Метод подбора параметров для пациент ориентированного моделирования кровотока

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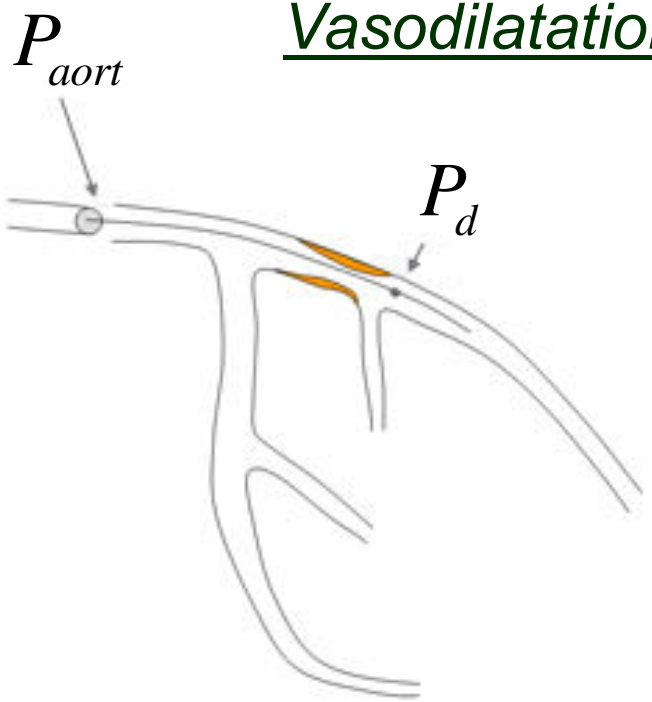
[3] Сеченовский Университет

10-я конференция “Математические модели и численные методы в биологии и медицине”,
ИВМ РАН, 6-8 ноября 2018 г.

Fractional flow reserve (FFR)



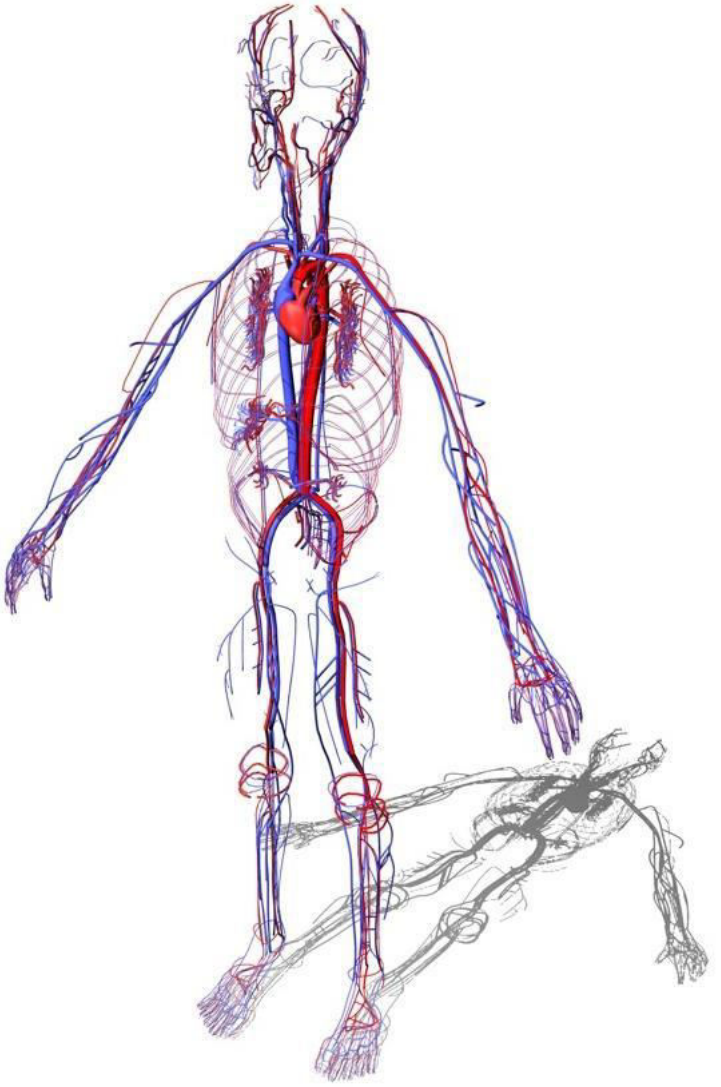
Vasodilatation

$$FFR = \frac{\text{pressure distal to the lesion } (P_d)}{\text{aortic pressure } (P_{aort})}$$

$FFR = 1 \longrightarrow$ healthy subject

$FFR \leq 0.75 \longrightarrow$ surgical treatment

Blood flow model



Model of coronary circulation

$$p_k - p_{veins} = R_k Q_k$$

arteries

$$\frac{\partial A}{\partial t} + \frac{\partial (uA)}{\partial x} = 0$$

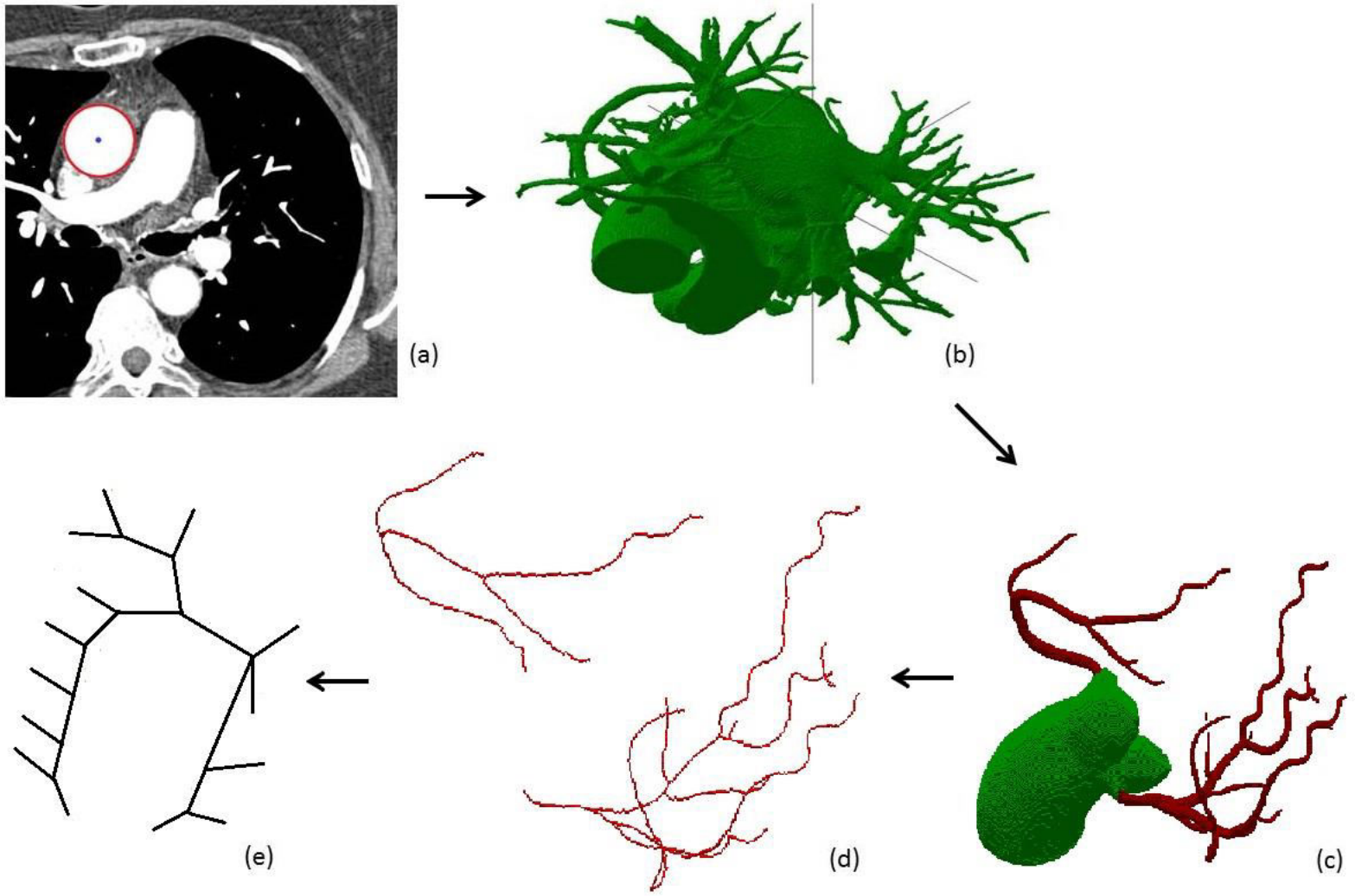
$$\frac{\partial u}{\partial t} + \frac{\partial}{\partial x}\left(\frac{u^2}{2} + \frac{P}{\rho}\right) = f_{fr}$$

p_{veins}

$$u_{in} A_{in} = Q(t)$$

$$p_k + \frac{\rho u_k^2}{2} = p_j + \frac{\rho u_j^2}{2}, \quad \forall j,k$$

Arterial structure. Patients 1-2.



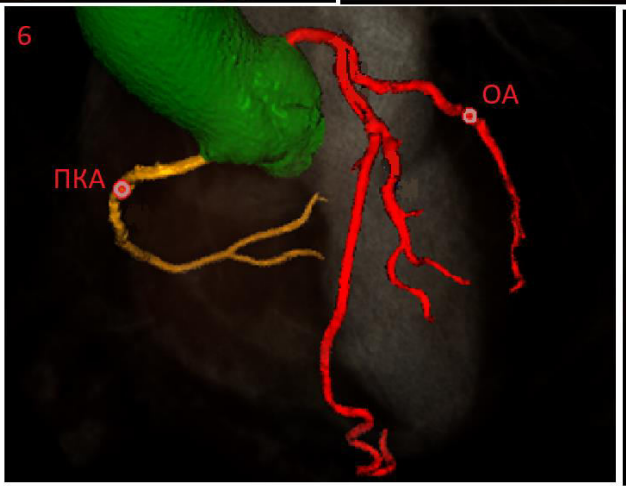
YuV Vassilevski, AA Danilov, TM Gamilov, YuA Ivanov, RA Pryamonosov, SS Simakov, Patient-specific anatomical models in human physiology, Russ. J. Num. Anal. Math. Mod., 2015

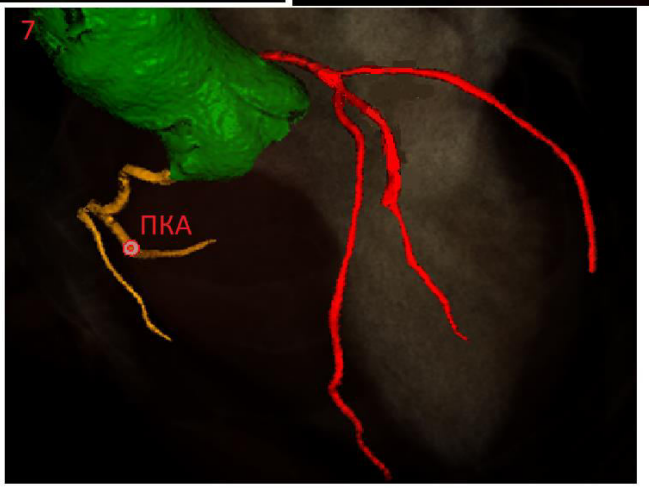
Arterial structure. Patients 1-2.

