Computational simulations of the results of carotid stenosis treatment based on patient-specific data

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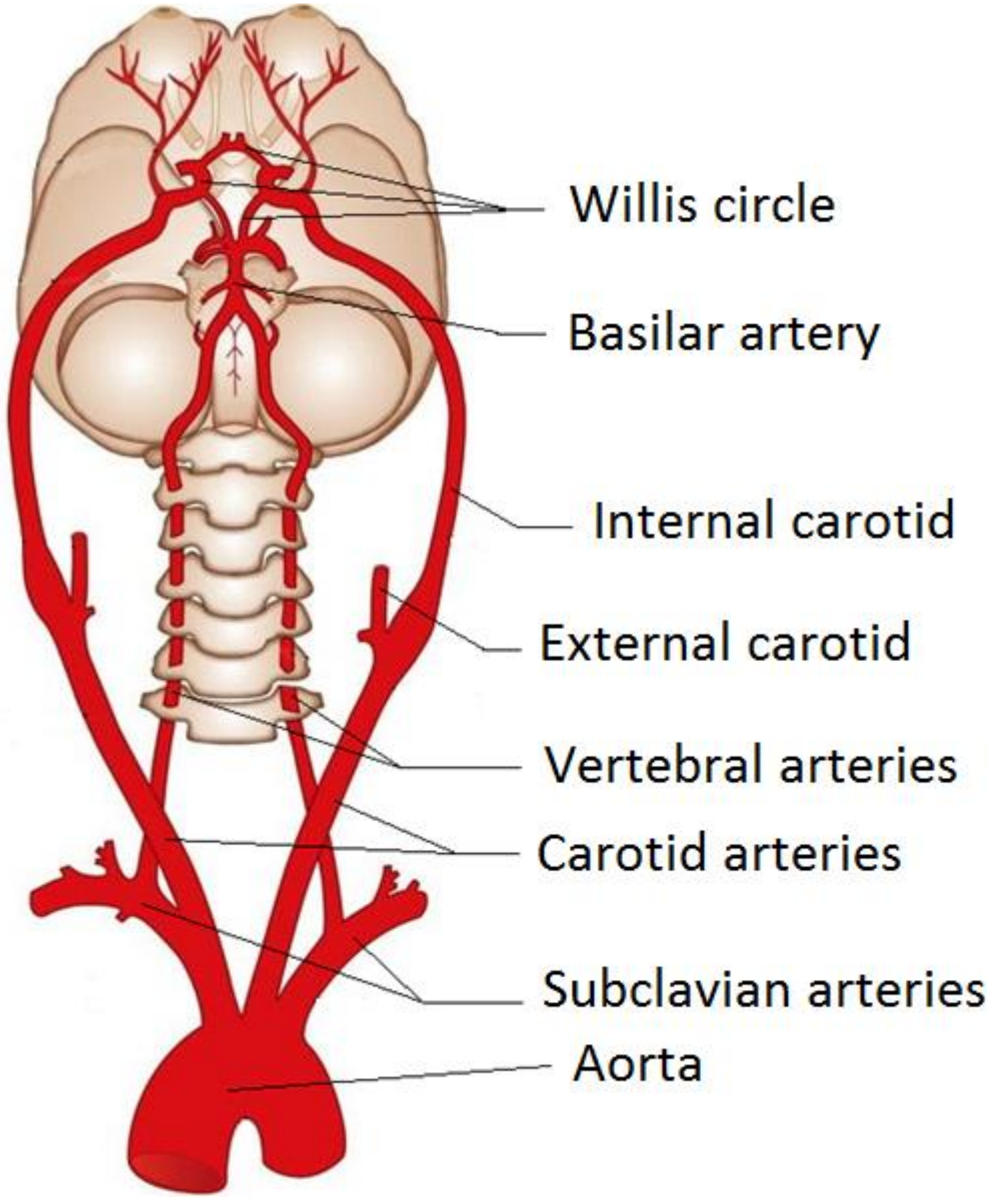
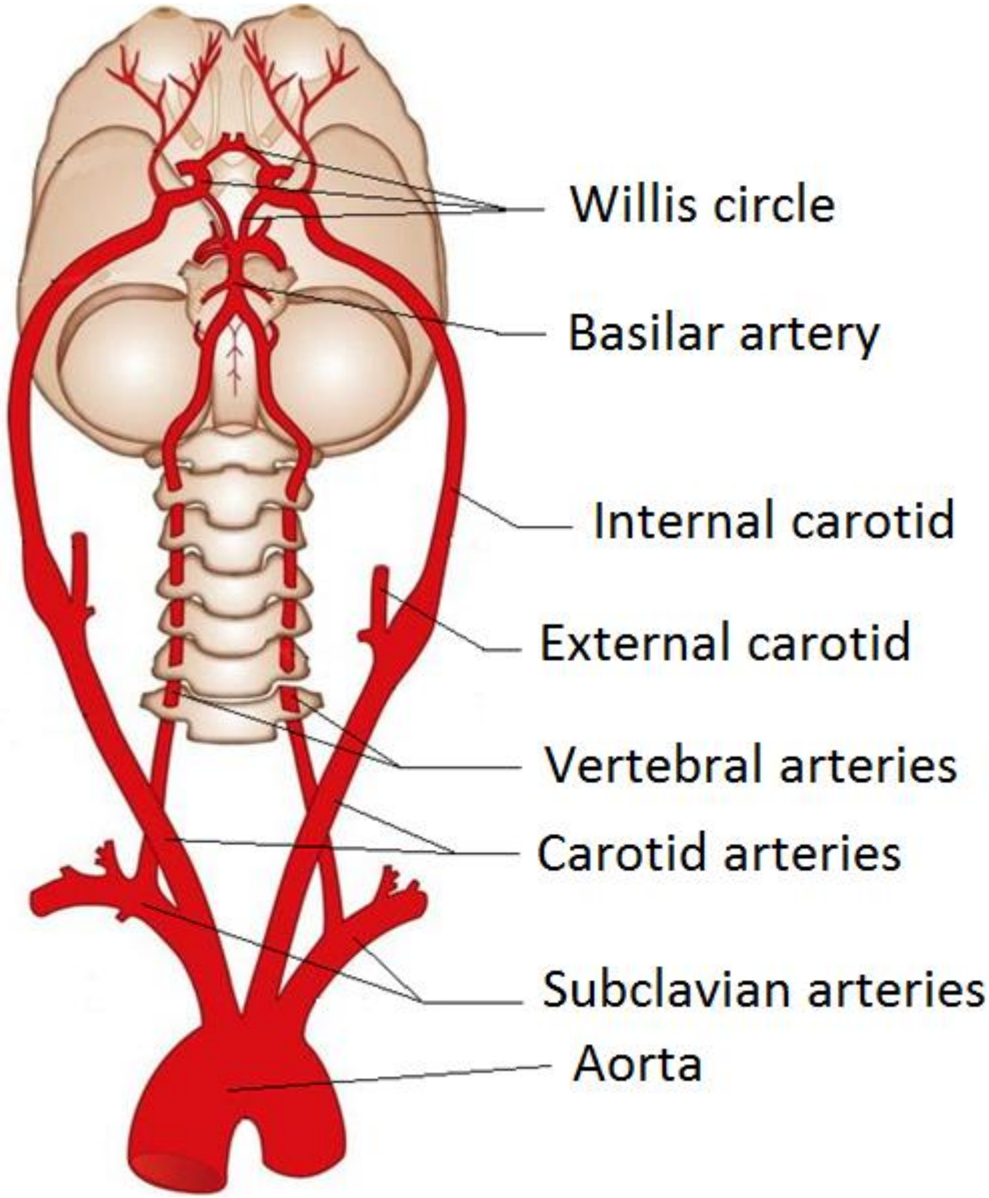
Moscow, INM RAS, 31.10.2016

