Elastic, dipole-dipole interaction and viscosity impact on vibrational properties of anisotropic hexagonal microtubule lattice

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Highlights

- The microtubule is modeled as a macroobject presenting a system of bound tubulins.
- Frequencies of the waves along the microtubule axis, along helix and anti-helix directions have been calculated.
- Dipole-dipole interaction impact on frequency values has been investigated.
- Three different wave polarization directions have been considered.
- Calculations have been performed for microtubules immersed in a viscous environment.

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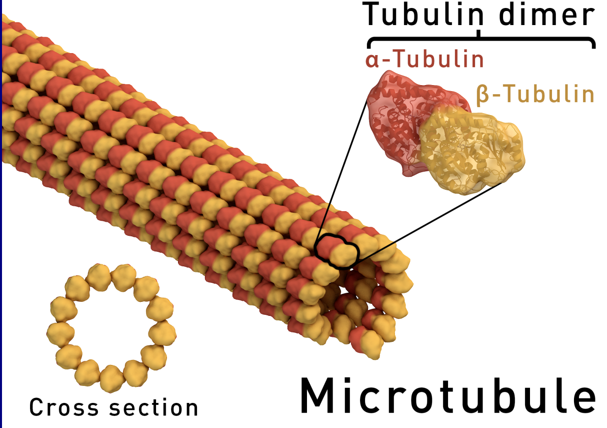
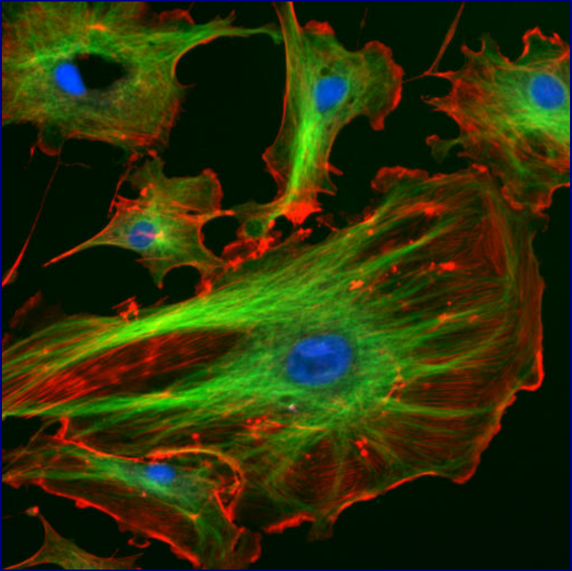
Microtubule



Structure of a microtubule. The ring shape depicts a microtubule in cross-section, showing the 13 protofilaments surrounding a hollow center. Microtubules are an important component of a cell cytoskeleton, including that of neurons.

Cytoskeleton

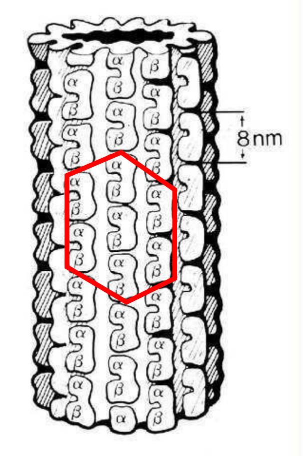


Components of the eukaryotic cytoskeleton. Actin filaments are shown in red, microtubules are in green, and the nuclei are in blue. The cystoskeleton provides the cell with an inner framework and enables it to move and change shape.

Motivation

Nowadays, there exists a supposition that microtubules are a system operating similarly to a kind of onboard computer of the cell, whose work is based on quantum calculations. However, this problem has not yet possessed an unambiguous solution since as a rule biological systems are considered to be too warm, wet and noisy for quantum processes to take place in them. However microtubules can have properties which allow them to avoid decoherence, thus depending functionally on quantum processes. In this connection, a microtubule can be presented as a macrosystem interacting with microobjects that are of quantum nature.

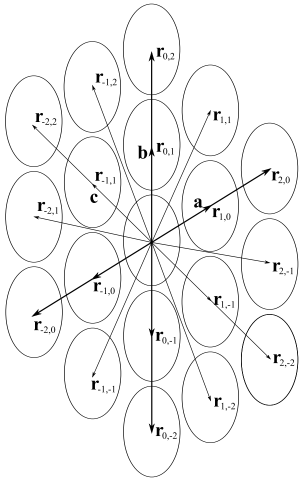
Motivation



Elementary cell of hexagonal microtubule lattice.