1

2

Arterial structure. Patients 3-7.

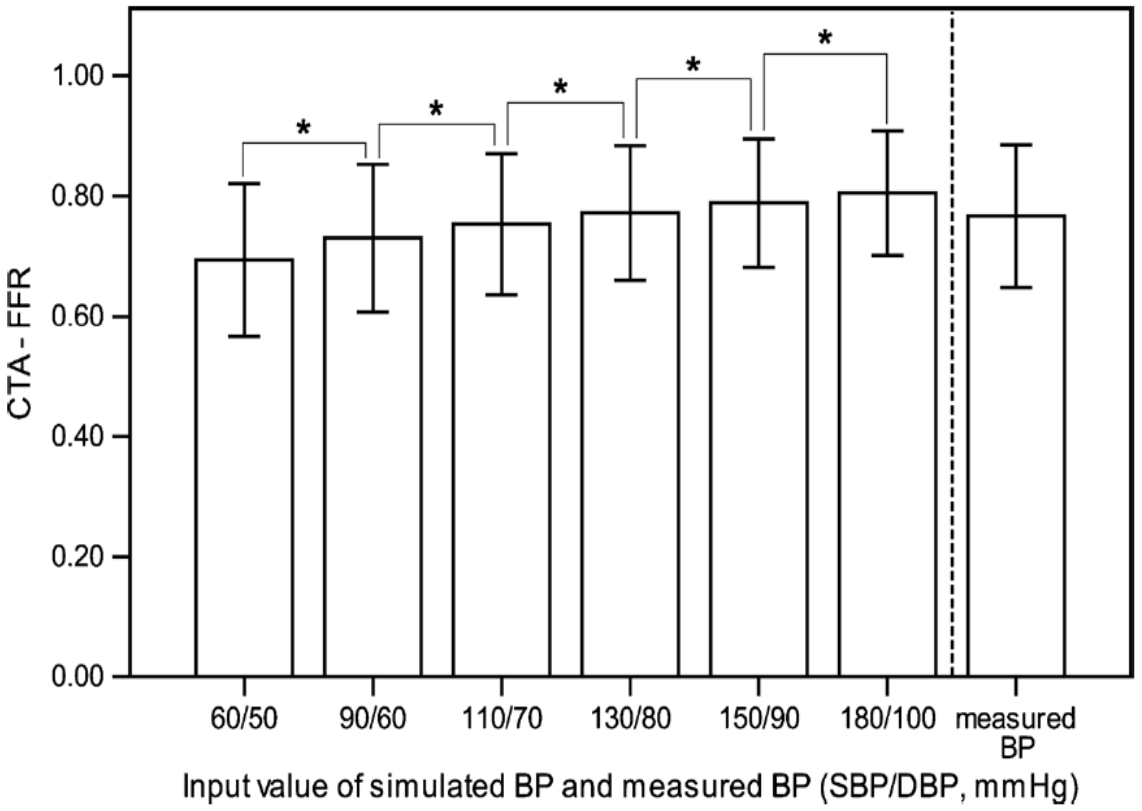


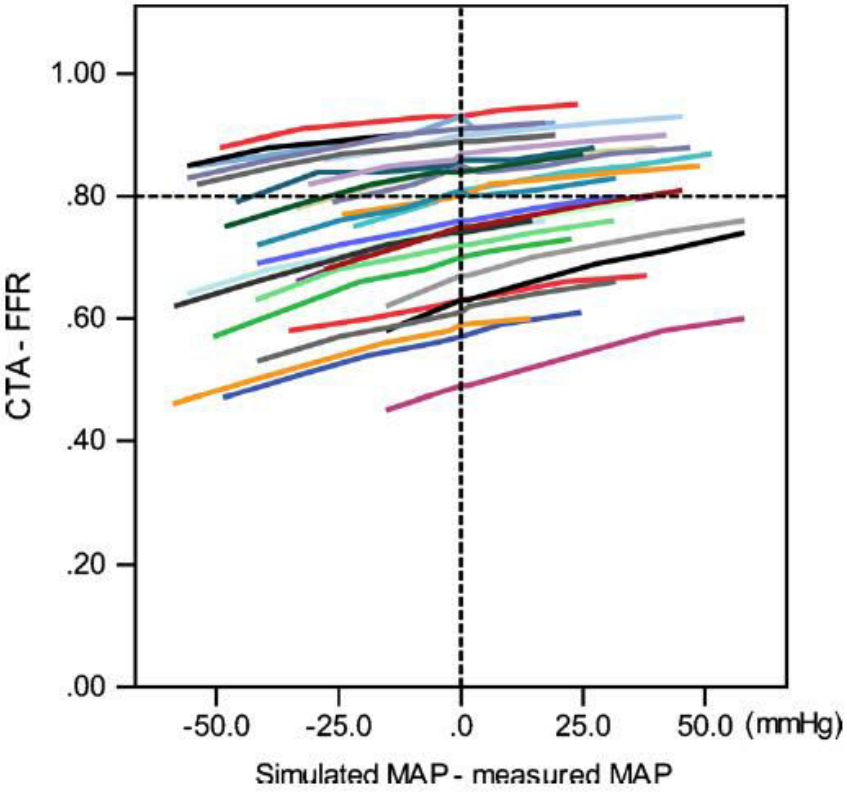


Results

Patient	Vessel	Stenosis	Length, mm	FFR	FFR model	Deviation
1	LAD	50%	40	0.58	0.51	14%
	LCA	55%	2	0.84	0.72	17%
	LCX	80%	10	0.61	0.59	3%
2	LAD	80%	20	0.78	0.74	5%
	RCA	55%	2	0.87	0.93	-5%
3	RCA	70%	20	0.77	0.81	-5%
4	RCA	60%	40	0.57	0.6	-5%
	LCX	50%	10	0.97	0.88	14%
5	LAD	50%	10	0.86	0.91	-5%
6	LCX	50%	10	0.83	0.89	-7%
	RCA	50%	10	0.93	0.91	2%
7	RCA	50%	3	0.89	0.85	5%

Blood pressure and FFR





Akira Kurata, Adriaan Coenen, Marisa Lubbers , etc (2016). The effect of blood pressure on non-invasive fractional flow reserve derived from coronary computed tomography angiography. European Radiology. 27. 10.1007/s00330-016-4541-x.

Parameters estimation

Parameters (stroke volume, elasticity, resistance,..)

Blood flow model

output values (systolic BP, diastolic BP)

Parameters estimation. Learning

1. “Base” model $M(\vec{p}) = \vec{u}$

$$M(\vec{p} + \Delta\vec{p}_k) = \vec{u} + \Delta\vec{u}_k$$

$$\Delta\vec{p}_k \longrightarrow \Delta\vec{u}_k$$

Parameters estimation. Learning

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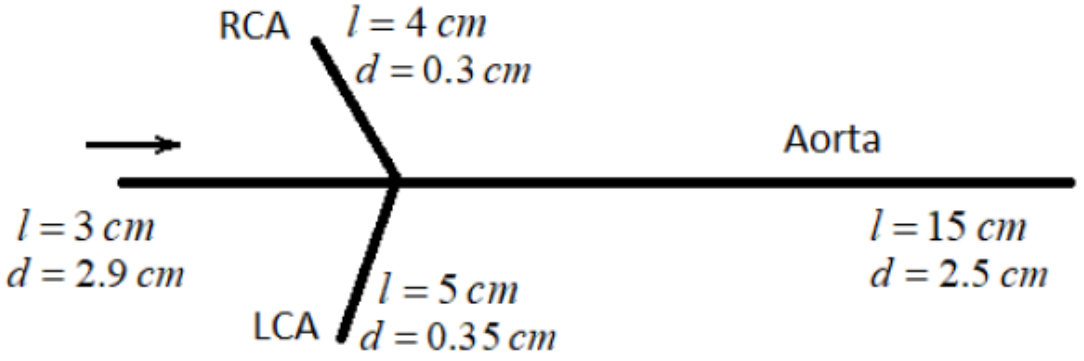

$$\vec{p} = \{R_a, c_a, SV\}^T$$

$$\vec{u} = \{P_s, P_d\}^T$$

Parameters estimation

2. Current model $\tilde{M}(\vec{p})$

$$\Delta\vec{p}_k \longrightarrow \Delta\vec{u}_k$$

Parameters estimation

2. Current model $\tilde{M}(\vec{p})$

$$\Delta\vec{p}_k \longrightarrow \Delta\vec{u}_k$$

$$\Delta\vec{p} = \sum_{k=1}^{K} x_k \Delta\vec{p}_k \qquad \Delta\vec{u} = \vec{u^*} - \tilde{M}(\vec{p})$$

$$\tilde{M}(\vec{p} + \Delta\vec{p}) - \tilde{M}(\vec{p}) = \Delta\vec{u}$$

$$\Delta\vec{u} = \sum_{k=1}^{K} x_k \Delta\vec{u}_k, \quad x_k^{min} \leqslant x_k \leqslant x_k^{max}$$

$$\sum_{k=1}^{K} x_k^2 \to min$$

$$\Delta\vec{u} = \sum_{k=1}^{K} x_k \Delta\vec{u}_k, \quad x_k^{min} \leqslant x_k \leqslant x_k^{max} \qquad \Delta\vec{u} \in R^M,\ M < K$$

$$\sum_{k=1}^{K} x_k^2 \to min \longrightarrow \sum_{k=1}^{K-M} x_i^2 + \sum_{i=1}^{M} \left(\left(\sum_{j=1}^{K-M} \alpha_{ij} x_j \right) + \beta_i \right) \to \min$$

$$\Delta\vec{u} = \sum_{k=1}^{K} x_k \Delta\vec{u}_k, \quad x_k^{min} \leqslant x_k \leqslant x_k^{max} \qquad \Delta\vec{u} \in R^M,\ M < K$$

$$\sum_{k=1}^{K} x_k^2 \to min \longrightarrow \sum_{k=1}^{K-M} x_i^2 + \sum_{i=1}^{M} \underbrace{\left(\left(\sum_{j=1}^{K-M} \alpha_{ij} x_j \right) + \beta_i \right)}_{x_i,\ i = K-M+1 \ldots K} \to \min$$

$$\Delta\vec{u} = \sum_{k=1}^{K} x_k \Delta\vec{u}_k, \quad x_k^{min} \leqslant x_k \leqslant x_k^{max} \qquad \Delta\vec{u} \in R^M, \; M < K$$

