Geometric modeling of a reconstructed aortic valve closure

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Collaborative work with

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- Alexander Danilov
- Roman Pryamonosov
- Pavel Karavaykin
- German Kopytov

Outline of the talk

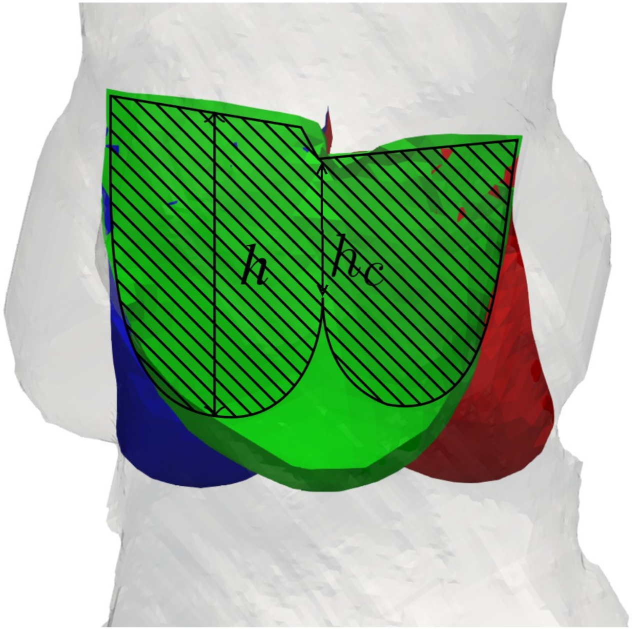
- Objective and input personalised data
- Algorithms used for modeling
- Benchmarks
- Numerical results

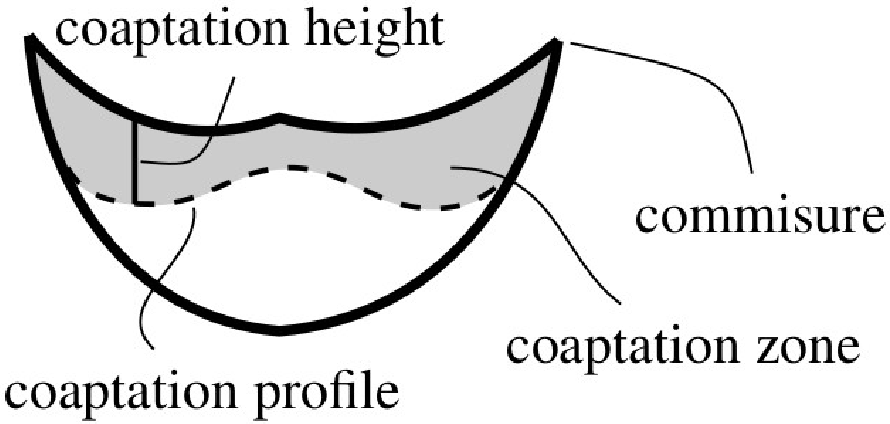
Objective

Develop a technology for assessing the area of coaptation (closure) of the reconstructed aortic valve.

Now we consider the following parameters of coaptation:

- **Height of the coaptation** is the maximum distance in the contact area of at least two leaflets along the blood flow line.
- **Maximum/minimum area of coaptation** is the maximum/minimum area of contact over leaflets on one leaflet.
- **Central coaptation** is the height of the contact area of the three leaflets along the bloodstream





Here h is height of coaptation and h_c is central coaptation

Input data

- Computer Tomography Angiography (ceCTA)
- Diastolic pressure in the aorta
- The parameters of the patient's pericardial stiffness

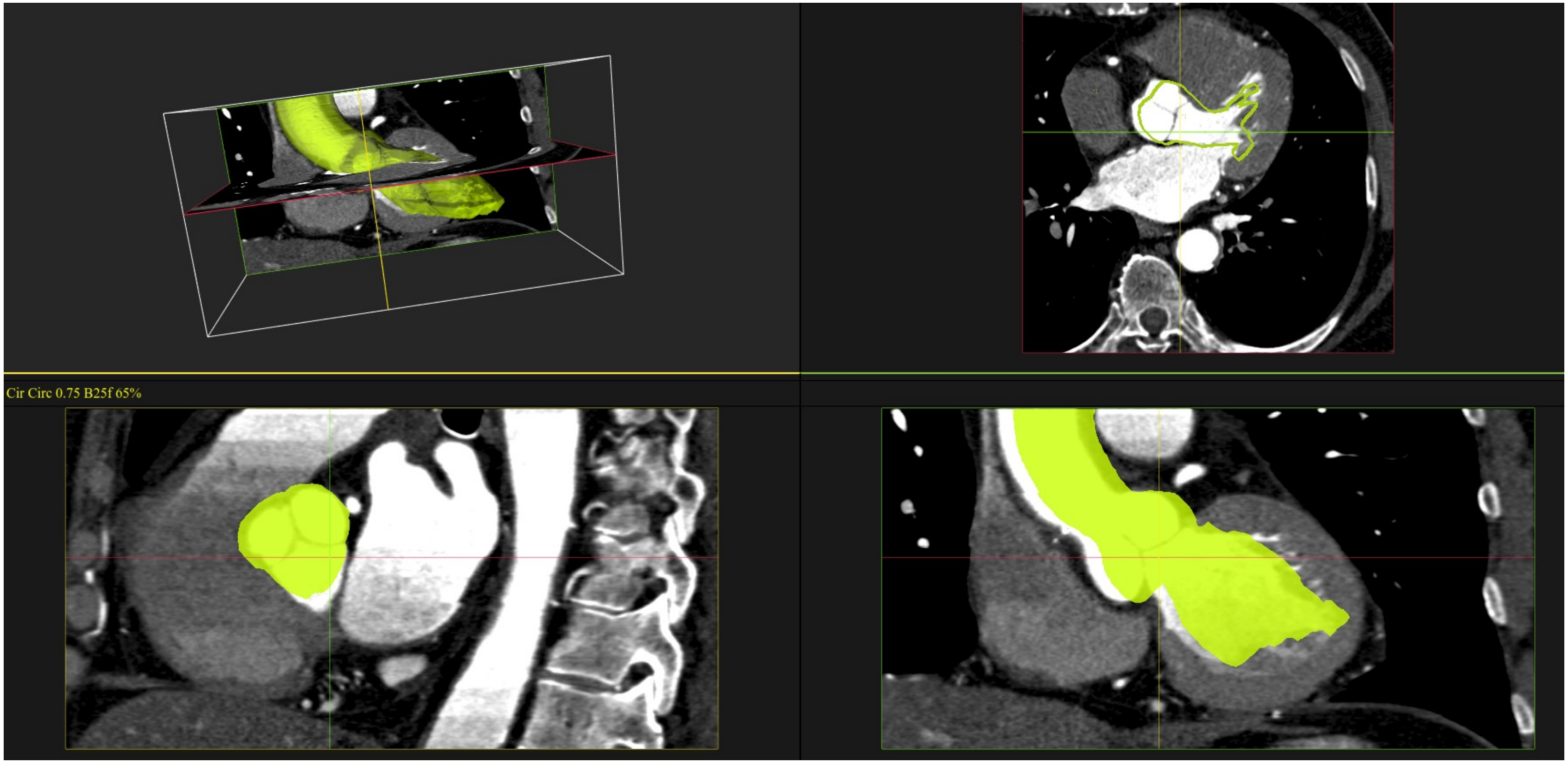
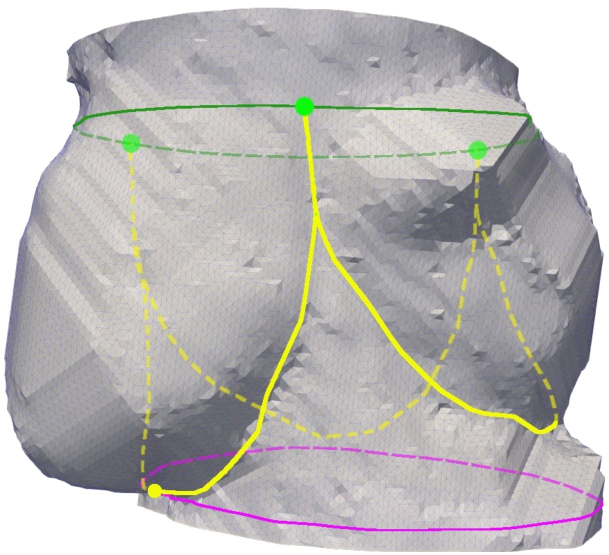
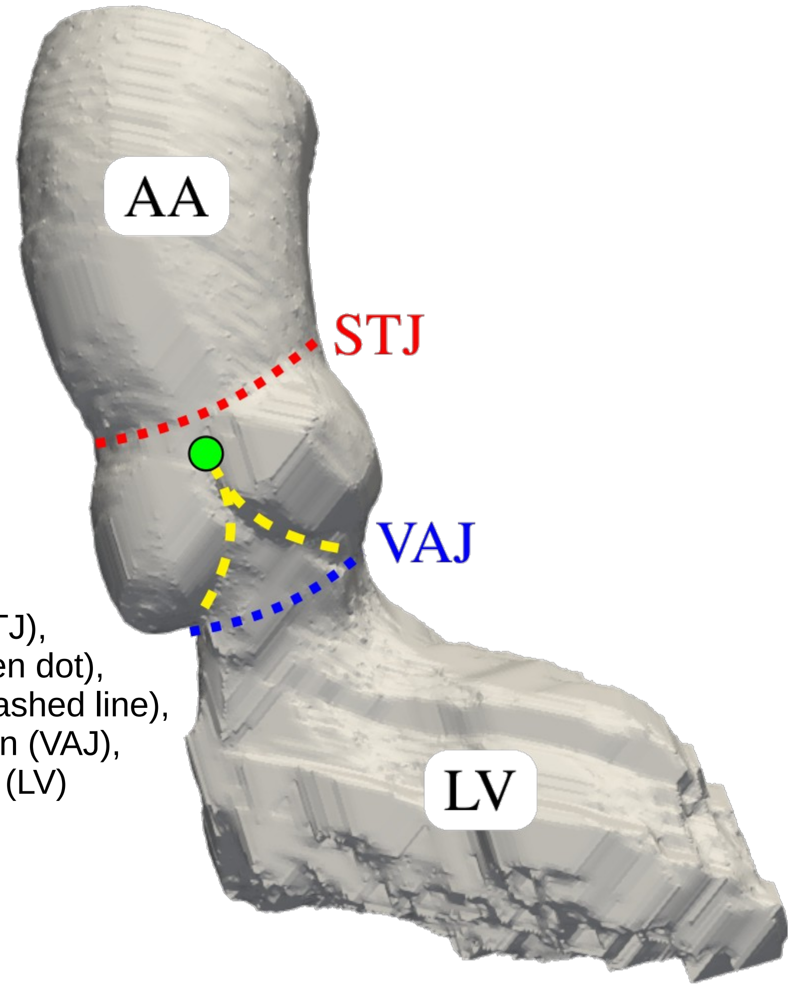


