Lumped dynamical model of the heart including heart functioning

Sergei Simakov[1,2,3]

[1]Moscow Institute of Physcis and technology, Institutskii Lane 9, Dolgoprudny, Russia

[2]Institute of Numerical Mathematics RAS, Gubkina St. 8, Moscow, Russia

[3]Sechenov University, Bolshajya Pirogovskajya 2/4, Moscow, Russia

Table of contents

1D-0D approximation in human physiology

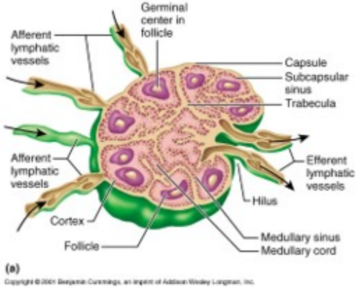

Figure 1: Lymph vessels and nodes

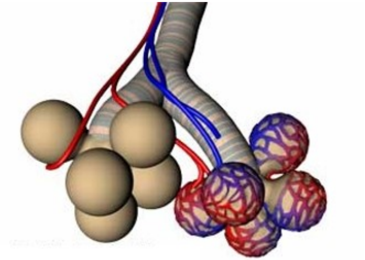

Figure 2: Bronchial tubes and alveoli

1D-0D approximation in human physiology

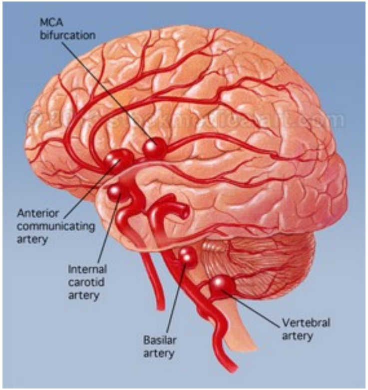

Figure 3: Blood vessels and aneurysms

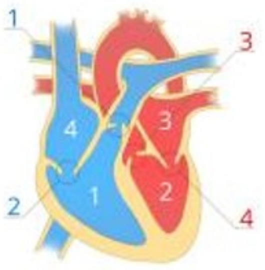

Figure 4: Heart chambers

Heart model

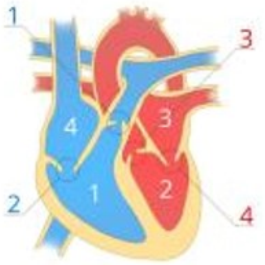

Figure 5: Heart chambers

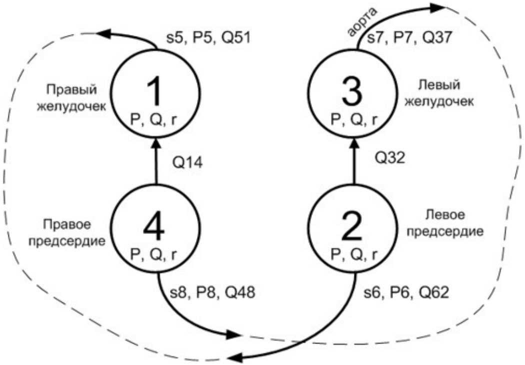

Figure 6: Scheme of the lumped compartments.

Heart model

- The heart chamber dynamics

$$I_j \frac{d^2 V_j}{dt^2} + R_j \frac{dV_j}{dt} + e_j(t) V_j = p_j, j = 1, \ldots, 4 \qquad (1)$$

- Myocardium activation function

$$e_j(t) = E_j^{syst} + \frac{E_j^{syst} + E_j^{diast}}{2} e(t) \qquad (2)$$

Heart model: activation function

- For the left ventricle

$$e(t) = \begin{cases} 1 - \cos\left(\frac{t}{T_{s1}}\pi\right), 0 \leqslant t \leqslant T_{s1} \\ 1 - \cos\left(\frac{t - T_{s2}}{T_{s1} - T_{s2}}\pi\right), T_{s1} < t < T_{s2} \\ 0, T_{s2} \leqslant t \leqslant T \end{cases} \tag{3}$$

- For the left auricle

$$e(t) = \begin{cases} 0, 0 \leqslant t \leqslant T_{pb} \\ 1 - \cos\left(\frac{t - T_{pb}}{T_{pw}}2\pi\right), T_{pb} < t < T_{pb} + T_{pw} \\ 0, T_{pb} + T_{pw} \leqslant t \leqslant T. \end{cases} \tag{4}$$

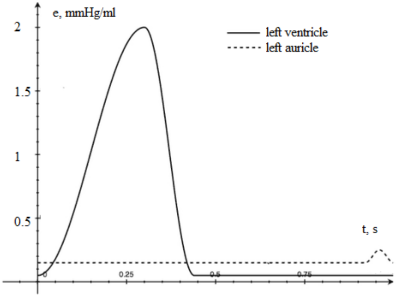

Figure 7: Chamber activation function.