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$$\vec{x}' = \{x_1, x_2, \ldots x_{K-M}\}^T$$

$$\Phi(\vec{x}') = \left\langle A\vec{x}', \vec{x}' \right\rangle + 2\left\langle \vec{b}, \vec{x}' \right\rangle + c \to \min, \quad A \geq 0$$

$$\Delta\vec{u} = \sum_{k=1}^{K} x_k \Delta\vec{u}_k, \quad x_k^{min} \leqslant x_k \leqslant x_k^{max} \qquad \Delta\vec{u} \in R^M, \; M < K$$

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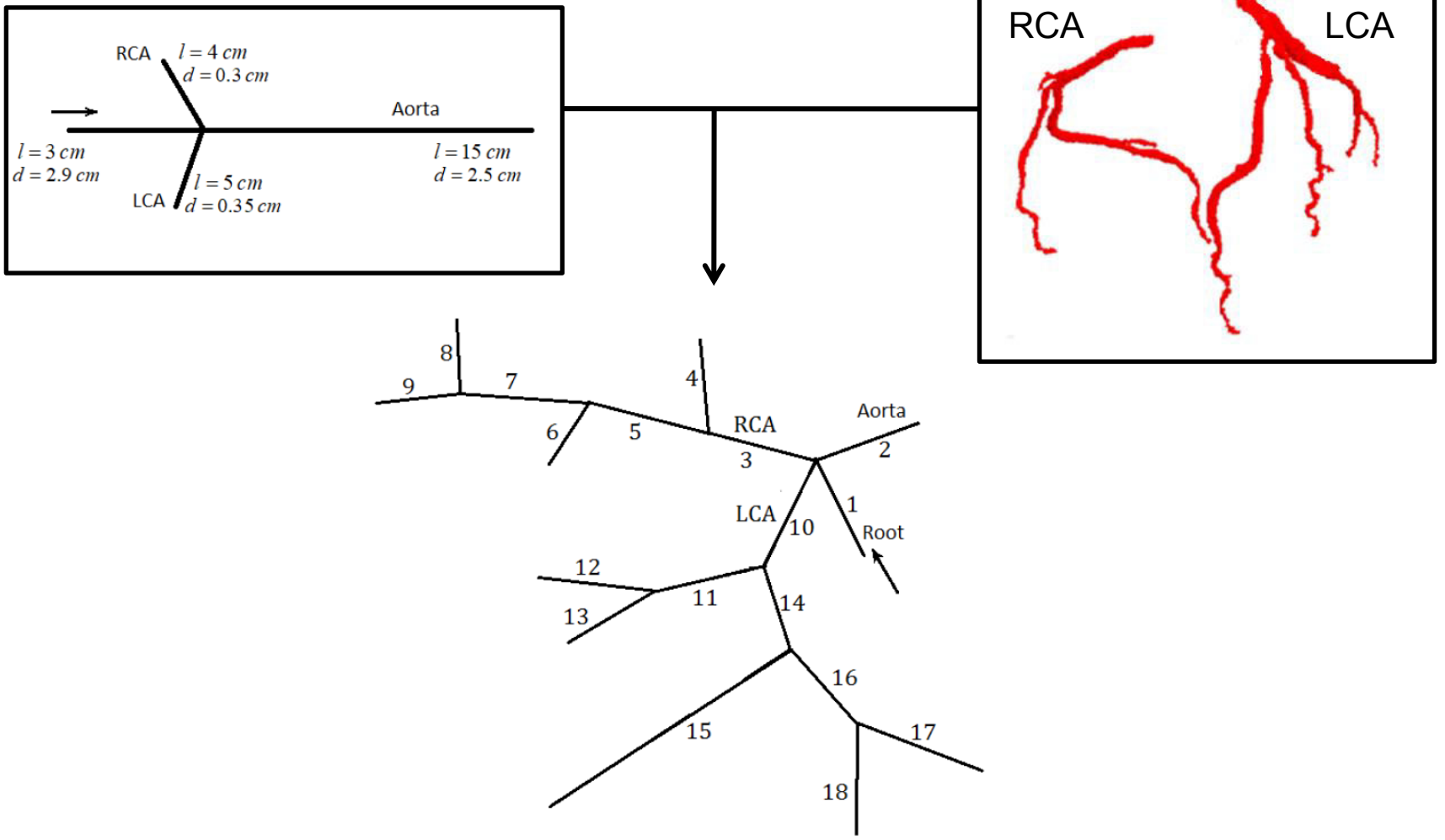
$$\Phi(\vec{x}') = \left\langle A\vec{x}', \vec{x}' \right\rangle + 2\left\langle \vec{b}, \vec{x}' \right\rangle + c \to \min, \quad A \geq 0$$

$$\vec{x}'_{\min}: \quad A\vec{x}' + \vec{b} = 0$$

Parameters estimation

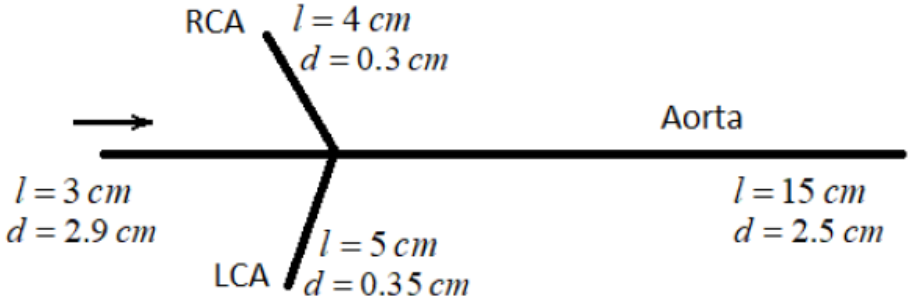
2. Current model $\tilde{M}(\vec{p})$

$$\Delta\vec{p}_k \longrightarrow \Delta\vec{u}_k$$



“Base” model and “new” model

“Base” model



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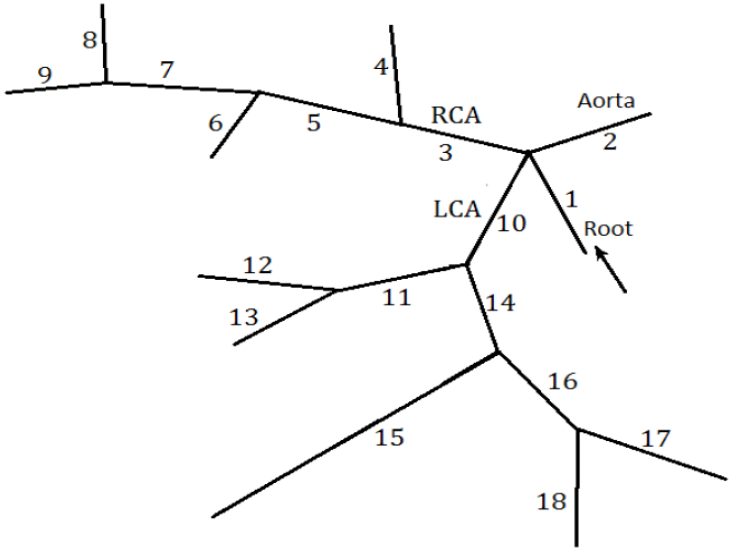
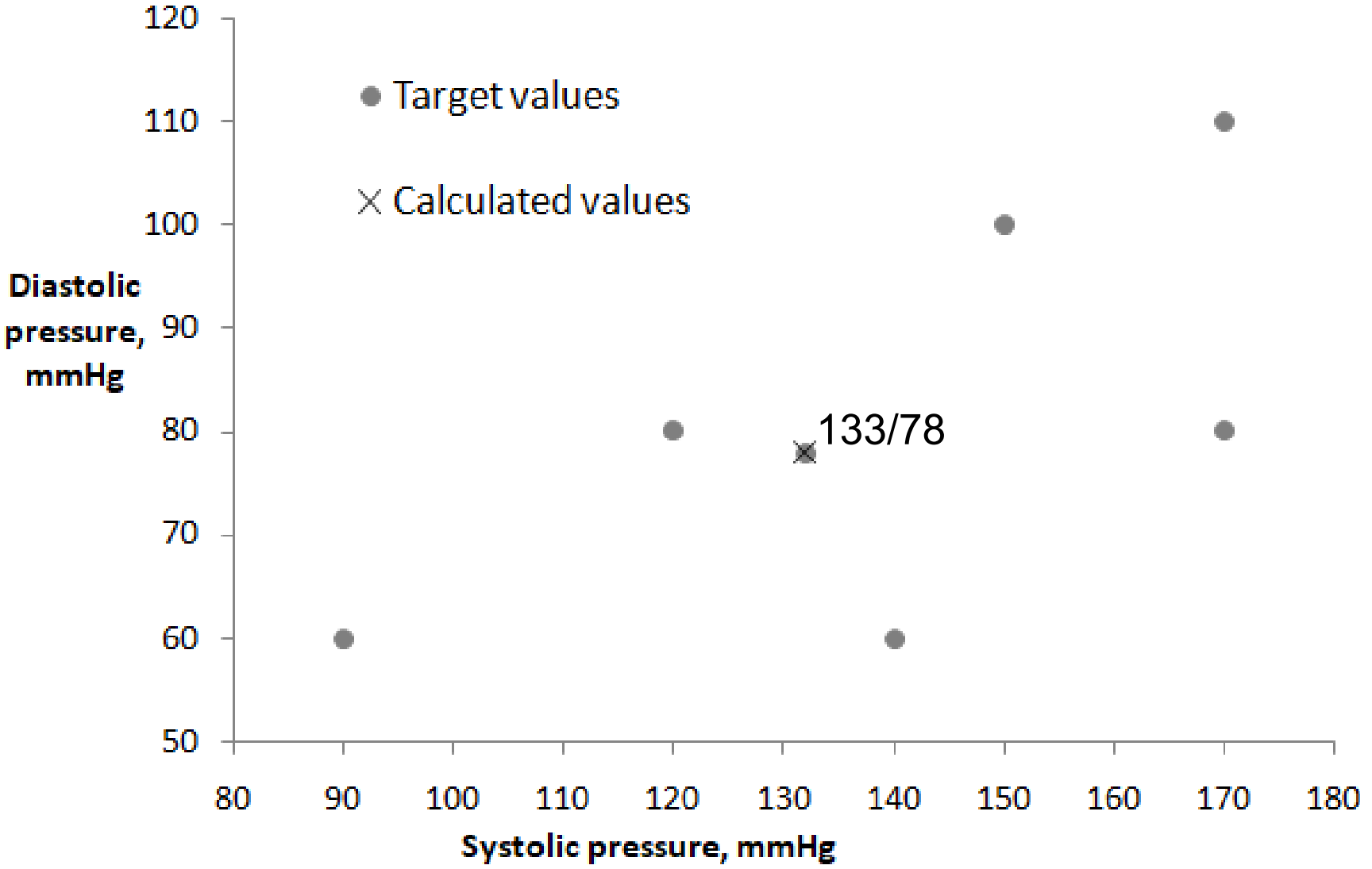
$$\Delta\vec{p}_k \longrightarrow \Delta\vec{u}_k$$