Image processing

- The aorta segmentation algorithm is based on Hough Circleness filter, Isoperimetric Distance Trees (IDT) algorithm, and mathematical morphology operators. ITK SNAP software and CGAL library generate a surface mesh by the marching cubes algorithm.
- Then on the image of the aorta, the points of commissure are selected and through them the lines of attachment (suturing path) of the future leaflets are constructed. Now this is being done manually; in the future, algorithms will be developed for doing this in a semi-automatic mode.



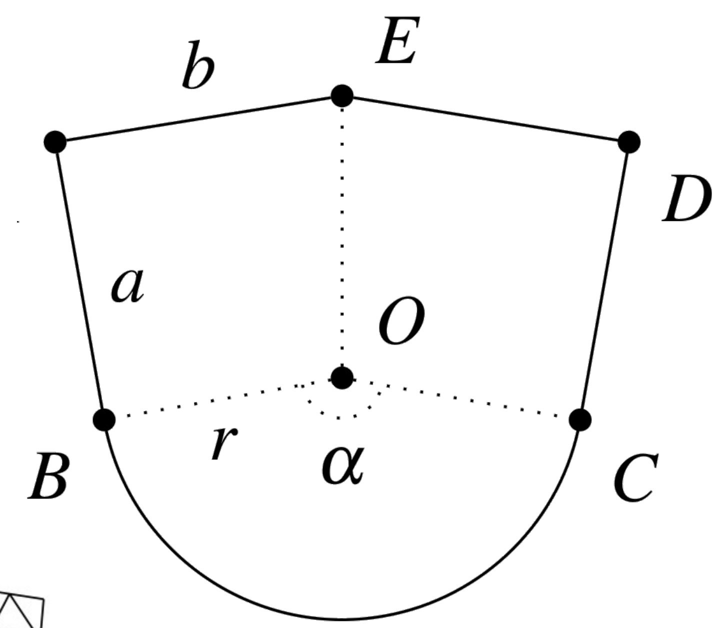
Surface mesh:
ascending aorta (AA),
sinotubular junction (STJ),
commissure point (green dot),
suturing path (yellow dashed line),
ventriculo-aortic junction (VAJ),
part of the left ventricle (LV)