Blood vessels of head and neck

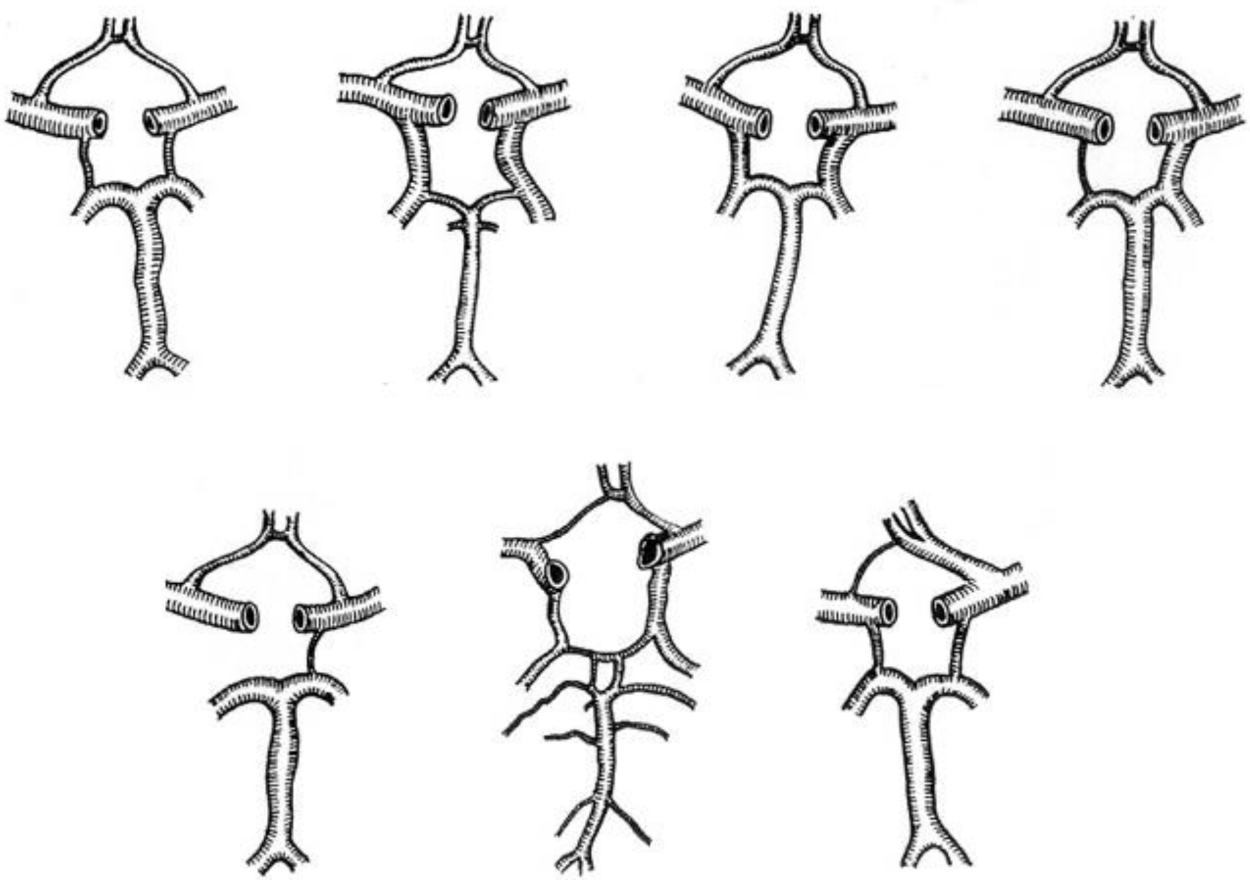


Blood vessels of head and neck



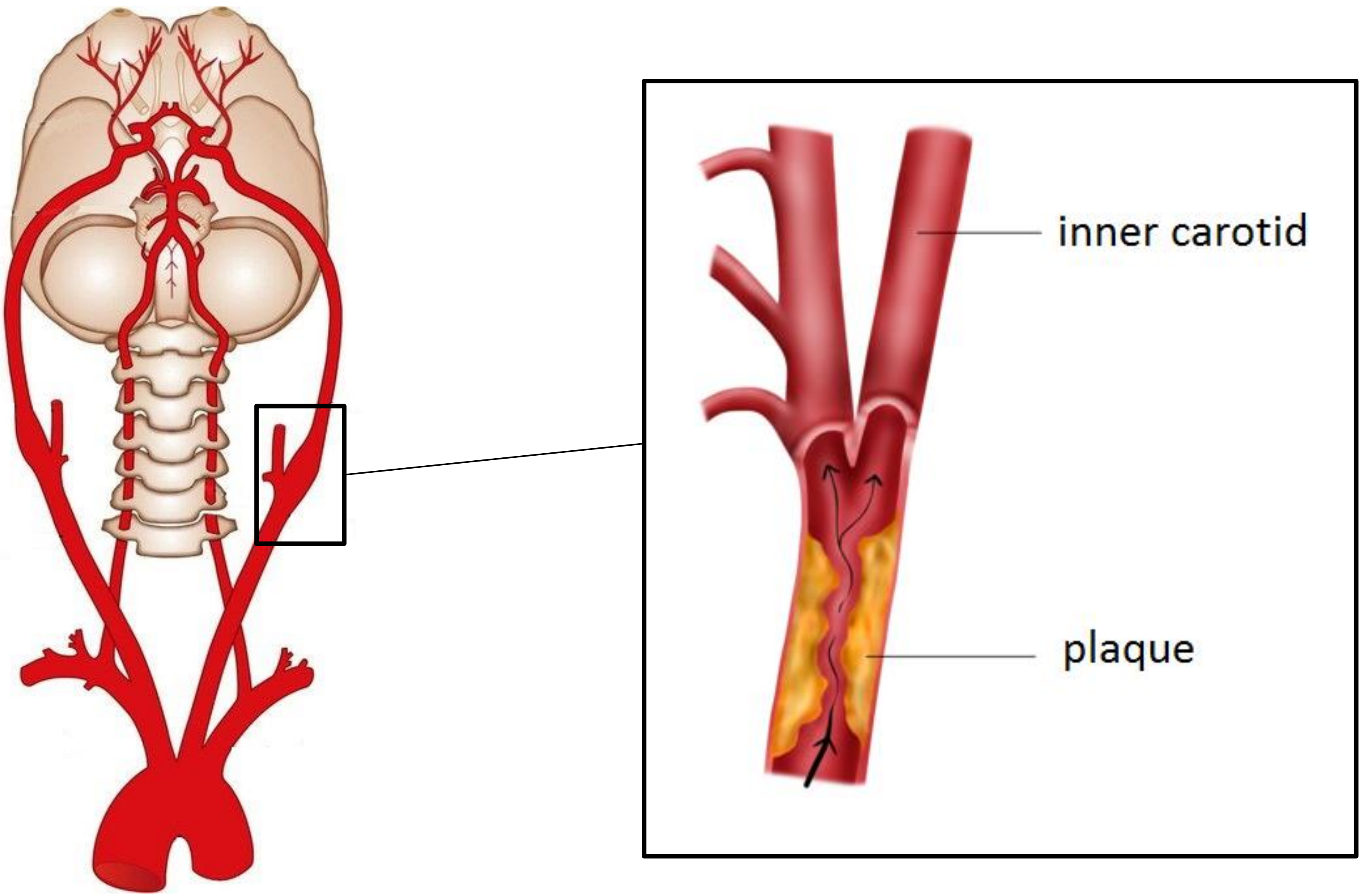
- 20 % blood volume
- high density
- regulatory mechanisms
- collateral paths
- individual