Heart model: interchamber flow

- Flow between chambers (Poiseuille pressure drop)

$$Q_{ij} = \frac{\alpha_{ij}}{r_{ij}}(P_j - P_i) \tag{5}$$

- Mass conservation for the lumped compartment (heart chamber)

$$\frac{dV_i}{dt} = \sum_j \alpha_{ij} Q_{ij}, i = 1, \dots, 4, \tag{6}$$

α_{ij} is the function of the valve opening $\longrightarrow$ to be shown later (see eq.(11))

Valves model

The valves are subjected by the friction force, pressure force and flow resistance force. The 2^{nd} Newton's law can be stated as

$$\frac{d^2\theta_{ij}}{dt^2} = K^{fr}\frac{d\theta_{ij}}{dt} + K^p\left(P_j - P_i\right)\psi_{ij}\cos\theta_{ij} + F^r_{ij}, \tag{7}$$

where

$$\psi_{ij} = \begin{cases} 0, P_i < P_j \text{ and } \theta_{ij} < \theta^{min}_{ij}, \\ 1, \text{ otherwise.} \end{cases} \tag{8}$$

$$F^r_{ij} = \begin{cases} 0, \theta^{max}_{ij}\theta_{ij} < \theta^{max}_{ij} \\ K^r \tan\left(10\left(\theta_{ij} - \theta^{max}_{ij}\right)\right), \theta_{ij} \geqslant \theta^{max}_{ij} \text{ and } \frac{d\theta_{ij}}{dt} > 0 \\ K^r \tan\left(10\left(\theta_{ij} - \theta^{min}_{ij}\right)\right), \theta_{ij} \leqslant \theta^{min}_{ij} \text{ and } \frac{d\theta_{ij}}{dt} < 0, \end{cases} \tag{9}$$

Valves model

- Instant valve opening

$$\alpha_{ij} = \begin{cases} 0, 0 \leqslant t < T_{open,ij}, \\ 1,\ T_{open,ij} \leqslant t \leqslant T_{close,ij}, \\ 0,\ T_{close,ij} < t \leqslant T. \end{cases} \quad (10)$$

- Not instant valve opening

$$\alpha_{ij} = \begin{cases} 0, -\infty < \theta \leqslant \theta_{ij}^{min}, \\ \dfrac{1-\cos\theta_{ij}}{1-\cos\theta_{ij}^{max}}, \theta_{ij}^{min} < \theta \leqslant \theta_{ij}^{max}, \\ 1, \theta > \theta_{ij}^{max}, \end{cases} \quad (11)$$

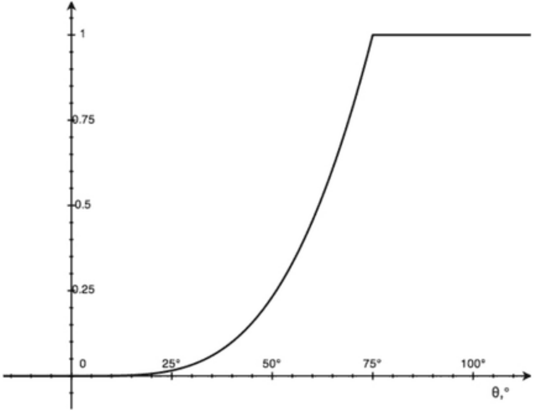

Figure 8: Valve function.

Numerical method: Obreshkov's pairs integration method

- Let's consider extended set

$$\frac{d\vec{y}}{dt} = \vec{f}\,(t, \vec{y}) \to \frac{d\vec{y}}{dt} = \vec{w}, \frac{d\vec{w}}{dt} = \frac{\partial \vec{f}}{\partial t} + \frac{\partial \vec{f}}{\partial \vec{y}}\vec{y} \tag{12}$$