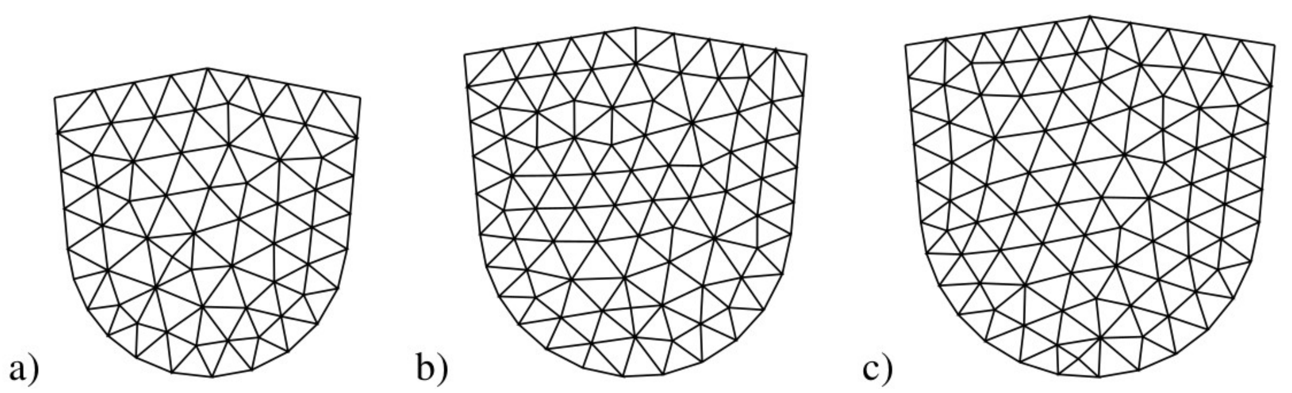


[1]Danilov, A., Ivanov Yu., Pryamonosov R., Vassilevski Yu.: Methods of graph network reconstruction in personalized medicine. Int. J. Numer. Method. Biomed. Eng. (2016) doi: 10.1002/cnm.2754
[2]Duda, R.O., Hart, P.E.: Use of the Hough Transformation to Detect Lines and Curves in Pictures. Commun. ACM (1972) doi: 10.1145/361237.361242
[3]Grady, L.: Fast, Quality, Segmentation of Large Volumes – Isoperimetric Distance Trees. Computer Vision – ECCV (2006), doi: 10.1007/11744078 35
[4]Yushkevich, P.A., Piven, J., Cody Hazlett, H., Gimpel Smith, R., Ho, S., Gee, J.C., Gerig, G.: User-Guided 3D Active Contour Segmentation of Anatomical Structures: Significantly Improved Efficiency and Reliability. Neuroimage (2006), doi: 10.1016/j.neuroimage.2006.01.015
[5]Rineau L., Yvinec M.: A generic software design for Delaunay refinement meshing. Computational Geometry (2007), doi: 10.1016/j.comgeo.2006.11.008

Parametrization and triangulation of leaflets

- A 4-parameter leaflet template is used with the following parameters:
  - the radius of circular arc **r**
  - the circular arc angle (BOC) **α** (is assumed to be fixed, α = 160˚)
  - the length of extension segments (B − A and C − D) **a**
  - the length of free segments (A − E and E − D) **b**.
- We use the advancing front triangular mesher from Ani2D package for uniform mesh generation.





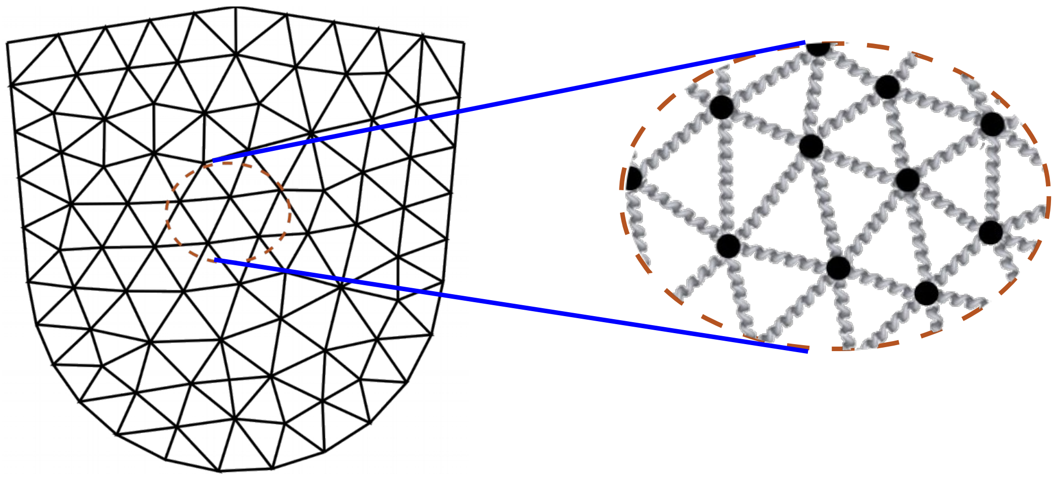
Leaflet uniform triangular meshes with mesh size h = 3 mm:
a) r = 11mm, a = 11mm, b = 11mm, b) r = 12mm, a = 13mm, b = 13mm,
c) r = 13mm, a = 13mm, b = 14mm, d) r = 14.5mm, a = 15.8mm, b = 13.5mm

Advanced Numerical Instruments 2D, Lipnikov, K. and Vassilevski, Yu. and Danilov, A. And others. https://sourceforge.net/projects/ani2d

Nodal-force model (NFM)

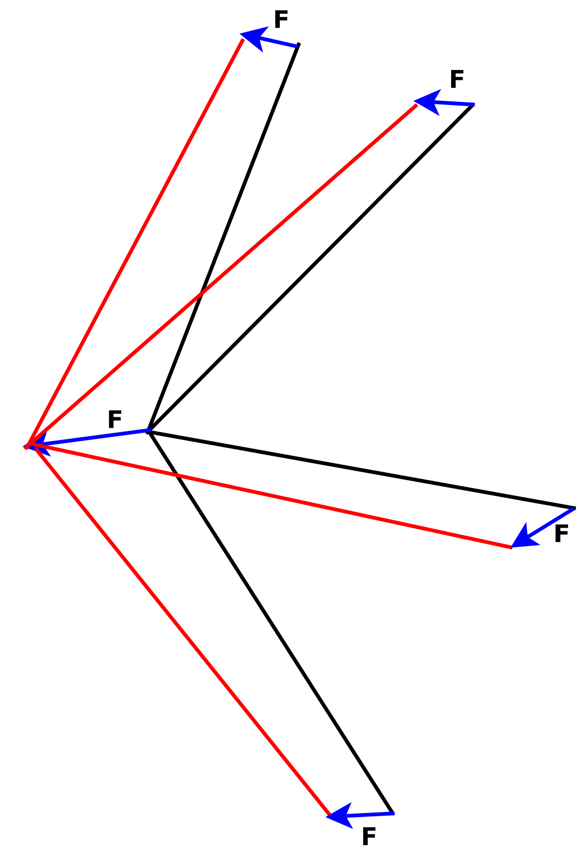
- Easy to implement
- High speed of calculations (about several mins using 1 CPU)
- The accuracy of the results is not very high

The basic idea of NFM: a triangulated surface is considered as a set of connected point masses (network nodes). All arising forces are considered to be applied to point masses. (The idea of the model is similar to the idea of mass-spring model)



General description of the computational scheme

- Calculate the net force acting on the node: $\mathbf{F_i^{res}}$
- Make the nodes shift along the direction of vector of the net force: $\mathbf{r_i} := \mathbf{r_i} + \delta \cdot \mathbf{F_i^{res}}$
- Repeat the process until has converged



We assume that the following forces act on the nodes:
- Elastic forces (from the side of the springs)
- Pressure forces (diastolic)
- Reaction forces (from other objects)

$$\vec{F}_i^{res} \equiv \vec{F}_i^{p} + \vec{F}_i^{s} + \vec{F}_i^{c}$$

Black lines are current position, **blue vectors** are the direction of the net force in the node, **red lines** are position at the next moment

Description of the scheme (pressure force)

$$\vec{F}_i^p = P\frac{\sum_i \vec{A}_i}{3}$$

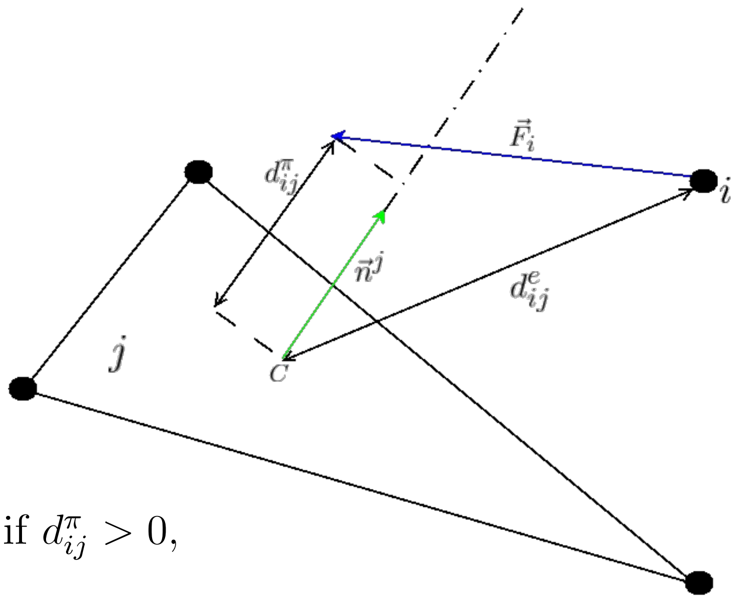
where *P* is the diastolic pressure, and in the numerator of the fraction is the total oriented area of the shared elements at this vertex