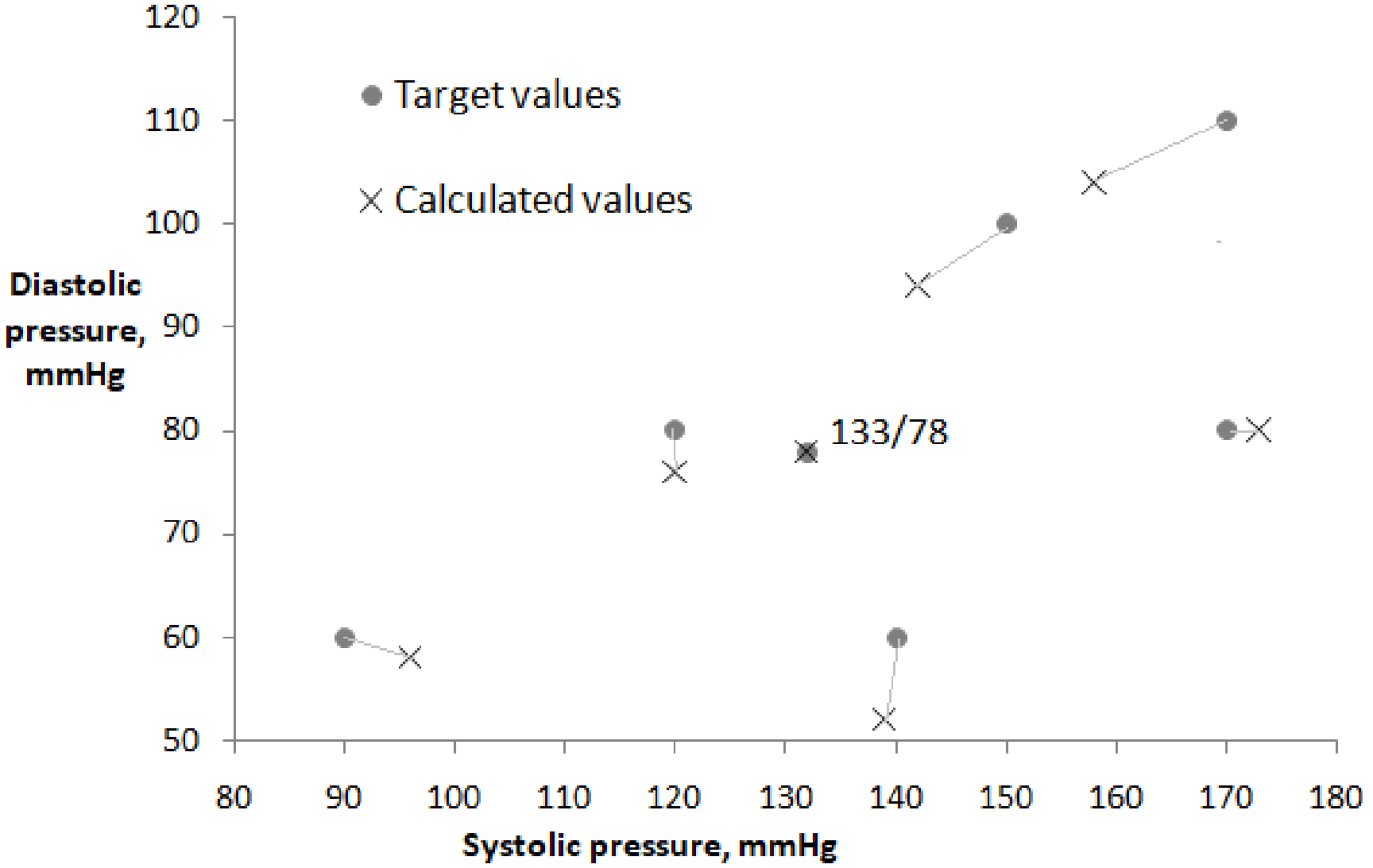
“New” model



Simulation results



Simulation results



$$\sigma_{\max} = 12\ mmHg$$

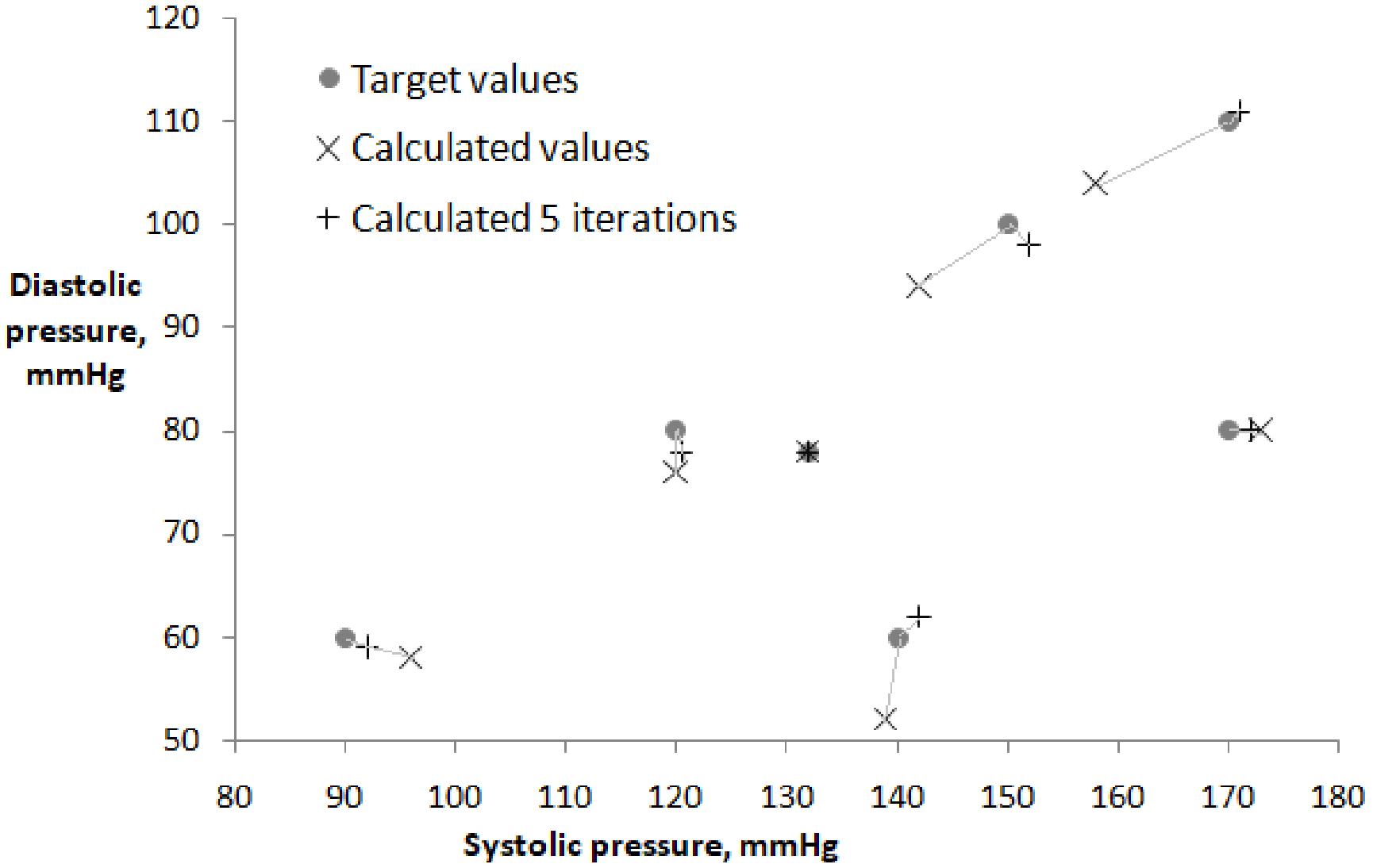
Additional iterations

$$\vec{p} = \vec{p}_0 \longrightarrow \tilde{M}(\vec{p}),\ \Delta\vec{p}_k \to \Delta\vec{u}_k,\ \vec{u}^* \longrightarrow \vec{p}^*$$

$$\vec{p} = \vec{p} + \Delta\vec{p}$$

$$\left\| \tilde{M}(\vec{p} + \Delta\vec{p}) - \vec{u}^* \right\| > \varepsilon$$

Simulation results



$$\sigma_{\max} = 2\ mmHg$$

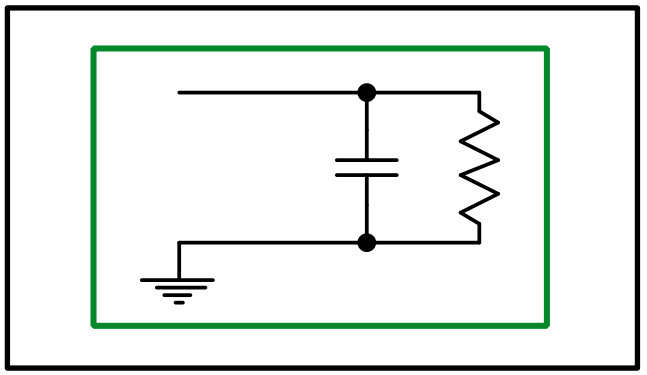
Parameters estimation

0-D: 2-element Windkessel

Parameters

P_{out}, C, τ

$\tau = RC$



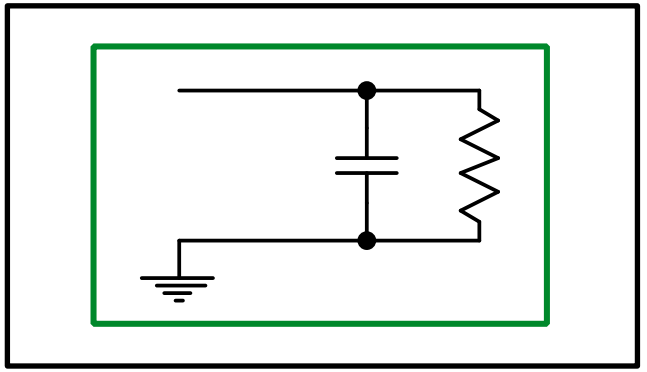
Output values
systolic BP,
diastolic BP

Virtual population

0-D: 2-element Windkessel

Parameters

P_{out}, C, τ



Output values
systolic BP,
diastolic BP

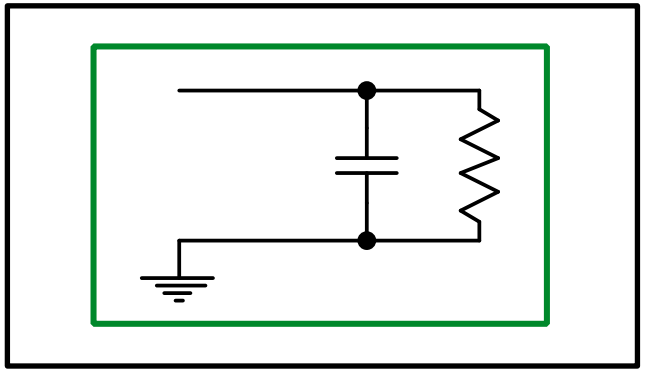
Parameters estimation
+
Virtual population

Virtual population

0-D: 2-element Windkessel

Parameters

P_{out}, C, τ



Output values
systolic BP,
diastolic BP

?

