Brain tissue displacement in

$$\gamma_{ce} = 1.0 * 10^{-2}\frac{D}{N\,s}, \gamma_{cv} = 1.0 * 10^{-2}\frac{D}{N\,s}, \gamma_{ac} = 1.06 * 10^{-2}\frac{D}{N\,s}, \gamma_{ev} = 4.5 * 10^{-2}\frac{D}{N\,s}$$

Conclusion

1. The model allows us to describe both a healthy state of an organism and a state of an organism at hydrocephalus
2. The model behavior correlates with the actual mechanisms of blood and CSF circulations
3. Calculations for simple geometry can be used to estimate the behavior of quantities in complex geometry