Description of the scheme (reaction force, 1 method)

Contact criteria: $d_{ij}^e \leq d^e \quad \mathrm{d}_{ij}^\pi \leq d^\pi$

where $d_{ij}^e = ||\vec{r}_i^n - \vec{r}_c^j|| \quad \mathrm{d}_{ij}^\pi = (\vec{r}_i^{n+1,*} - \vec{r}_c^j, \vec{n}^j)$

$$d^e = 1.1 \cdot \underset{e_j \in E}{average}\left(\max_{k=1,2,3}\left(\left|\left|\vec{r}_c^j - \vec{X}_k^j\right|\right|\right)\right)$$



Contact force:

$$\vec{F}_{ij}^c = \begin{cases} ||\vec{F}_i|| \exp\left(-\dfrac{k_1 d_{ij}^\pi}{||\vec{F}_i||}\right) \cdot \vec{n}^j, & \text{if } d_{ij}^\pi > 0, \\ (||\vec{F}_i|| - k_2 d_{ij}^\pi) \cdot \vec{n}^j, & \text{if } d_{ij}^\pi \leq 0, \end{cases}$$

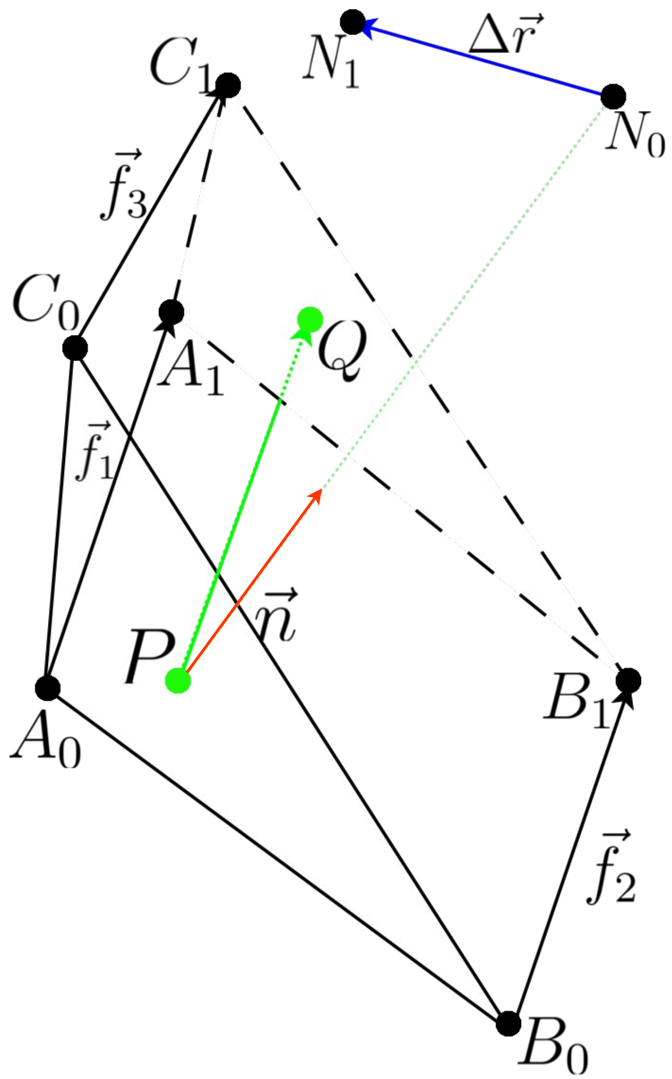
Pappalardo, O.A., Sturla, F., Onorati, F., Puppini, G., Selmi, M., Luciani, G.B., Faggian, G., Redaelli, A., Votta, E.: Mass-spring models for the simulation of mitral valve function: Looking for a trade-off between reliability and time-efficiency. Med Eng Phys. (2017) doi: 10.1016/j.medengphy.2017.07.001

Description of the scheme (reaction force, 2 method)

Let $A_0B_0C_0$ and N_0 are triangle and node belonging to different leaflets correspondingly;
P is an exact projection of N_0 to triangle $A_0B_0C_0$;
$\vec{w}$ is barycentric coordinates of point P;
$A_1B_1C_1$ and N_1 are the same triangle and node after applying elastic and pressure forces, but before taking into account the contact force;
Q is point of triangle $A_1B_1C_1$ with barycentric coordinates $\vec{w}$;

$\boldsymbol{F_i}=[\vec{A}_i,\vec{B}_i,\vec{C}_i]$ is a matrix of coordinates of nodes of the triangular face $A_iB_iC_i$, i = 0,1;

$margin$ is pre-selected contact force action distance;



$$d=PN_0;\quad \vec{n}=\frac{\overrightarrow{PN_0}}{PN_0}$$

$$m=margin+2\|\Delta\vec{r}\| \qquad \vec{v}_r=\Delta\vec{r}-\vec{PQ}$$

$$Contact\ criteria: d\leqslant m \ \ and\ \ \vec{v}_r\uparrow\downarrow\vec{n}$$

$$Contact\ shift: \vec{j}=0.5\cdot(m-(\vec{n},\vec{QN}_1))\vec{n}$$

$$Contact\ conversion\ formulas: \quad \tilde{\vec{r}}_1=\vec{r}_1+\vec{j} \qquad \widetilde{\boldsymbol{F}}_1=\boldsymbol{F}_1-\vec{j}\,\vec{w}^T$$

Description of the scheme (elastic force, 1 method) mass-spring model (MSM)

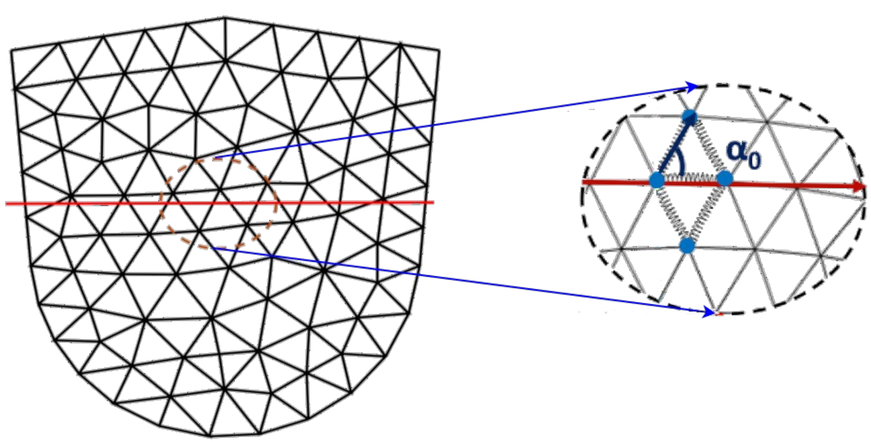
$$\mathbf{F}_i^{s_{ij}} = k_{ij}\left(\left\|\mathbf{r}_j - \mathbf{r}_i\right\| - l_0^{s_{ij}}\right)\frac{\mathbf{r}_j - \mathbf{r}_i}{\left\|\mathbf{r}_j - \mathbf{r}_i\right\|}, \quad k_{ij} = \frac{E(\varepsilon, \alpha_0) h \sum_s A_s}{(l_0^{s_{ij}})^2}$$