Parameters

$\tilde{P}_{out}, \tilde{C}, \tilde{\tau}$

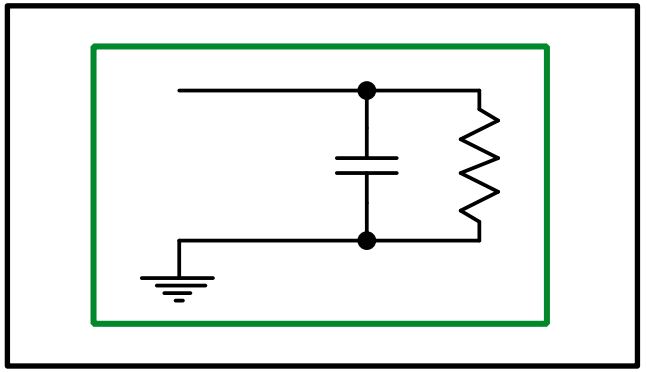
Parameters estimation
+
Virtual population

Virtual population

0-D: 2-element Windkessel

Parameters

P_{out}, C, τ



Output values
systolic BP,
diastolic BP

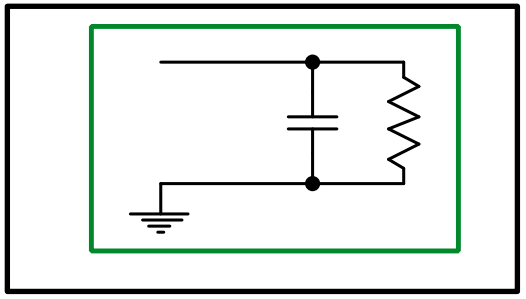
?

Parameters

$\tilde{P}_{out}, \tilde{C}, \tilde{\tau}$

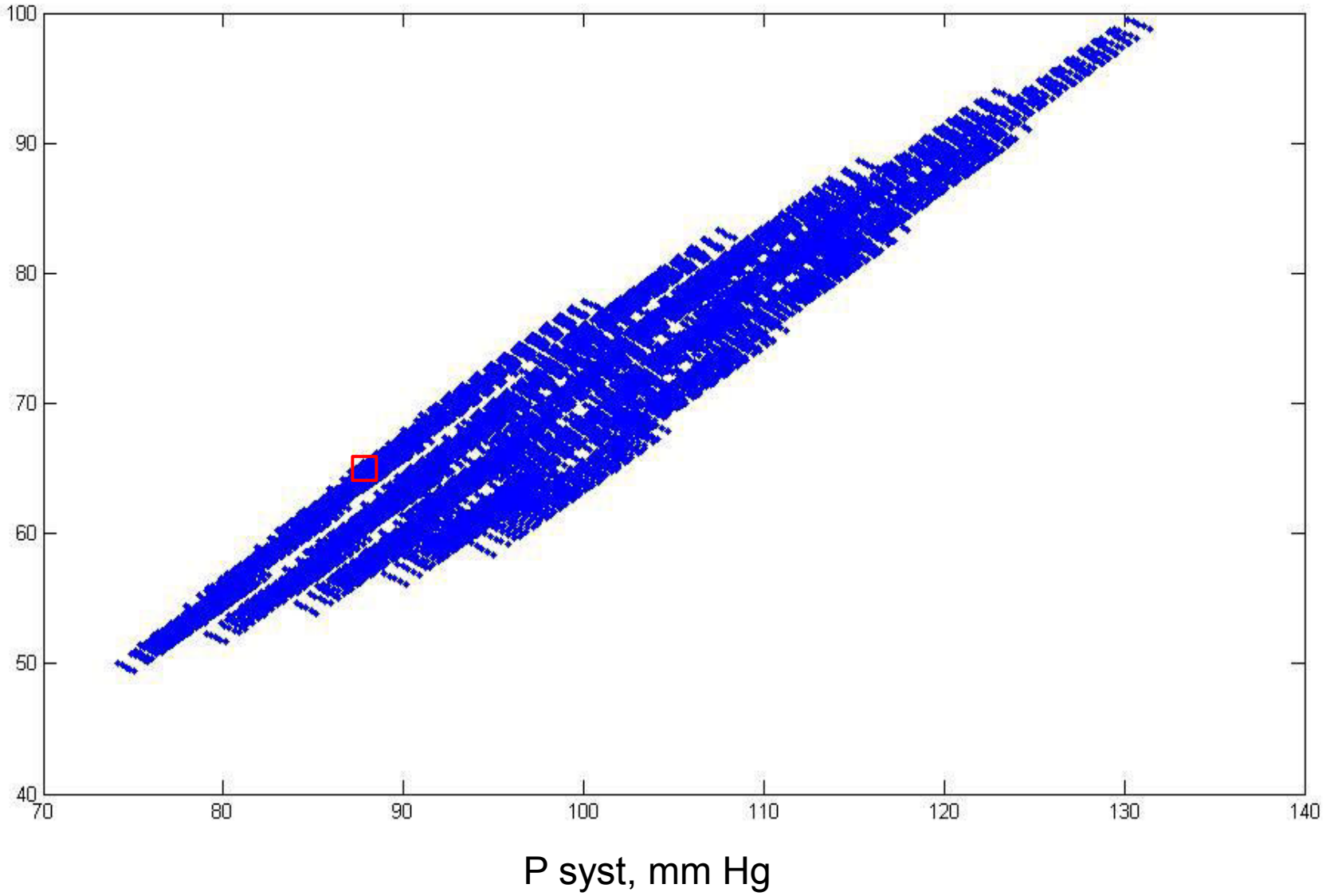
Parameters estimation
+
Virtual population

?