Circle of Willis

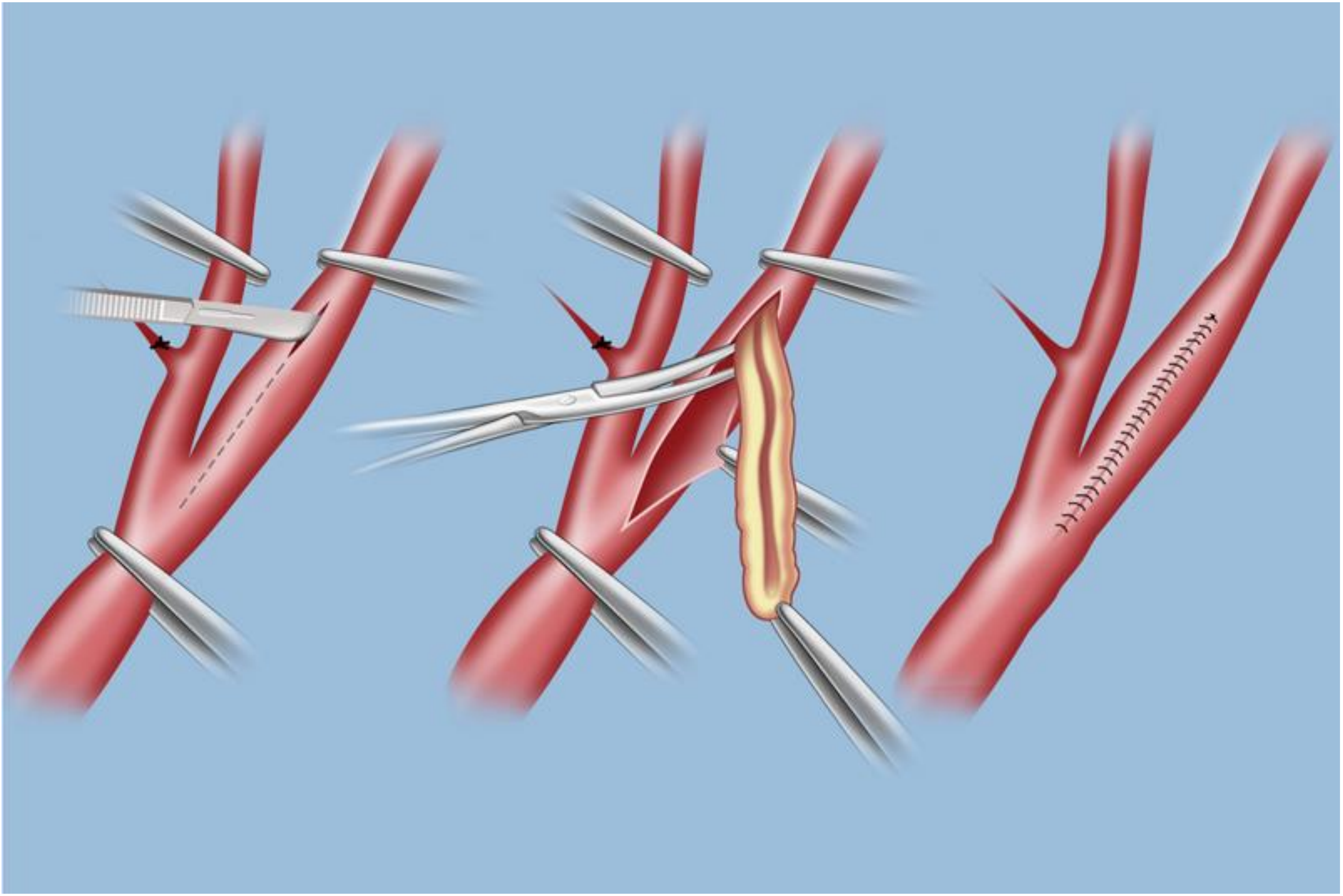


Full circle:
20-25 %

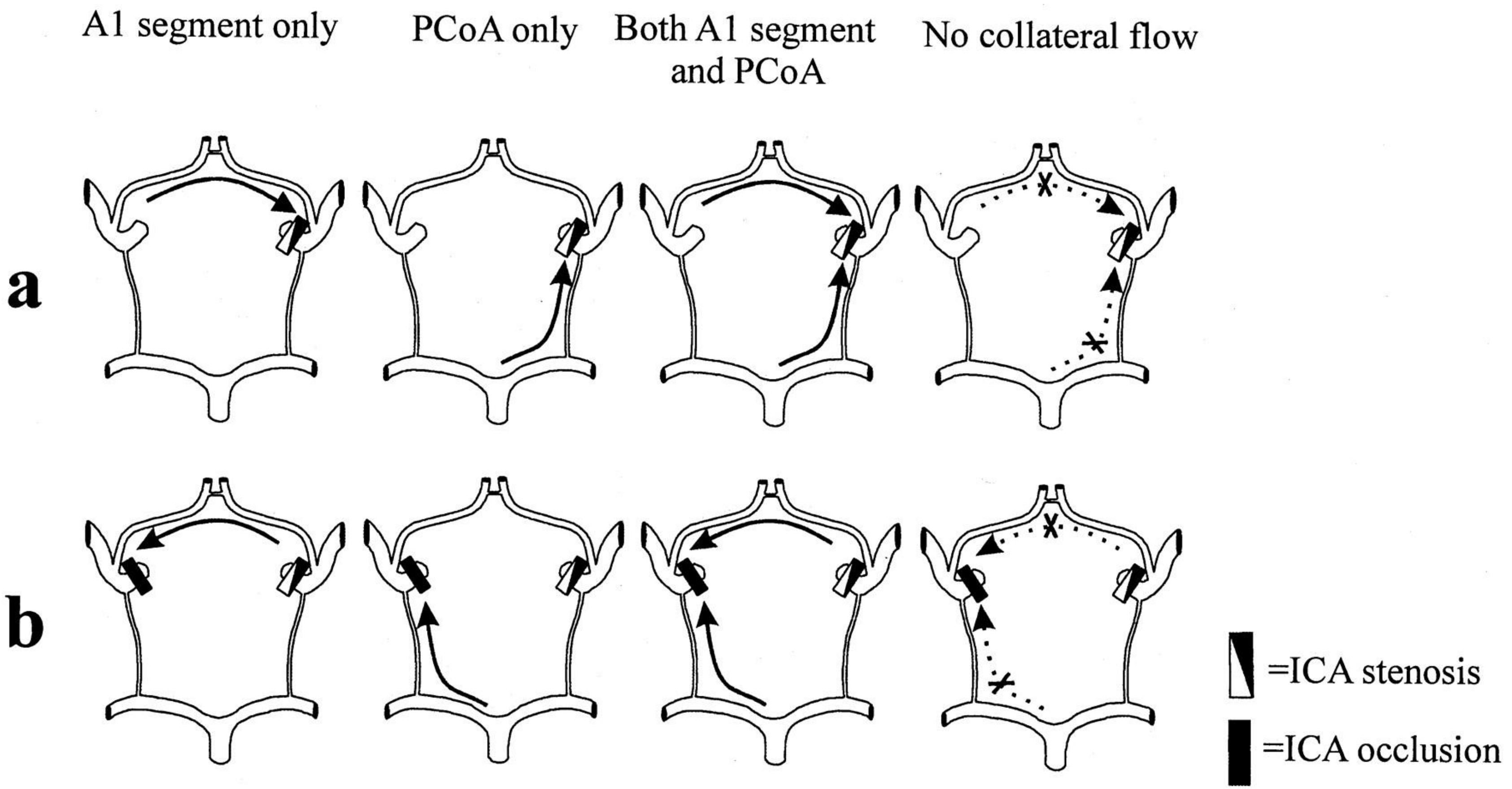
Stenosis



Carotid endarterectomy

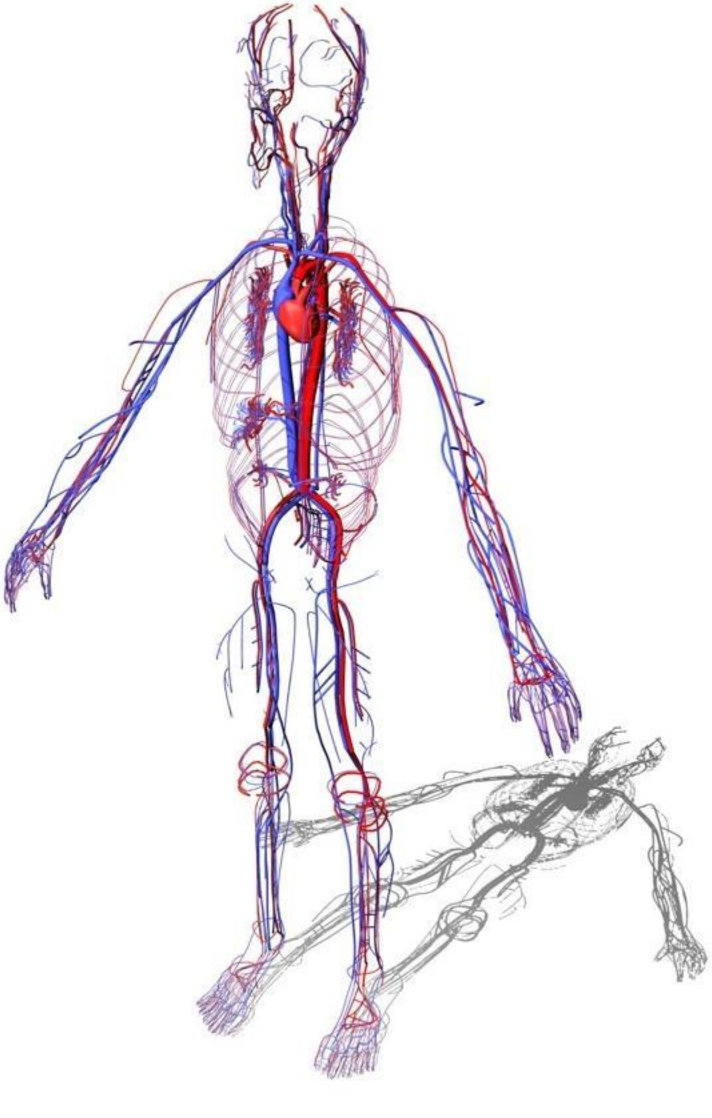


Blood flow patterns



Effect of Carotid Endarterectomy on Primary Collateral Blood Flow in Patients With Severe Carotid Artery Lesions
J. Hendrikse, D.R. Rutgers, C.J.M. Klijn, et.al. 2003

Blood flow model



Blood flow circulation model

1) Mass balance $$\frac{\partial A}{\partial t}+\frac{\partial (uA)}{\partial x}=0$$

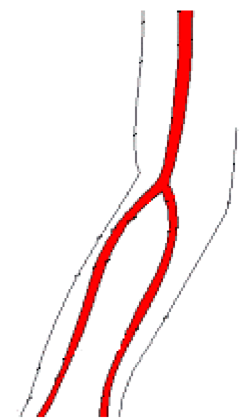
2) Momentum balance

$$\frac{\partial u}{\partial t}+\frac{\partial}{\partial x}\left(\frac{u^2}{2}+\frac{P}{\rho}\right)=-16\mu u\frac{\eta(A)}{Ad^2}+\psi(\dots),\ \eta(A)=\begin{cases}2, A>A_0\\ \dfrac{A}{A_0}+\dfrac{A_0}{A}, A\le A_0\end{cases}$$

3) Bifurcations

3.1 $$\sum_{k=k_1,\dots,k_M}\varepsilon_k^m Q_k=0,\ \varepsilon_k^m=\pm 1,\ Q_k=u_k A_k$$