Thus, investigating the role of quantum effects in microtubules in this case calls for building a model of a microtubule as a macroobject in the first place, whose mechanical properties are to be further investigated on the base of approaches thoroughly considering the influence of some factors, the most important of which are dipole-dipole interaction of tubulins and the viscosity of intracellular environment. Besides, the investigation should be conducted with the hexagonal two-dimensional microtubule structure and its anisotropic properties taken into consideration. At present, there are no models, in which all of these factors would be determined with a precision required for this.

Model: Elastic interaction



A microtubule lattice section.

The expression for the force acting on the central node from the neighboring nodes sides is as follows

$$
\begin{aligned}
\mathbf{F}^{el}_{n,m} = \varkappa_1 \big(\mathbf{e}_1 \cdot (\mathbf{u}_{n+1,m} - 2\mathbf{u}_{n,m} + \mathbf{u}_{n-1,m})\big)\mathbf{e}_1 + \\
+\varkappa_2 \big(\mathbf{e}_2 \cdot (\mathbf{u}_{n,m+1} - 2\mathbf{u}_{n,m} + \mathbf{u}_{n,m-1})\big)\mathbf{e}_2 + \\
+\varkappa_3 \big(\mathbf{e}_3 \cdot (\mathbf{u}_{n-1,m+1} - 2\mathbf{u}_{n,m} + \mathbf{u}_{n+1,m-1})\big)\mathbf{e}_3,
\end{aligned}
$$

with $\mathbf{u}_{n,m}$ being the deviation of an (n, m) lattice node from the state of equilibrium; the unit vectors $\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3$ being directed from the node $(0, 0)$ to the nodes $(1, 0), (0, 1), (-1, 1)$; $\varkappa_1, \varkappa_2, \varkappa_3$ being the elastic constants between $(0, 0)$ and $(1, 0), (0, 1), (-1, 1)$ lattice nodes.

Model: Elastic interaction

Wavelike displacement of tubulin dimers has the form of

$$\begin{aligned}\mathbf{u}_{n,m}(t) &= \mathbf{u}_{0,0}(0)e^{-i\omega t}e^{i\mathbf{k}\cdot\mathbf{r}_{n,m}} = \\ &= \mathbf{u}_{0,0}(0)e^{-i\omega t}e^{i(k_1 r^1_{n,m}+k_2 r^2_{n,m})} = \\ &= \mathbf{u}_{0,0}(0)e^{-i\omega t}e^{i(k_1 na+k_2 mb)}, \end{aligned} \quad (1)$$

with $\mathbf{u}_{0,0}$ is a vector amplitude, ω is a frequency, and $\mathbf{k}$ is a two-dimensional wave vector with a components k_1 and k_2. According to Newton's second law and with regard for (1)

$$\mathbf{F}^{el}_{n,m} = m_0\frac{d^2\mathbf{u}_{n,m}}{dt^2} = -m_0\omega^2\mathbf{u}_{n,m}, \quad (2)$$

Model: Elastic interaction

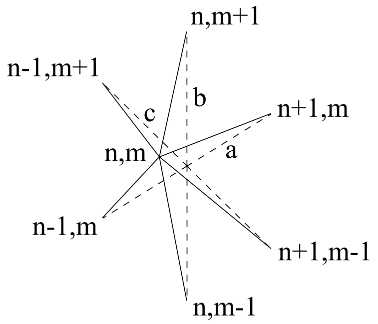
Solving the system of equations (2), we obtain an expression for the frequency

$$\begin{aligned}\omega^2 &= \frac{1}{2}\Big\{\omega_1^2+\omega_2^2+\omega_3^2 \pm \Big[\omega_1^4+\omega_2^4+\omega_3^4+ \\ &\quad +2\omega_1^2\omega_2^2\cos(2\alpha_{12})+2\omega_2^2\omega_3^2\cos(2\alpha_{23})+2\omega_3^2\omega_1^2\cos(2\alpha_{13})\Big]^{1/2}\Big\},\end{aligned}$$

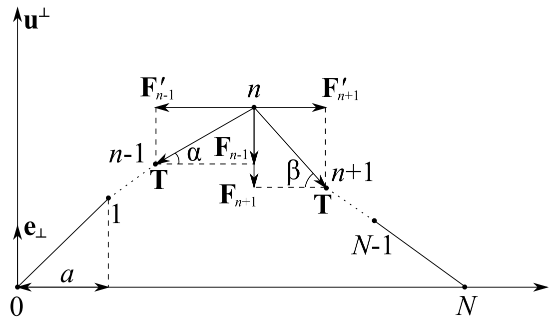
with ω_1^2, ω_2^2, ω_3^2 are determined by the equalities

$$\begin{aligned}\omega_1^2 &= 4\frac{\varkappa_1}{m_0}\sin^2\left(\frac{k_1 a}{2}\right),\\ \omega_2^2 &= 4\frac{\varkappa_2}{m_0}\sin^2\left(\frac{k_2 b}{2}\right),\\ \omega_3^2 &= 4\frac{\varkappa_3}{m_0}\sin^2\left(\frac{k_1 a-k_2 b}{2}\right).\end{aligned}$$

Model: Transverse lattice oscillations



Hexagonal lattice node transverse oscillations.