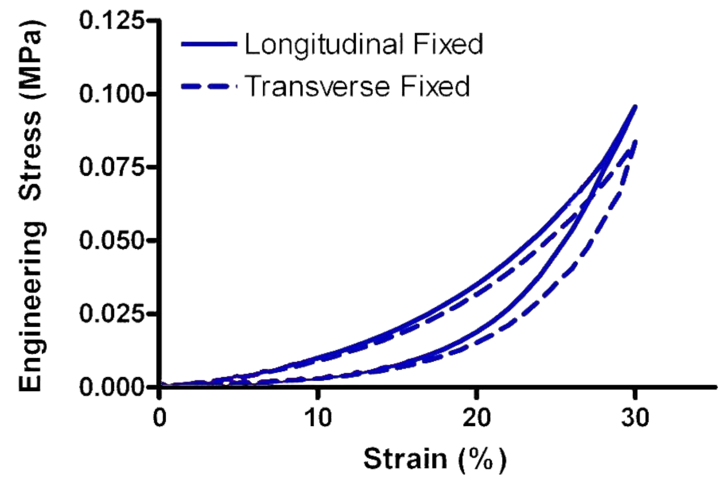
- force acting on the i-th node from the side of the spring connecting the i-th and j-th nodes

$$\mathbf{F}_\mathbf{i}^\mathbf{S} = \sum_\mathbf{j} \mathbf{F}_\mathbf{i}^{\mathbf{S_{ij}}}$$

Isotropic MSM: $E(\varepsilon, \alpha_0) = E_{iso}(\varepsilon)$

Anisotropic MSM: $E(\varepsilon, \alpha_0) = \sqrt{E_f^2(\varepsilon)\sin^2\alpha_0 + E_{cf}^2(\varepsilon)\cos^2\alpha_0}$





Averaged engineering stress-strain curves of fixed pericardium tissue, in the longitudinal (–) and transverse (- - -) directions.

[1]Hammer, P. E., Sacks, M. S., Pedro, J., Howe, R. D.: Mass-spring model for simulation of heart valve tissue mechanical behavior. Ann Biomed Eng (2011) doi: 10.1007/s10439-011-0278-5
[2]Zigras, T. C. (2007): Biomechanics of Human Pericardium: A Comparative Study of Fresh and Fixed Tissue (Doctoral dissertation, McGill University).

Description of the scheme (elastic force, 2 method) hyperelastic nodal force method (HNFM)

Considering the deformation of a triangular element with a given surface elastic potential U, one can derive the formulas of forces acting from the deforming triangle to each vertex

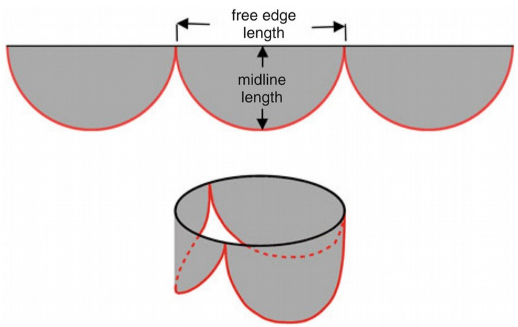
$$U(\mathbb{C}) \overset{discretization}{\rightarrow} U_d(\mathbf{r}_i, \mathbf{r}_j, \mathbf{r}_k) \quad \rightarrow \quad \mathbf{F}_\mathbf{i}^\mathbf{e}(T) = -A_T \frac{\partial U_d(\mathbf{r}_i, \mathbf{r}_j, \mathbf{r}_k)}{\partial \mathbf{r}_i}$$

We considered the following materials:

- St-Venant-Kirchhoff $\quad U_{TBS} = \frac{\lambda}{2}(tr\mathbb{E})^2 + \mu(tr\mathbb{E}^2)$
- Incompressible neo-Hookean material $\quad U_{NH} = H\frac{\mu}{2}(I_1 + 1/I_2 - 3)$
- Incompressible Gent $\quad U_{Gent} = -H J_m \frac{\mu}{2} \ln(1 - (I_1 + 1/I_2 - 3)/J_m)$

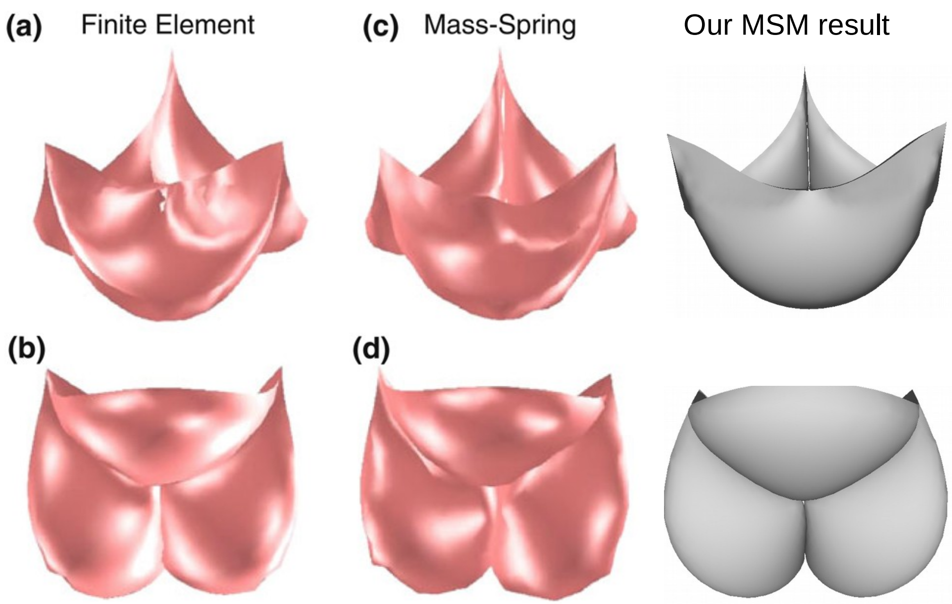
[1]Delingette H. Triangular springs for modeling nonlinear membranes. IEEE Trans Vis Comput Graph. 2008 Mar-Apr; 14(2):329-41.
[2]Vassilevski, Y.V., Salamatova, V.Y. and Lozovskiy, A.V., 2017. Concise formulas for strain analysis of soft biological tissues. Differential Equations, 53(7), pp.908-915.
[3]Salamatova, V.Y., 2019. Finite Element Method for 3D Deformation of Hyperelastic Materials. Differential Equations, 55(7), pp.990-999.