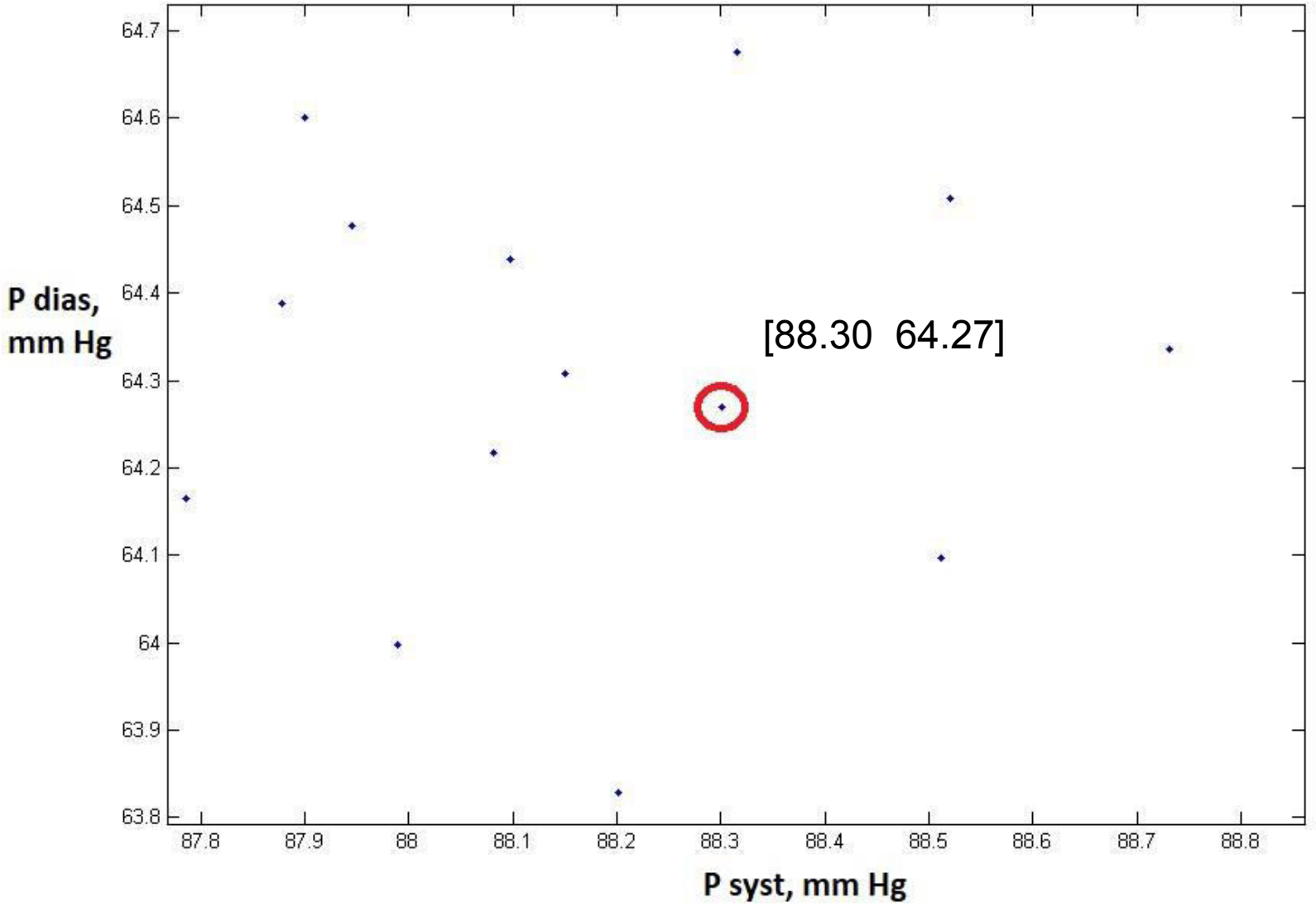


New output values
systolic BP,
diastolic BP

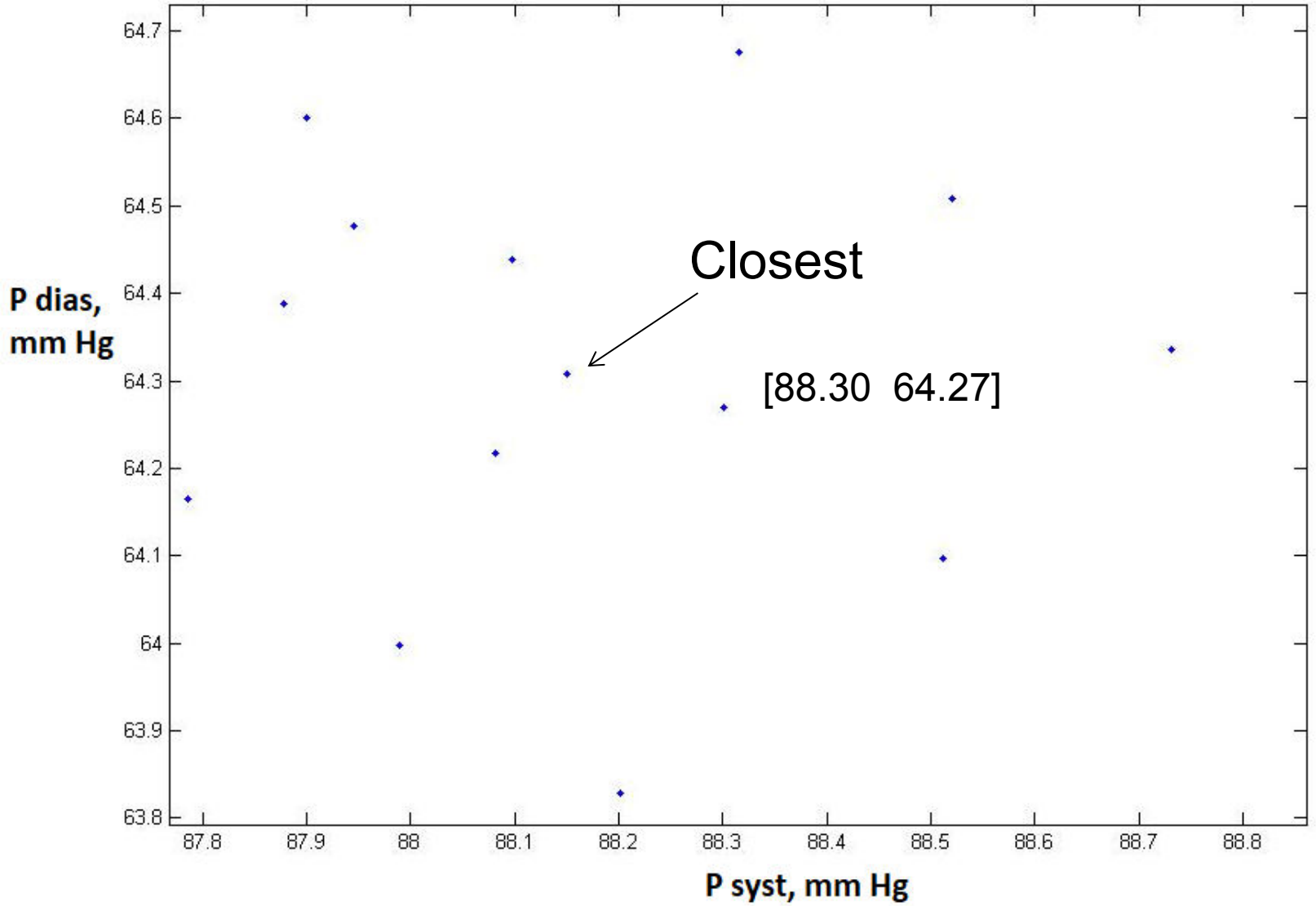
Virtual population pressures distribution



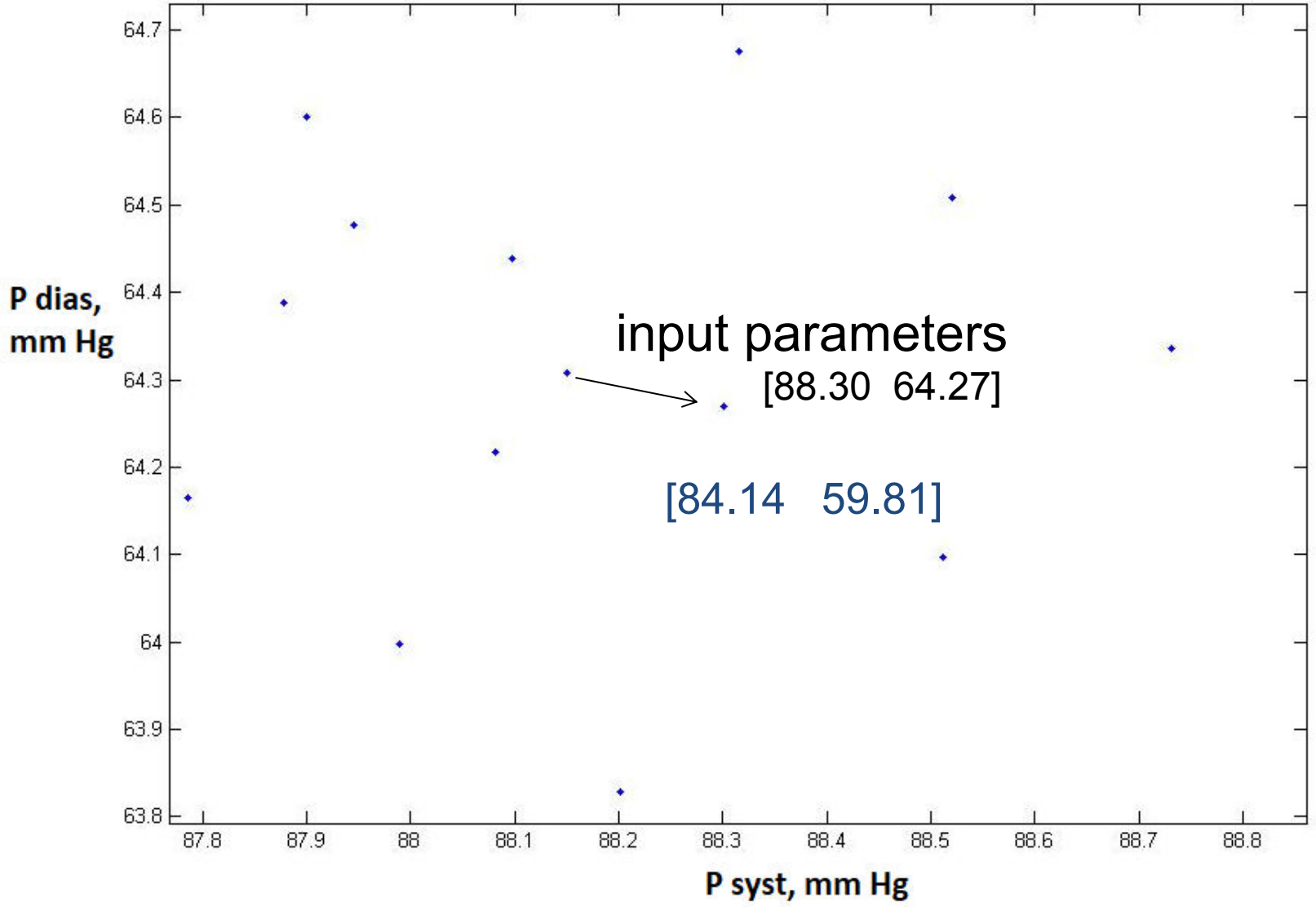
Virtual population pressures distribution



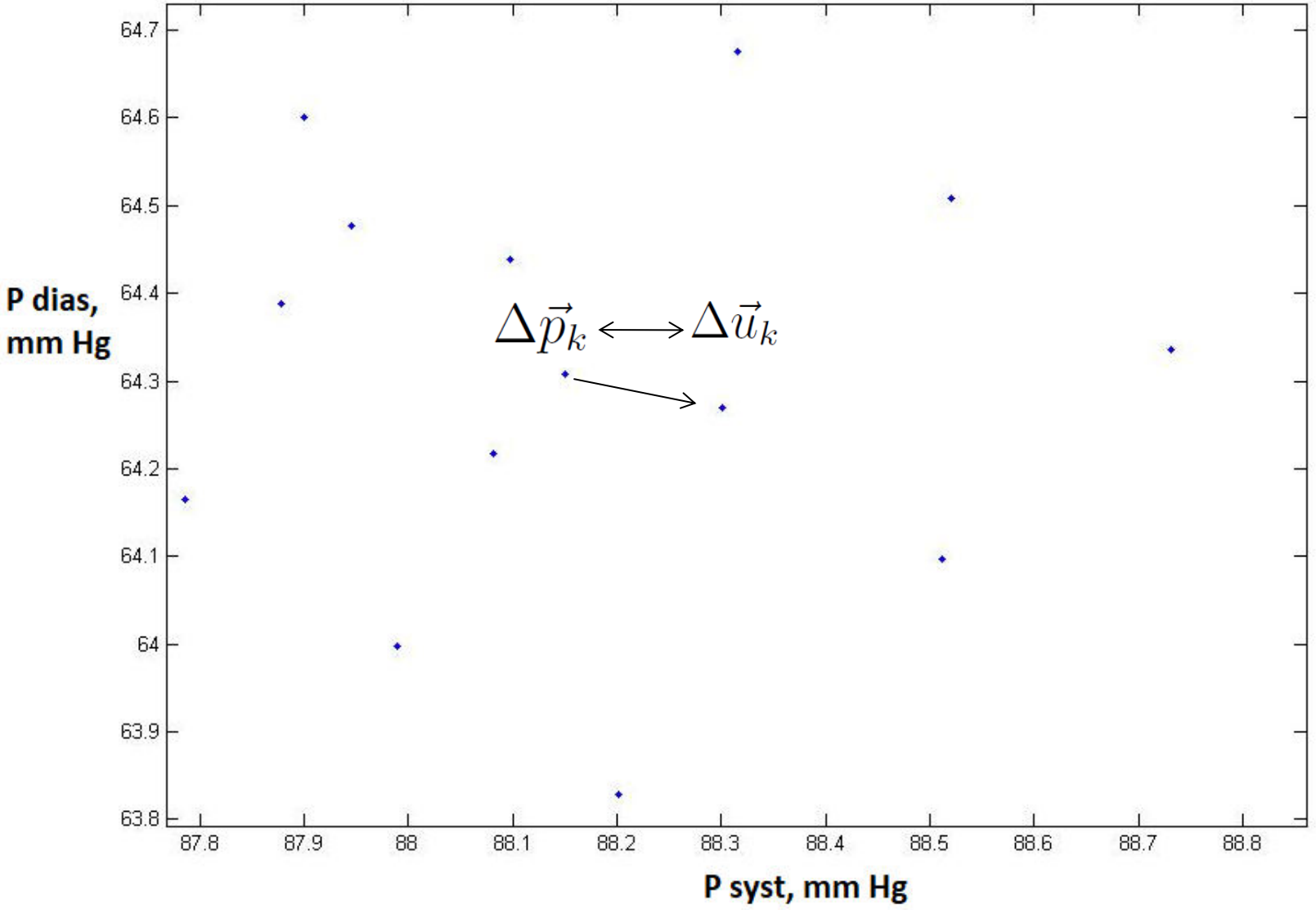
Virtual population pressures distribution