Forces acting on a lattice node, with transverse oscillations occurring.

Similarly, we obtain an expression for the frequency for the transverse vibrations of the microtubule lattice

$$(\omega_{\perp}^{el})^2 = \frac{T_1}{a\varkappa_1}\omega_1^2 + \frac{T_2}{b\varkappa_2}\omega_2^2 + \frac{T_3}{c\varkappa_3}\omega_3^2$$

with T_1, T_2, T_3 are tension forces along helix, protofilament, anti-helix directions respectively.

Model: Viscosity impact

We consider viscosity supposing that the viscous friction force $\mathbf{F}^{vis}_{n,m}$ acting on (n, m) node is directly proportional to the velocity

$$\mathbf{F}^{vis}_{n,m} = -\gamma \dot{\mathbf{u}}_{n,m}, \tag{3}$$

with γ being the damping coefficient. We write down Newton's second law with regard for viscosity

$$m_0 \ddot{\mathbf{u}}_{n,m} = \mathbf{F}^{el}_{n,m} + (\mathbf{F}^{el}_{n,m})^{\perp} + \mathbf{F}^{vis}_{n,m}. \tag{4}$$

Let us suggest that the viscosity impact on the microtubule results in the damped oscillations of the lattice nodes from law

$$\mathbf{u}_{n,m}(t) = \mathbf{v}_{n,m}(t) e^{-\frac{\gamma}{2m_0} t}, \tag{5}$$

with the oscillations amplitude $\mathbf{v}_{n,m}(t)$ depending on time

$$\mathbf{v}_{n,m}(t) = \mathbf{v}_{n,m}(0) e^{-i\omega t}. \tag{6}$$

Model: Viscosity impact

Solving the systems of equations (4) under assumptions (5), (6), we obtain expressions for the squared frequency with regard for elastic oscillations in the plane of the lattice and viscosity

$$(\omega_{1,2}^{el,vis})^2 = \\ = \frac{-\left(f_{11}^{el} + f_{22}^{el} + \frac{\gamma^2}{2m_0}\right) \pm \sqrt{(f_{11}^{el} - f_{22}^{el})^2 + 4f_{12}^{el}f_{21}^{el}}}{2m_0}, \tag{7}$$

as well as an expression for the squared frequency with regard for transverse oscillations and viscosity

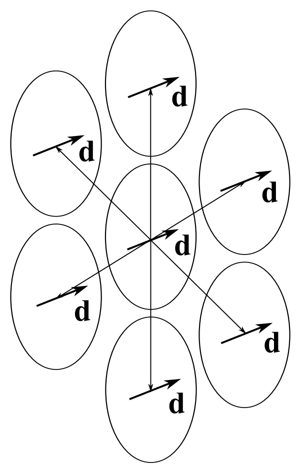
$$(\omega_{\perp}^{el,vis})^2 = -\frac{\left(f_{\perp\perp}^{el} + \frac{\gamma^2}{4m_0}\right)}{m_0}, \tag{8}$$

Model: Viscosity impact

with

$$
\begin{aligned}
f^{el}_{11} &= -4\varkappa_1 \sin^2\left(\frac{k_1 a}{2}\right) + 4\varkappa_3 \sin^2\left(\frac{k_1 a - k_2 b}{2}\right)\frac{\cos\alpha_{13}\sin\alpha_{23}}{\sin\alpha_{12}},\\
f^{el}_{12} &= -4\varkappa_1 \sin^2\left(\frac{k_1 a}{2}\right)\cos\alpha_{12} + 4\varkappa_3 \sin^2\left(\frac{k_1 a - k_2 b}{2}\right)\frac{\cos\alpha_{23}\sin\alpha_{23}}{\sin\alpha_{12}},\\
f^{el}_{21} &= -4\varkappa_2 \sin^2\left(\frac{k_2 b}{2}\right)\cos\alpha_{12} - 4\varkappa_3 \sin^2\left(\frac{k_1 a - k_2 b}{2}\right)\frac{\cos\alpha_{13}\sin\alpha_{13}}{\sin\alpha_{12}},\\
f^{el}_{22} &= -4\varkappa_2 \sin^2\left(\frac{k_2 b}{2}\right) - 4\varkappa_3 \sin^2\left(\frac{k_1 a - k_2 b}{2}\right)\frac{\cos\alpha_{23}\sin\alpha_{13}}{\sin\alpha_{12}}.
\end{aligned}
$$