Simple benchmark

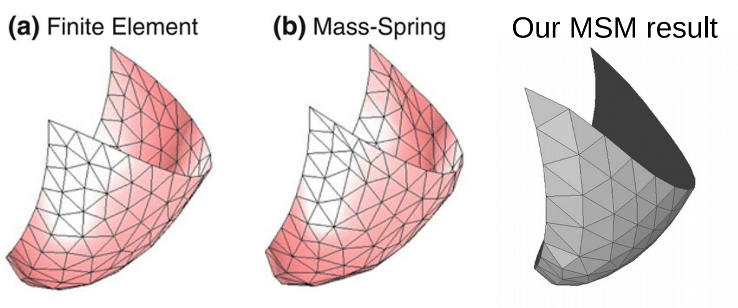


The initial geometry of the leaflets

Three leaflets:



One leaflet:



Free edge length (mm)	26.2 +/- 0.02	26.3 +/- 0.2	26.5 +/- 0.3

Hammer, P. E., Sacks, M. S., Pedro, J., Howe, R. D.: Mass-spring model for simulation of heart valve tissue mechanical behavior. Ann Biomed Eng (2011) doi: 10.1007/s10439-011-0278-5

Closure benchmark

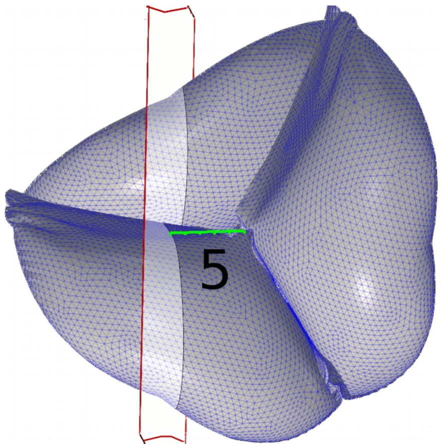
Used model`s parameters:

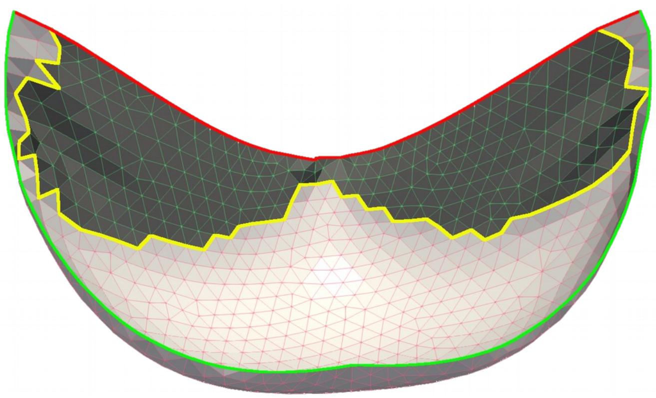
FSI, lin.elastisity[1]	MSM	TBS	neo-Hookean	Gent
E = 1 MPa, $\nu = 0.45$	E = 1 MPa	E = 1 MPa, $\nu = 0.45$	E = 1 MPa, μ = E / 3	E = 1 MPa, μ = E/3, $J_m = 2.3$

Comparison of coaptation parameters:

Model	h_E, mm	h_{C-C}, mm	h_{avr}, mm	NCCA, %	CPU time, sec
FSI, lin. elasticity[1]	10.5	1.5	2.7	21	n/a
MSM	10.8	3.8	3.3	25	44
TBS	10.8	3.1	2.9	24	58
neo-Hookean	10.4	3.0	2.5	21	136
Gent	10.8	3.4	3.1	24	203

Here was used template from [1] with d_{AA}= 24 mm, thickness H = 0.3 mm; h_{C-C}, h_{avr}, NCCA, CPU time in MSM, TBS, neo-Hookean and Gent models was measured at pressure P = 80 Hg mm, but h_E at P = 3 Hg mm.





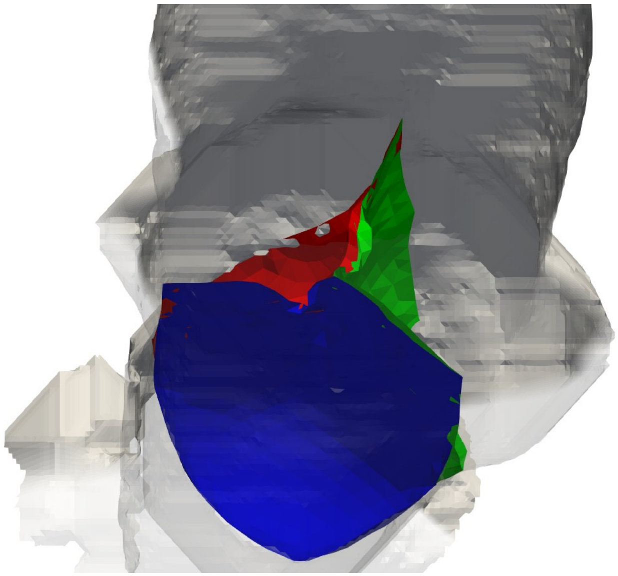
h_E is defined as the valve height of the closed valve.
h_{C-C} is the coaptation height measured in the C−C plane.
h_{avr} is defined as the ratio of the coaptation area (bounded by the yellow-red curve) and the free-edge length (the red curve).
NCCA (normalized cusp coaptation area) is defined as the ratio of the coaptation area and the total cusp surface area (bounded by the green-red curve).

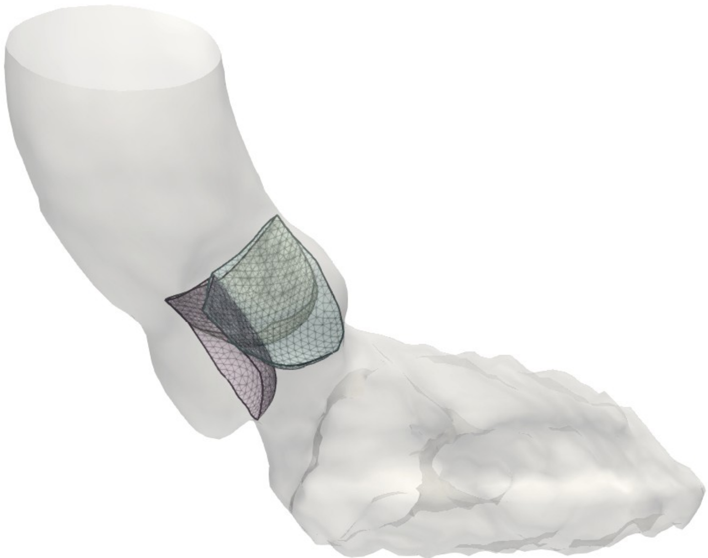
[1]Marom, G., Haj-Ali, R., Rosenfeld, M., Schäfers, H.J. and Raanani, E., 2013. Aortic root numeric model: Annulus diameter prediction of effective height and coaptation in post-aortic valve repair. The Journal of thoracic and cardiovascular surgery, 145(2), pp.406-411.
[2]Marom, G., Haj-Ali, R., Raanani, E., Schäfers, H.J. and Rosenfeld, M., 2012. A fluid-structure interaction model of the aortic valve with coaptation and compliant aortic root. Medical & biological engineering & computing, 50(2), pp.173-182.