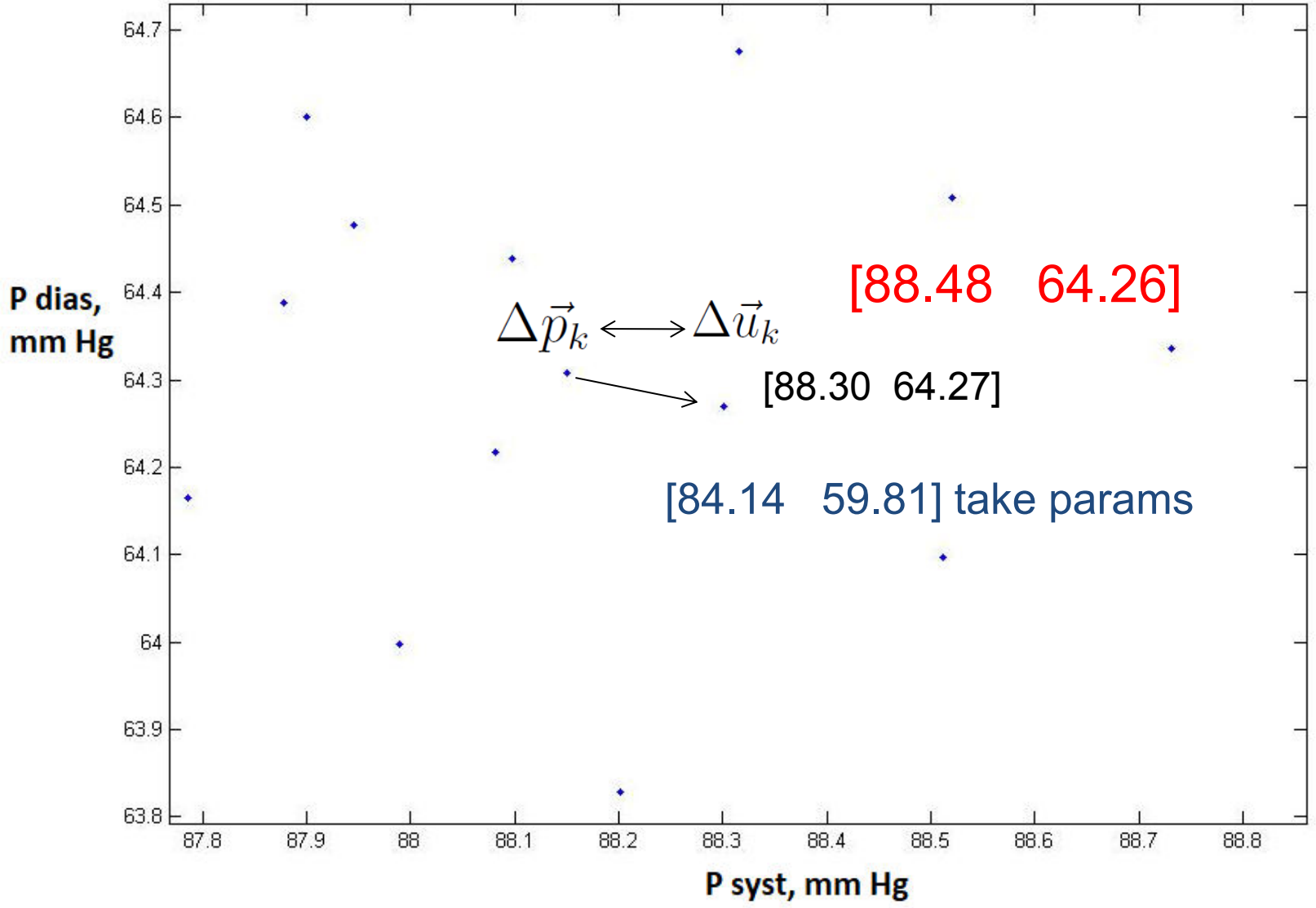
Take parameters from the closest point



Use parameters estimation algorithm



Use parameters estimation algorithm

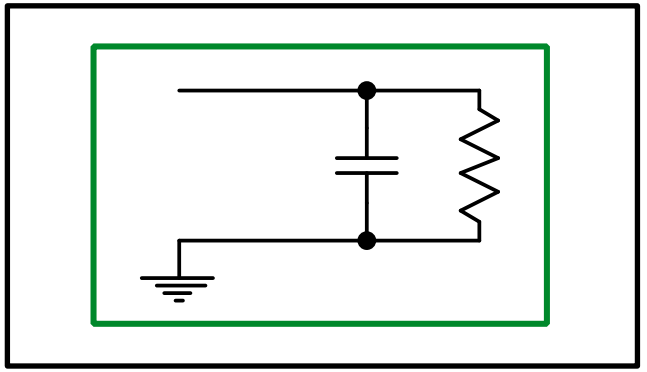


Virtual population

0-D: 2-element Windkessel

Parameters

P_{out}, C, τ



Output values
systolic BP,
diastolic BP

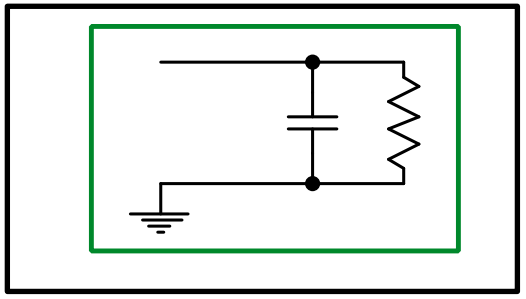
?

Parameters

$\tilde{P}_{out}, \tilde{C}, \tilde{\tau}$

Parameters estimation
+
Virtual population

?



New output values
systolic BP,
diastolic BP

All virtual population

	Relative deviation	
	max	average
Output values (P syst, P diast)		
Transfer parameters	10 %	2 %
Parameters estimation	0.8 %	0.2 %
Input parameters P_{out}, C, τ		
Transfer parameters	17 %	7 %
Parameters estimation	16 %	5 %

Conclusion

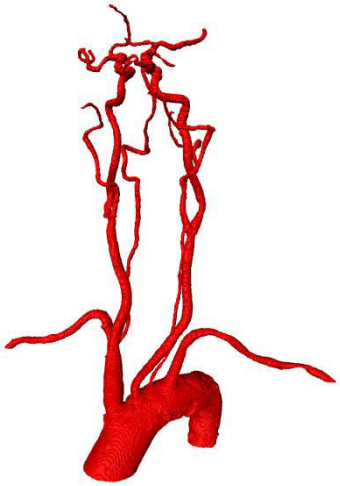
Pros

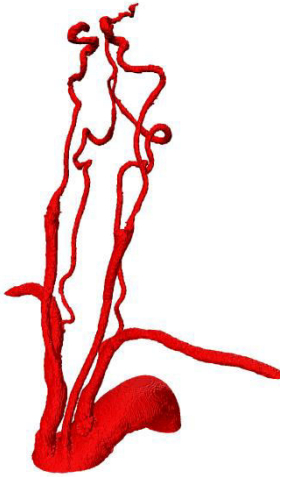
- fast
- can handle problems with a lot of parameters
- good for similar problems

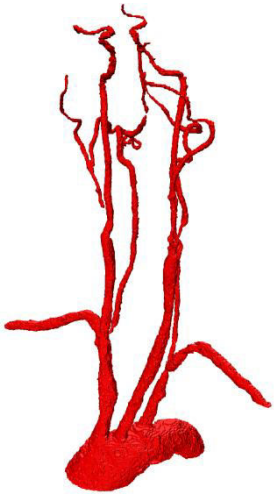
Cons

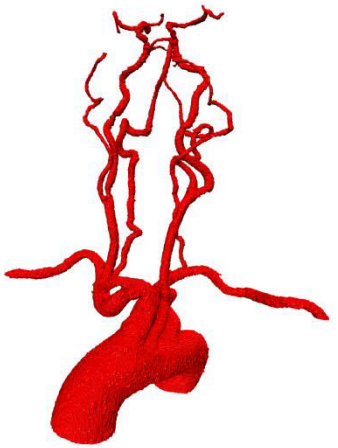
- nonlinearity
- low accuracy

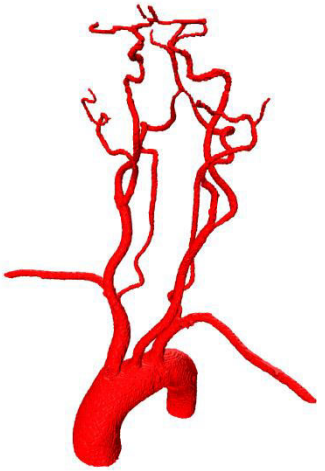
Thank You!