- General solution for a single-step implicit Runge-Kutta method can be found as

$$\vec{y}_{s+1}^{n+1} = \vec{y}_{s}^{n+1} - \mathbf{B}^{-1}\left(t^{n+1}, y^{n+1}\right)\vec{R}\left(\vec{y}_{s}^{n+1}\right), s = 1, 2, \ldots, \vec{y}_{0}^{n+1} = \vec{y}^{n} \tag{13}$$

$$\vec{R} = \sum_{k=0}^{1}\left[a_k \vec{y}^{n+k} - \tau b_k \vec{f}^{n+k} - \tau^2 c_k \left(\frac{\partial \vec{f}}{\partial t} + \frac{\partial \vec{f}}{\partial \vec{y}} \cdot \vec{y}\right)_{n+k}\right] \tag{14}$$

$$\mathbf{B} = \frac{\partial \vec{R}}{\partial \vec{y}^{n+1}} = \mathbf{E} - \tau b \frac{\partial \vec{f}}{\partial \vec{y}} - \tau^2 c \left(\frac{\partial}{\partial \vec{y}}\left(\frac{\partial \vec{f}}{\partial t}\right) + \left(\frac{\partial \vec{f}}{\partial \vec{y}}\right)^2 + \mathbf{C}\right), \mathbf{C} = \left\{\left(\frac{\partial}{\partial y_j}\left(\frac{\partial \vec{f}}{\partial \vec{y}}\right)\right) \cdot \vec{f}\right\}_j \tag{15}$$

Numerical method: Obreshkov's pairs integration method

It was shown[1], that the set

$$a_1 = 1, a_0 = -1, b_0 = \frac{1}{2} + c_0 + c_1, b_1 = \frac{1}{2} - c_0 - c_1, c_1 = c_0 - \frac{1}{6}, c_1 = 0 \qquad (16)$$

provides the methos, which is **A and L stable,** 3^{rd} **order approximation**.

[1]Kholodov, Lobanov, Evdokimov, 2001

Aortic regurgitation: valves

Aortic valve not fully closed $\Longrightarrow$ reverse flow to the ventricle (regurgitation):

$\theta_{ao}^{min} : 0^\circ \to 25^\circ \Longrightarrow$ Early aortic valve opening $(0.05s)$.

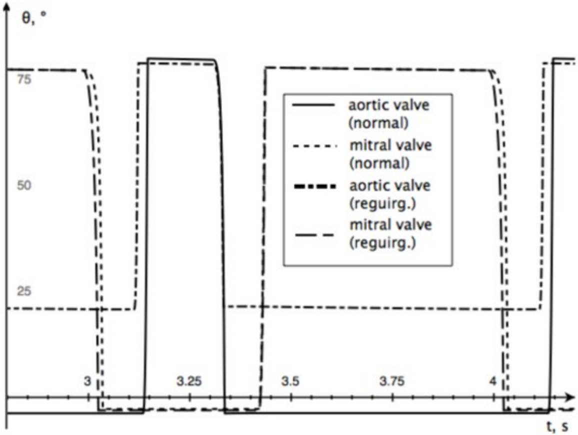

Figure 9: Aortic and mitral valve angles.

Aortic regurgitation: flow

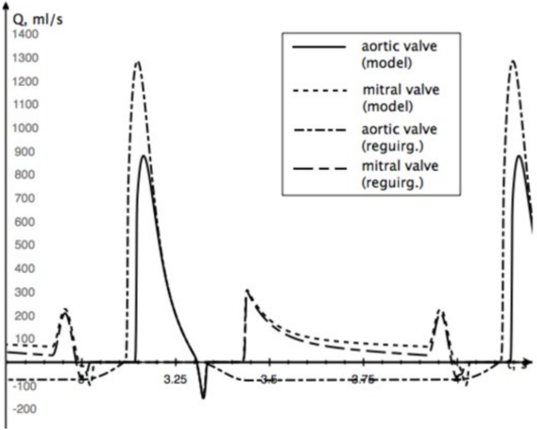

Figure 10: Flow through the valves.

Aortic valve not fully closed:

$$\theta_{ao}^{min} : 0^\circ \to 25^\circ \Longrightarrow$$

- Peak aortic valve flow increase 45%;
- Backward aortic valve flow $70 ml/s$.

Aortic regurgitation: volume

Aortic valve not fully closed:

$\theta_{ao}^{min} : 0^\circ \rightarrow 25^\circ \Longrightarrow$
Peak ventricle volume
increase 30%
Overexpansion!

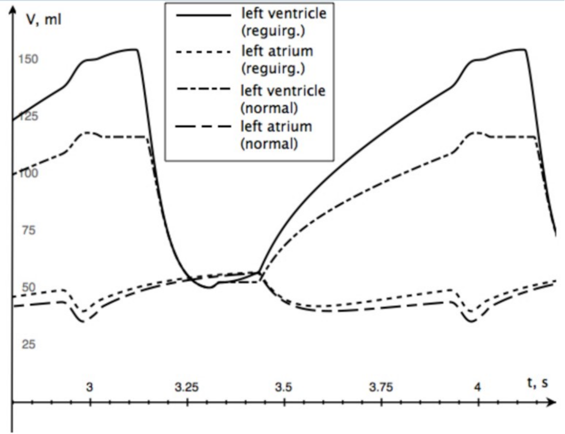

Figure 11: Left ventricle and atrium volume.