MSM results (1)

- Results obtained from real patient data.
- When calculating, 3 templates (a, b, c) of leaflets were used.
- The pressure was assumed to be 80 mm Hg
- Mass-spring model (MSM) was used
- Young's effective modulus was approaching a three-step function

$$E(\varepsilon) = \begin{cases} E_1 & \text{if } \varepsilon \leq \lambda_1^* \\ E_2 & \text{if } \lambda_1^* < \varepsilon < \lambda_2^* \\ E_3 & \text{if } \varepsilon \geq \lambda_2^* \end{cases}$$





An example of the result of calculations

MSM results (2)

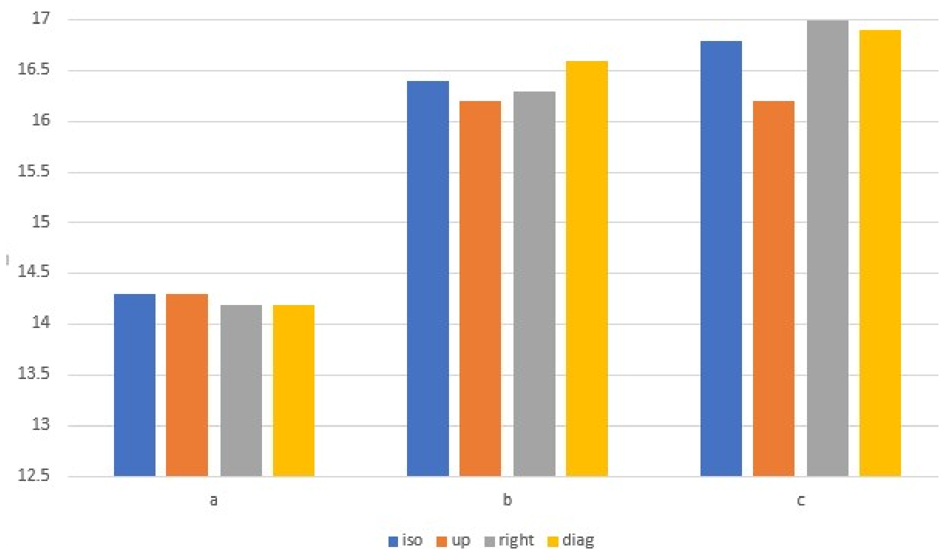
Anisotropy sensitivity:

Elastic moduli for MSM derived from experimental data for treated human pericardium

	E_1 (kPa)	E_2 (kPa)	E_3 (kPa)	λ_1^*	λ_2^*
Longitudinal direction	137	568	968	0.175	0.3
Transverse direction	63	570	1400	0.175	0.3
Average isotropic case	106	569	1200	0.175	0.3

	params	iso	$\uparrow$	$\rightarrow$	$\nearrow$
a	h, mm	14.3	14.3	14.2	14.2
	h_c, mm	0	0	0	0
	S_{max}, mm^2	355	354	350	358
	S_{min}, mm^2	287	286	286	289
b	h, mm	16.4	16.2	16.3	16.6
	h_c, mm	11.6	10	10.1	12.2
	S_{max}, mm^2	461	451	454	463
	S_{min}, mm^2	382	382	379	381
c	h, mm	16.8	16.2	17.0	16.9
	h_c, mm	11.9	13.4	12.7	13.2
	S_{max}, mm^2	502	511	516	506
	S_{min}, mm^2	420	416	423	417

Values of coaptation parameters for the isotropic case and three anisotropic ones with horizontal, vertical and diagonal anisotropy directions for templates a, b, c



Height of coaptation values for templates a, b, c

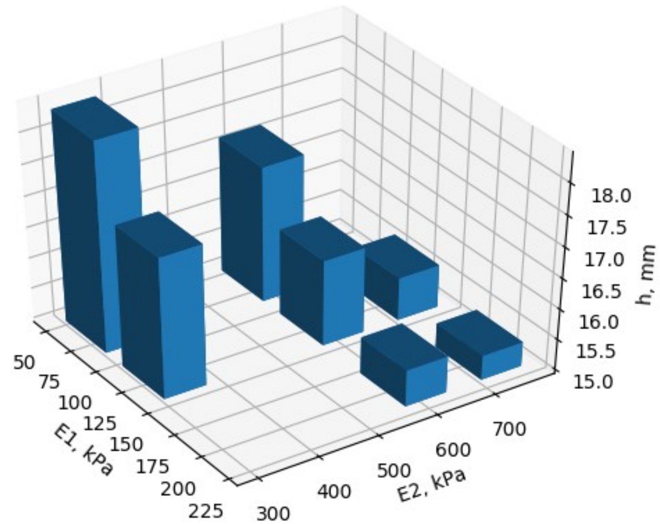
Zigras, T. C. (2007): Biomechanics of Human Pericardium: A Comparative Study of Fresh and Fixed Tissue (Doctoral dissertation, McGill University).

MSM results (3)

Sensitivity of coaptation parameters to stiffness parameters:

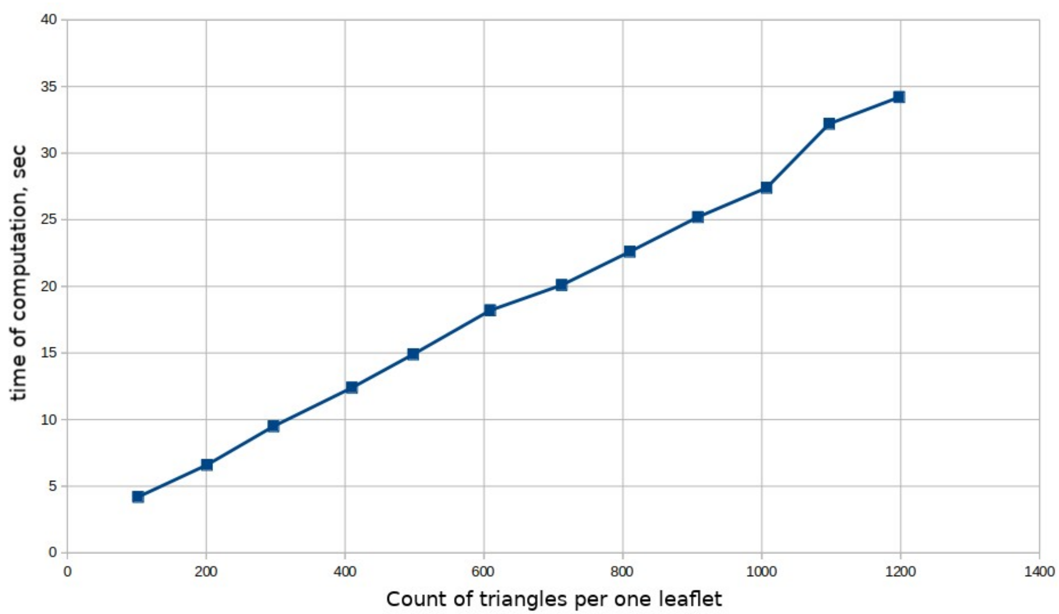
Sensitivity of coaptation parameters to stiffness moduli. Here the material was considered isotropic, the template is b. E_3 = 1200 kPa, λ_1 = 0.175, λ_2 = 0.3

E_1 (kPa)	E_2 (kPa)	h (mm)	h_c (mm)	S_{max} (mm^2)	S_{min} (mm^2)
106	569	16.4	11.6	461	382
180	569	15.6	9.7	453	374
106	700	15.7	11.6	457	375
180	700	15.4	9.2	441	371
50	569	17.2	10.9	463	386
106	300	17.3	12.9	476	394
50	300	18.4	13.6	489	407



The dependence of the height of coaptation on the elastic moduli.