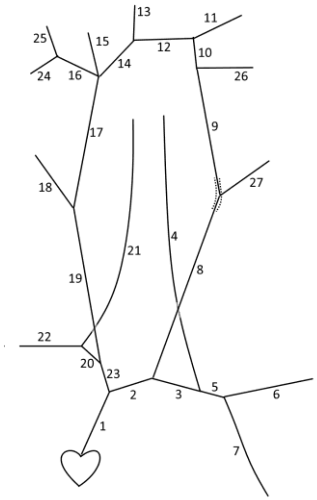








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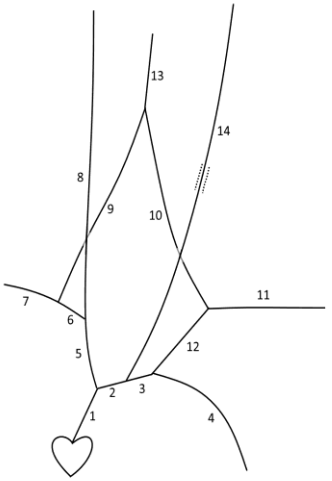
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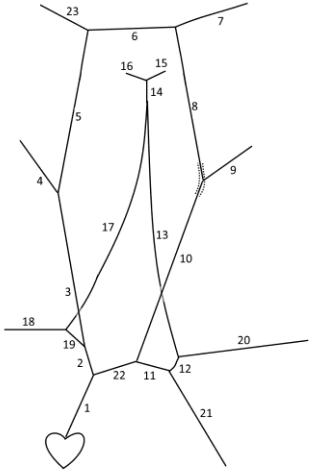
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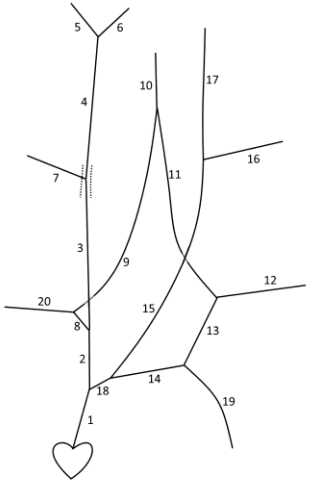
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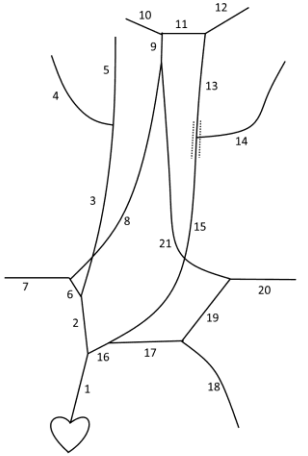
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Blood flow circulation model

1) Mass balance $$\frac{\partial A}{\partial t}+\frac{\partial (uA)}{\partial x}=0$$

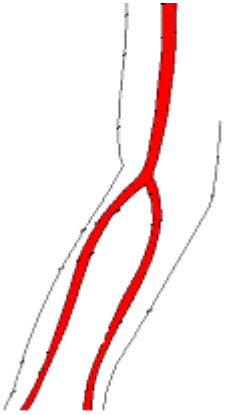
2) Momentum balance

$$\frac{\partial u}{\partial t}+\frac{\partial}{\partial x}\left(\frac{u^2}{2}+\frac{P}{\rho}\right)=-16\mu u\frac{\eta(A)}{Ad^2}+\psi(\ldots),\ \eta(A)=\begin{cases}2, A>A_0\\ \frac{A}{A_0}+\frac{A_0}{A}, A\le A_0\end{cases}$$

3) Bifurcations

3.1 $$\sum_{k=k_1,\ldots,k_M}\varepsilon_k^m Q_k=0,\ \varepsilon_k^m=\pm 1,\ Q_k=u_k A_k$$

3.2 $$p_k+\frac{\rho u_k^2}{2}=p_j+\frac{\rho u_j^2}{2},\quad \forall j,k$$



3.3 Compatibility conditions

Wall state equation

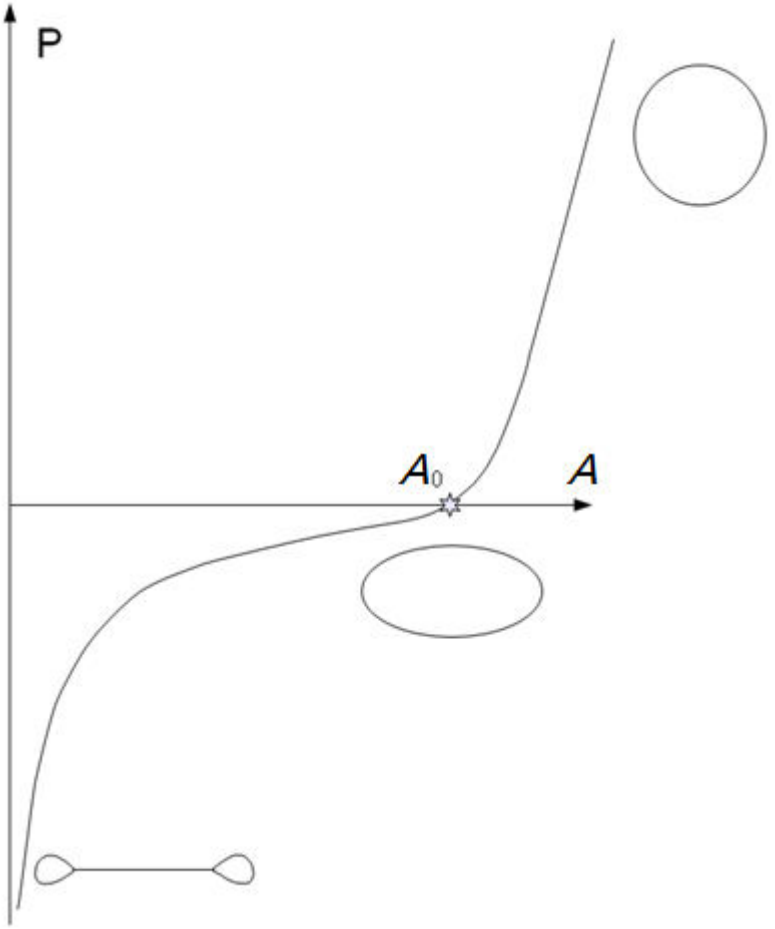
4) Vessel wall elasticity

Analytic approximation

$$P(A) = P^{ext}(t,x) + \rho c^2 f(A)$$

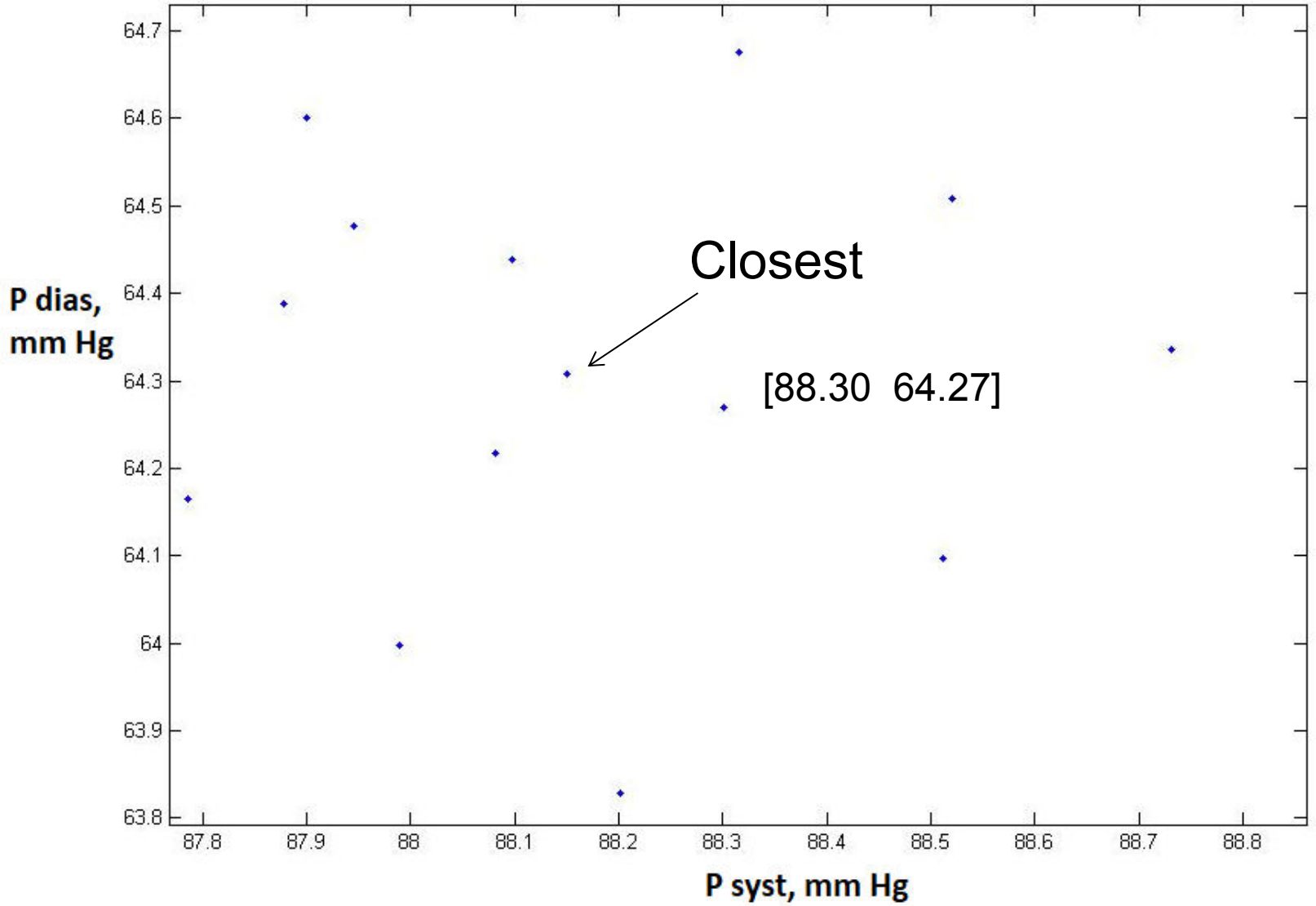
$$f(A) = \begin{cases} \exp(A/A_0 - 1) - 1, A > A_0 \\ \ln(A/A_0), A \le A_0 \end{cases}$$

$P^{ext}(t,x)$ — external pressure



Pedley, Luo, 1998

Virtual population pressures distribution



Closest:

{P syst, P diast, CA}

All virtual population

	Relative deviation	
	max	average
Output values (P syst, P diast)		
Transfer parameters	10 %	2 %
Parameters estimation	0.8 %	0.2 %
Input parameters P_{out}, C, τ		
Transfer parameters	17 %	7 %
Parameters estimation	16 %	5 %

All virtual population 3Wk

	Relative deviation	
	max	average
Output values (P syst, P diast)		
Transfer parameters	5 %	0.4 %
Parameters estimation	5 %	0.4 %
Input parameters P_{out}, C, τ, Z_0		
Transfer parameters	15 %	6 %
Parameters estimation	15 %	6 %

All virtual population 3Wk (R > 10Rmin)

	Relative deviation	
	max	average
Output values (P syst, P diast)		
Transfer parameters	16 %	3 %
Parameters estimation	7 %	2 %
Input parameters P_{out}, C, τ, Z_0		
Transfer parameters	40 %	9 %
Parameters estimation	50 %	6 %

- CA from pressure wave (paper)
- Response surface model