Model: Dipole-dipole interaction



The force acting on the side of the $(1,0)$ dipole on the $(0,0)$ dipole is determined by the equality

$$\mathbf{F}_{10} = \frac{3k}{r_{10}^5}\left\{\left[d^2 - 5\left(\mathbf{d}\cdot\mathbf{r}_{10}\right)^2/r_{10}^2\right]\mathbf{r}_{10} + \right.$$
$$\left. +2\left(\mathbf{d}\cdot\mathbf{r}_{10}\right)\mathbf{d}\right\},$$

with $k = 1/(4\pi\varepsilon\varepsilon_0)$ being the Coulomb constant; **d** – tubulin dipole moment.

Model: Dipole-dipole interaction

The solution of Newton's equations taking into account elasticity, viscosity and dipole-dipole interaction

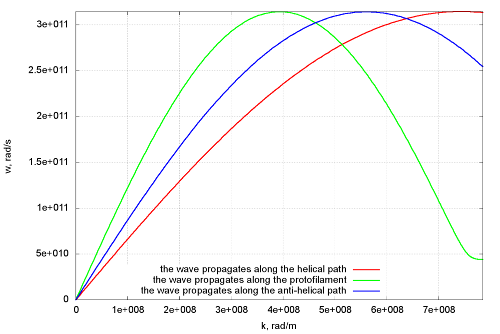
$$m_0 \ddot{\mathbf{u}}_{n,m} = \mathbf{F}^{el}_{n,m} + (\mathbf{F}^{el}_{n,m})^{\perp} + \mathbf{F}^{vis}_{n,m} + \mathbf{F}^{dip}_{n,m}.$$

leads to the cubic equation for the squared frequency ω^2

$$\begin{vmatrix} f^{el}_{11} + f^{dip}_{11} + \frac{\gamma^2}{4m_0} + m_0\omega^2 & f^{el}_{12} + f^{dip}_{12} & f^{dip}_{1\perp} \\ f^{el}_{21} + f^{dip}_{21} & f^{el}_{22} + f^{dip}_{22} + \frac{\gamma^2}{4m_0} + m_0\omega^2 & f^{dip}_{2\perp} \\ f^{dip}_{\perp 1} & f^{dip}_{\perp 2} & f^{el}_{\perp\perp} + f^{dip}_{\perp\perp} + \frac{\gamma^2}{4m_0} + m_0\omega^2 \end{vmatrix} = 0,$$

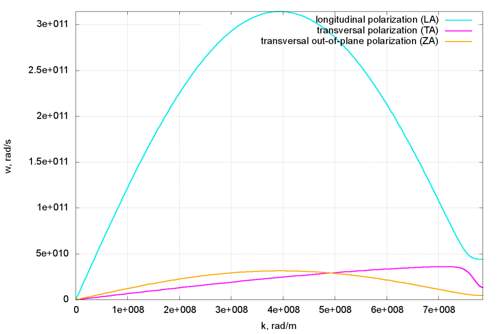
from which we obtain a result for the frequencies.

Results: Different directions of wave propagation

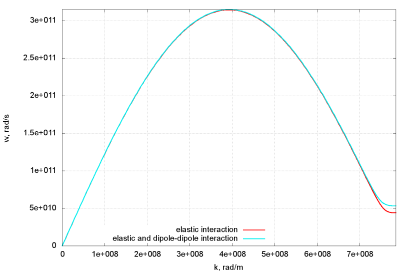


The figure presents the dependence of frequency on the wave vector module for three different directions of wave propagation, namely helix, along the protofilament and anti-helix ones. In this case, the lattice node oscillations along the microtubule axis were considered (i. e. the wave is polarized along the microtubule axis) and elastic interaction between the lattice nodes was only taken into account. Figure demonstrates that the first frequency peak is reached at the wave length λ_{max} amounting to the doubled distance between the lattice nodes in this propagation path. The frequency peak being the same for all of the directions under consideration and amounting to 3.15×10^{11} rad/s.