3.2 $$p_k+\frac{\rho u_k^2}{2}=p_j+\frac{\rho u_j^2}{2},\quad \forall j,k$$



3.3 Compatibility conditions

Wall state equation

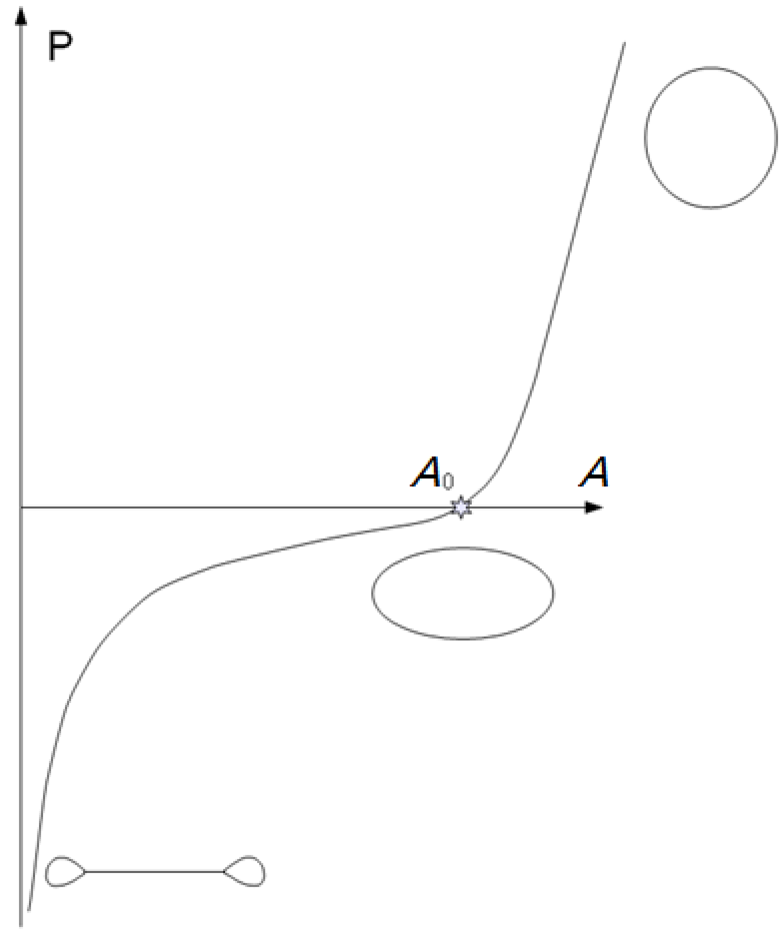
4) Vessel wall elasticity

Analytic approximation

$$P(A) = P^{ext}(t,x) + \rho c^2 f(A)$$

$$f(A) = \begin{cases} \exp(A/A_0 - 1) - 1, & A > A_0 \\ \ln(A/A_0), & A \le A_0 \end{cases}$$

$P^{ext}(t,x)$ — external pressure



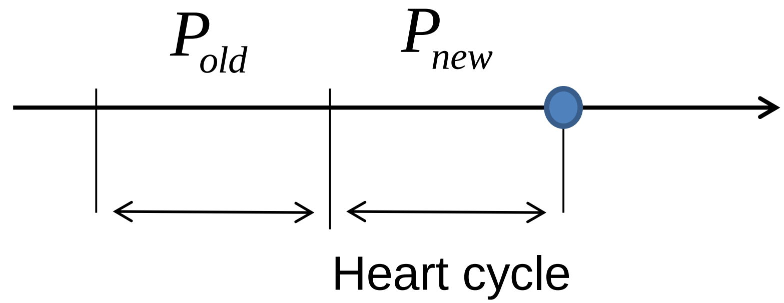
Pedley, Luo, 1998

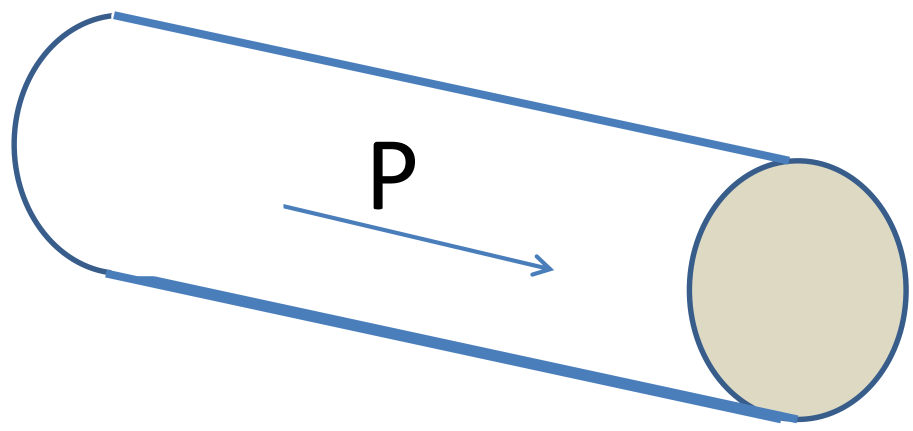
Autoregulation

Response to external pressure:

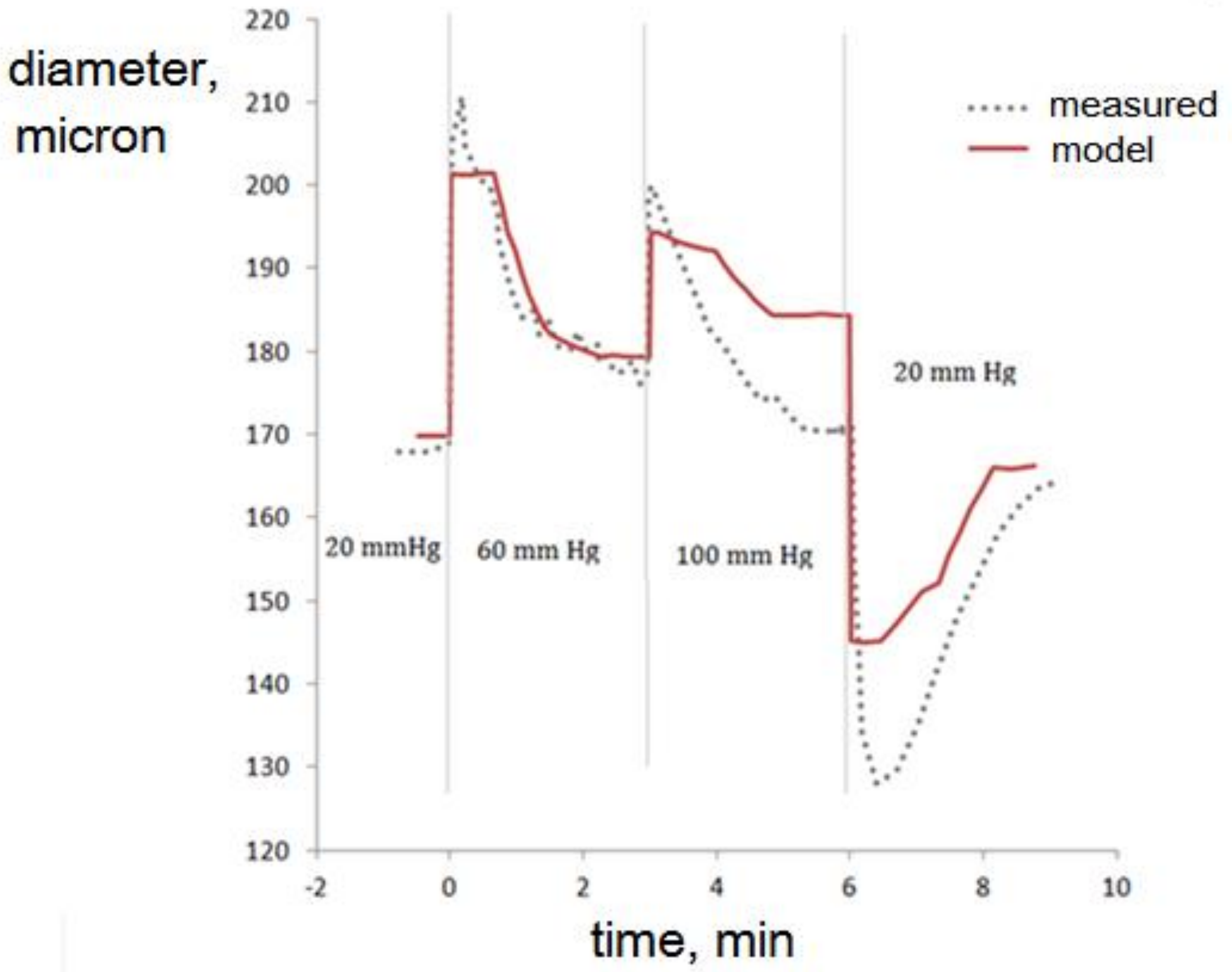
$$P = \rho c^2 \left(\exp\left(\frac{A}{A_0} - 1 \right) - 1 \right)$$

$$\boxed{\frac{c_{new}}{c_{old}} = \left(\frac{P_{new}}{P_{old}} \right)^{1/2}}$$