Performance:

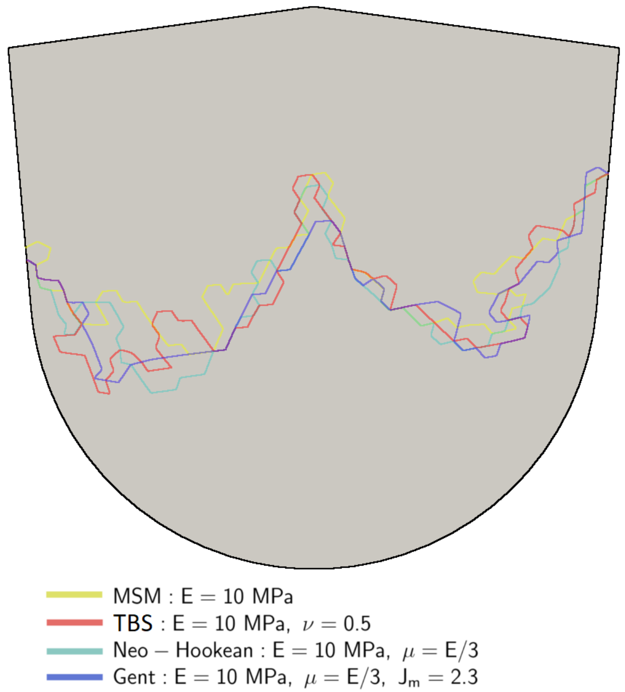


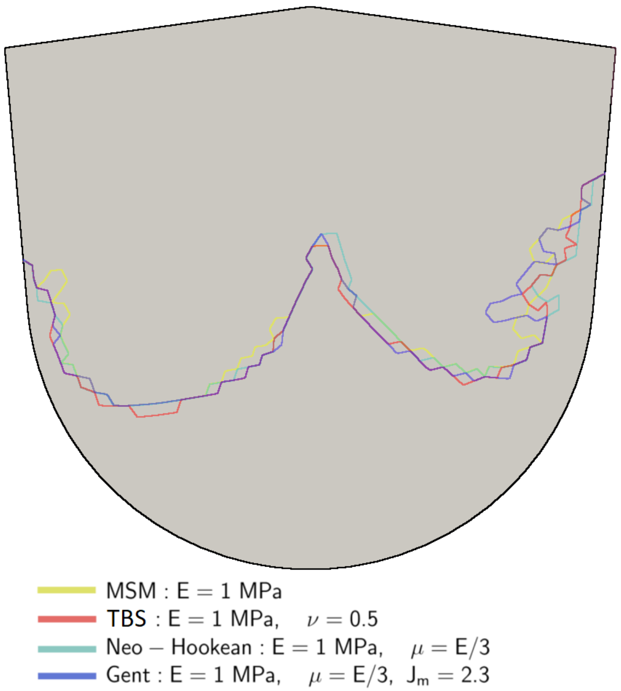
The influence of the material model on the coaptation heights (1)

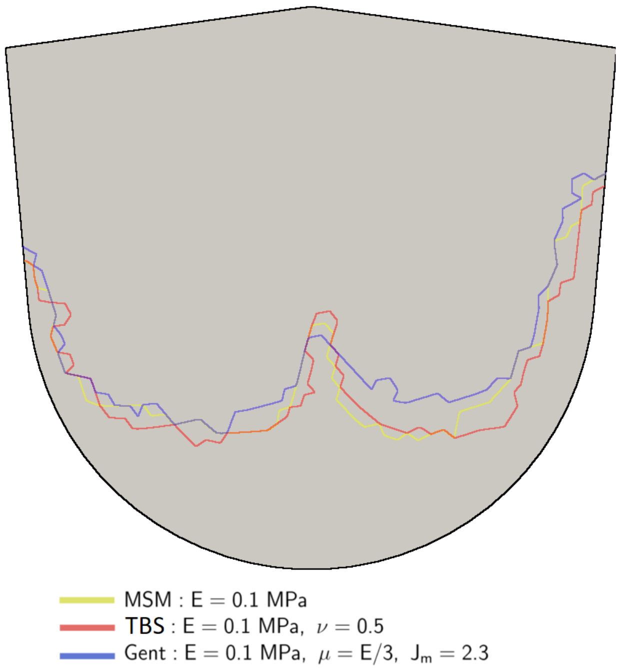
- Results obtained from real patient data
- Template (d) of leaflets were used.
- The pressure was assumed to be 80 mm Hg

model		h, mm	h_{C-C}, mm	CPU time, sec
MSM	$E = 10$ MPa	15.4	13.1	282
TBS	$E = 10$ MPa, $\nu = 0.5$	15.4	15.1	578
Neo-Hookean	$E = 10$ MPa, $\mu = E/3$	13.0	12.9	527
Gent	$E = 10$ MPa, $\mu = E/3$, $J_m = 2.3$	15.5	13.5	2072
MSM	$E = 1$ MPa	17.3	16.0	143
TBS	$E = 1$ MPa, $\nu = 0.5$	16.7	16.2	426
Neo-Hookean	$E = 1$ MPa, $\mu = E/3$	17.4	15.8	464
Gent	$E = 1$ MPa, $\mu = E/3$, $J_m = 2.3$	16.3	15.9	2612
MSM	$E = 0.1$ MPa	21.5	20.3	166
TBS	$E = 0.1$ MPa, $\nu = 0.5$	23.7	23.4	191
Gent	$E = 0.1$ MPa, $\mu = E/3$, $J_m = 2.3$	23.6	23.2	546

The influence of the material model on the coaptation heights (2)







In the experiments the coaptation profile is determined primarily by the elastic modulus, and not the elastic model

Conclusion

- Mass-spring model:
  - Under the experimental conditions, the effect of anisotropy on the maximum height of the coaptation and the area of the coaptation was not great. However, anisotropy influenced the magnitude of central coaptation.
  - The ultimate value of the maximum height of the coaptation simulation used by the method is mainly determined by the geometry of the problem
  - High speed of computation is reached
- MSM and HNFM:
  - Under the experimental conditions, the coaptation profile is determined mainly by the elastic modulus, and not by the material model
  - Time of computation significantly depend on using material model

Thanks for attention

Links to materials used in the presentation:

[1] Hammer, P. E., Sacks, M. S., Pedro, J., Howe, R. D.: Mass-spring model for simulation of heart valve tissue mechanical behavior. Ann Biomed Eng (2011) doi: 10.1007/s10439-011-0278-5
[2] Pappalardo, O.A., Sturla, F., Onorati, F., Puppini, G., Selmi, M., Luciani, G.B., Faggian, G., Redaelli, A., Votta, E.: Mass-spring models for the simulation of mitral valve function: Looking for a trade-off between reliability and time-efficiency. Med Eng Phys. (2017) doi: 10.1016/j.medengphy.2017.07.001
[3] Zigras, T. C. (2007): Biomechanics of Human Pericardium: A Comparative Study of Fresh and Fixed Tissue (Doctoral dissertation, McGill University).
[4] Advanced Numerical Instruments 2D, Lipnikov, K. and Vassilevski, Yu. and Danilov, A. And others. https://sourceforge.net/projects/ani2d
[5] Marom, G., Haj-Ali, R., Rosenfeld, M., Schäfers, H.J. and Raanani, E., 2013. Aortic root numeric model: Annulus diameter prediction of effective height and coaptation in post-aortic valve repair. The Journal of thoracic and cardiovascular surgery, 145(2), pp.406-411.
[6] Marom, G., Haj-Ali, R., Raanani, E., Schäfers, H.J. and Rosenfeld, M., 2012. A fluid-structure interaction model of the aortic valve with coaptation and compliant aortic root. Medical & biological engineering & computing, 50(2), pp.173-182.
[7] Delingette H. Triangular springs for modeling nonlinear membranes. IEEE Trans Vis Comput Graph. 2008 Mar-Apr; 14(2):329-41.
[8] Vassilevski, Y.V., Salamatova, V.Y. and Lozovskiy, A.V., 2017. Concise formulas for strain analysis of soft biological tissues. Differential Equations, 53(7), pp.908-915.
[9] Salamatova, V.Y., 2019. Finite Element Method for 3D Deformation of Hyperelastic Materials. Differential Equations, 55(7), pp.990-999.