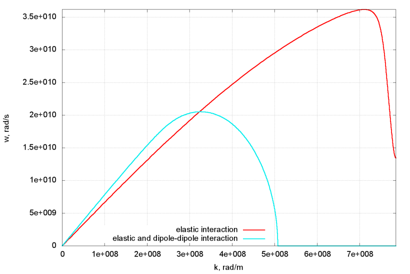
Results: Different polarization directions



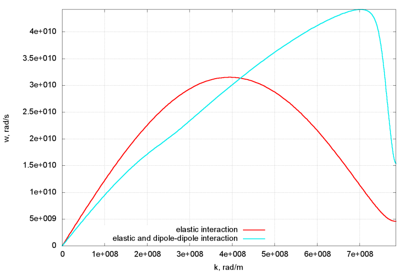
The figure presents the frequency dependence on the wave vector for different polarization directions, with the wave propagating along the protofilament. The figure suggests that the frequency peak is reached in the case of the longitudinally polarized waves. For two other cases (TA and ZA), the frequency is considerably lower. Thus, for the transversal oscillations (TA), the frequency peak is reached at $k = 7.15 \times 10^8$ rad/m and is $\omega = 3.6 \times 10^{10}$ rad/s. For the oscillations occurring out-of-plane (ZA), the frequency peak is reached at $k = 3.95 \times 10^8$ rad/m and is $\omega = 3.2 \times 10^{10}$ rad/s.



1) Calculations for the longitudinally polarized waves, with the wave propagating along the protofilament.

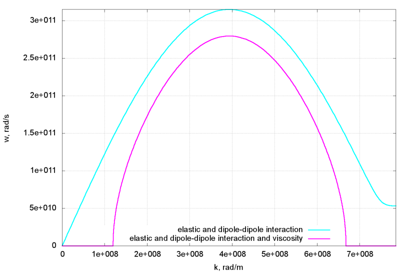


2) Calculations for the transversely polarized waves, with the wave propagating along the protofilament.

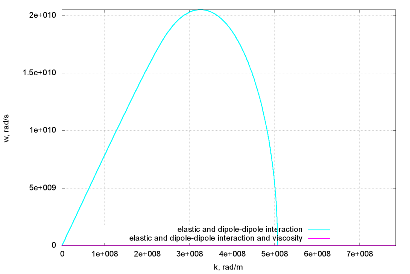


3) Calculations for the out-of-plane polarized waves, with the wave propagating along the protofilament.

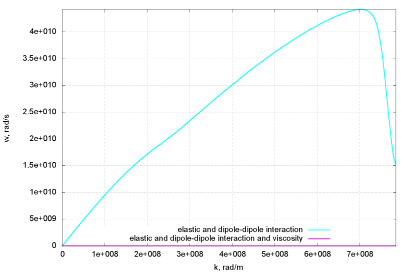
The figures present dipole-dipole impact on the wave frequency. The dipole-dipole interaction impact depends on the mutual orientation of the wave polarization and the dipole moment direction. There is the range of wave lengths for the transversely polarized waves (figure 2) in wich the frequency becomes an absolutely imaginary value, which may correspond to the lattice destruction.



1) The longitudinally polarized waves, with the wave propagating along the protofilament.



2) The transversely polarized waves, with the wave propagating along the protofilament.



3) The out-of-plane polarized waves, with the wave propagating along the protofilament.

The figures present viscosity impact on the wave frequency. Viscosity decreases the oscillation frequency value significantly for the longitudinally polarized waves. In the areas $0 < k < 1.2 \times 10^8$ rad/m and $k > 6.7 \times 10^8$ rad/m there are no damping harmonic oscillations. In these areas, the lattice nodes deviation from the state of equilibrium decreases exponentially. Viscosity dampens the oscillations occurring in the TA and ZA directions respectively. In these cases there are no solutions corresponding to the damping harmonic oscillations.

Conclusion

- It has been shown that the direction of the wave polarization influences the frequency velocity values in the lattice considerably.
- The impact of dipole-dipole interaction greatly depends on the direction of the wave polarization; thus, it is only moderate for the longitudinally (LA) polarized waves while it is sufficient for the transversely (TA), and out-of-plane (ZA) polarized waves. Moreover dipole-dipole interaction may result in the waves which are able to cause the rupture of microtubules.
- With viscosity considered, lattice oscillations become harmonically damped only over a certain wavelength range when longitudinal polarization occurs. Out of this range as well as for the other polarization directions, lattice deviations from equilibrium are dampened exponentially.

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Publications

The results were published in the articles:

- S.Eh. Shirmovsky, D.V. Shulga, Elastic, dipole-dipole interaction and viscosity impact on vibrational properties of anisotropic hexagonal microtubule lattice, **Biosystems** 166 (2018) 1-18.
- S.Eh. Shirmovsky, D.V. Shulga, Microtubules lattice equal-frequency maps: The dynamics of relief changes in dependence on elastic properties, tubulins' dipole-dipole interaction and viscosity, **Physica A** 534 (2019) 122165.

Thank you for your attention