Mitral valve stenosis: valves

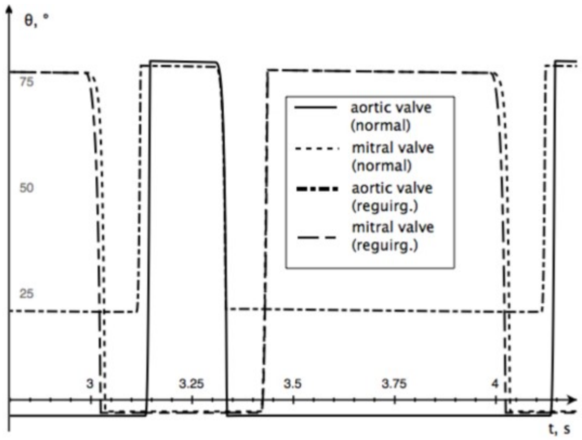

Figure 12: Aortic and mitral valve angles.

Decreased opening of the mitral valve:

$\theta_{mi}^{max} : 75^\circ \to 50^\circ \Longrightarrow$

- Late aortic valve opening (0.05s);
- Peak lumen decrease 25%.

Mitral valve stenosis: flow

Decreased opening of the mitral valve:

$\theta_{mi}^{max}: 75^{\circ} \rightarrow 50^{\circ} \Longrightarrow$

- Peak aortic valve flow decrease 40%;
- Peak mitral valve flow decrease 75%.

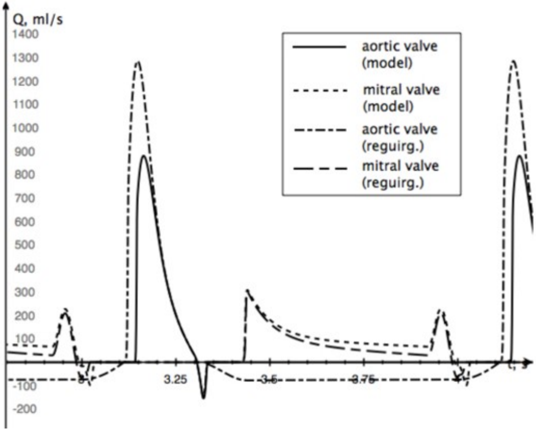

Figure 13: Flow through the valves.

Mitral valve stenosis: volume

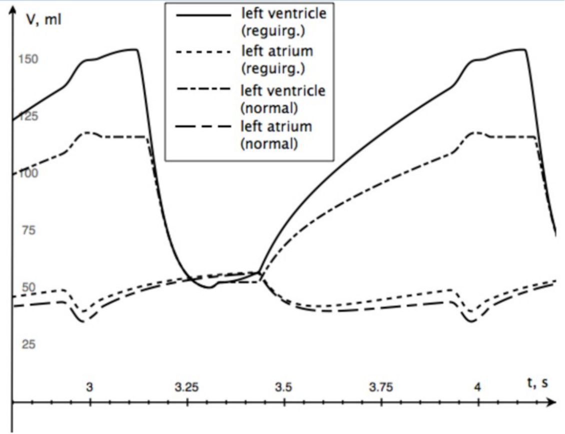

Figure 14: Left ventricle and atrium volume.

Decreased opening of the mitral valve:

θ_{mi}^{max} : $75^\circ \to 50^\circ \Longrightarrow$

- Peak ventricle volume decrease 25%;
- Peak auricle volume **increase** 25%. **Overexpansion!**

Conclusions

- **Benefits**: The model reproduces basic known physiological behaviour of the heart dynamics with the valves and their pathologies.
- **The drawback**: The systole and diastole duration are still fixed parameters.
- **Future work**: Patient-specfic validation is required.
- **Future work**: Integration with the vascular network is required[2].

[1] Partly done, but not presented here

Acknowledgements

The work was supported by Russian Science Foundation Grant No. 14-31-00024.

Thank you!


MEDICAL
MEDICAL
MEDICAL
MEDICAL
MEDICAL
MEDICAL
Health Care
Doctor
Hospital
Pharmacist
Nurse
Dentist
First Aid
Surgeon
Emergency