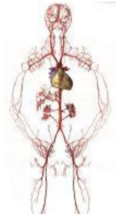




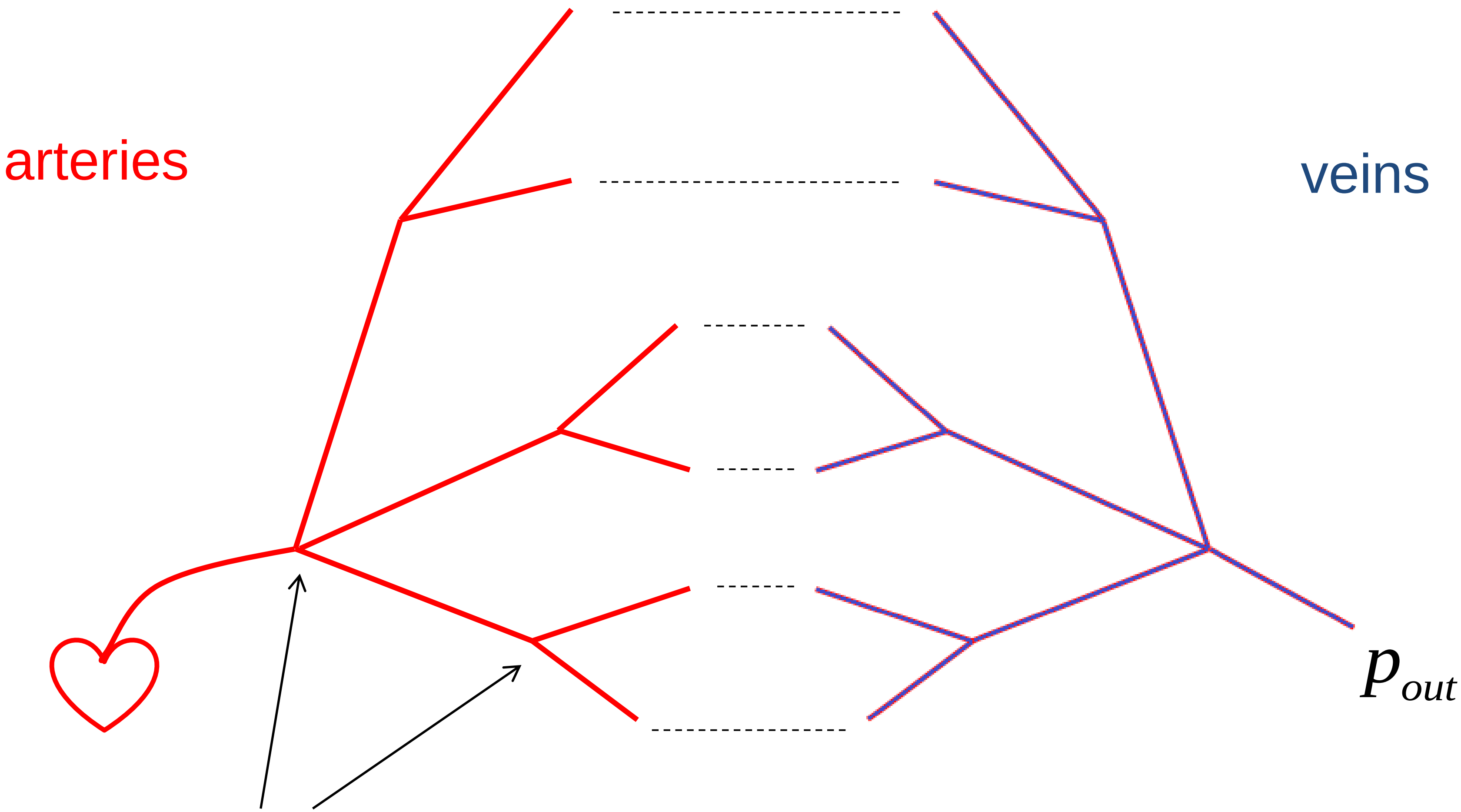
Autoregulation. Rat artery



Bifurcations. Virtual vessels



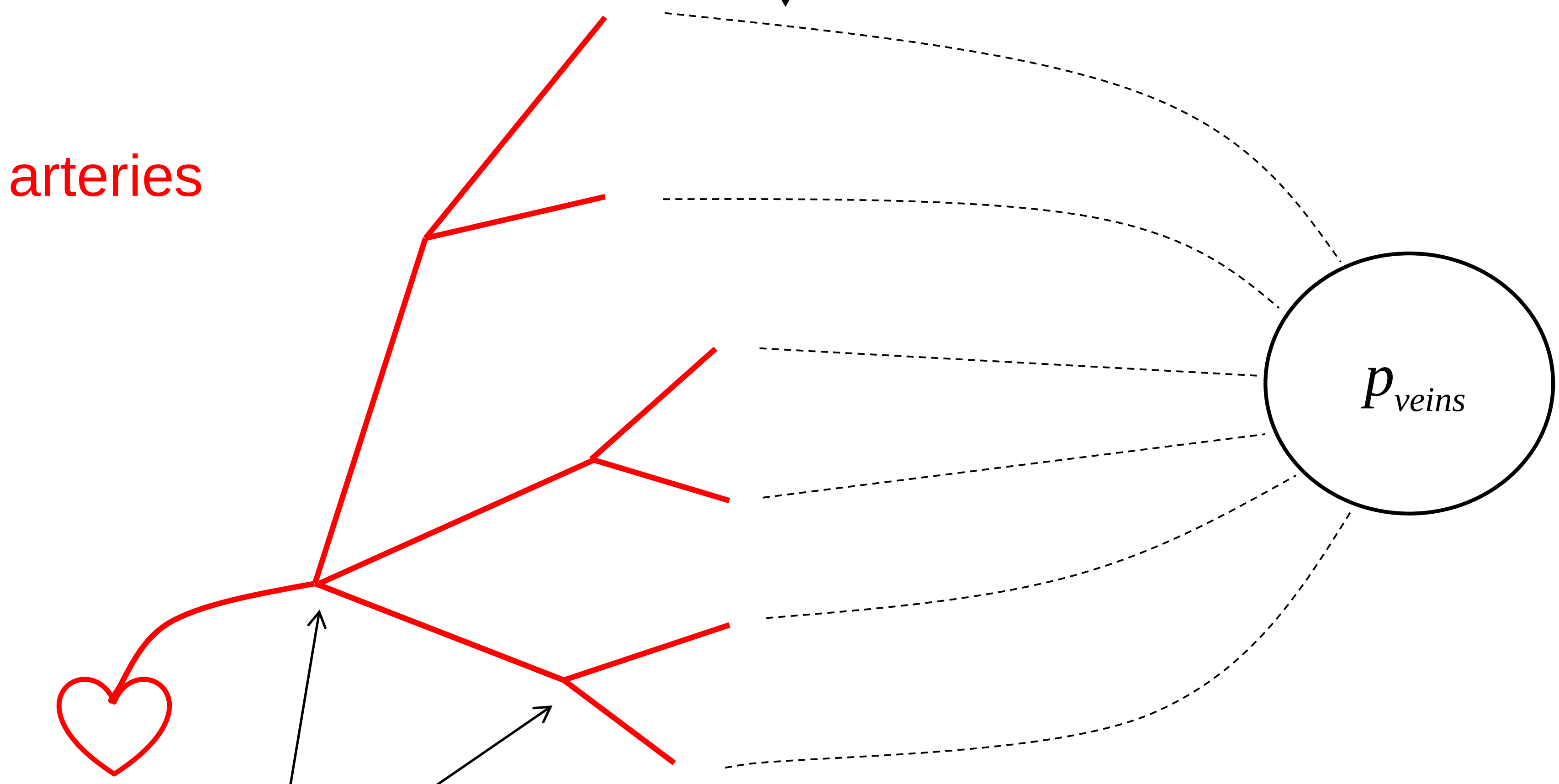
$$p_k\left(t, x_k\right) - p_m^{node}\left(t\right) = \alpha_k R_k^m Q_k$$



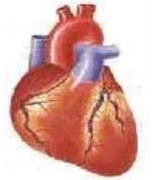
$$p_k\left(t, x_k\right) - p_m^{node}\left(t\right) = 0$$

Bifurcations. Resistance

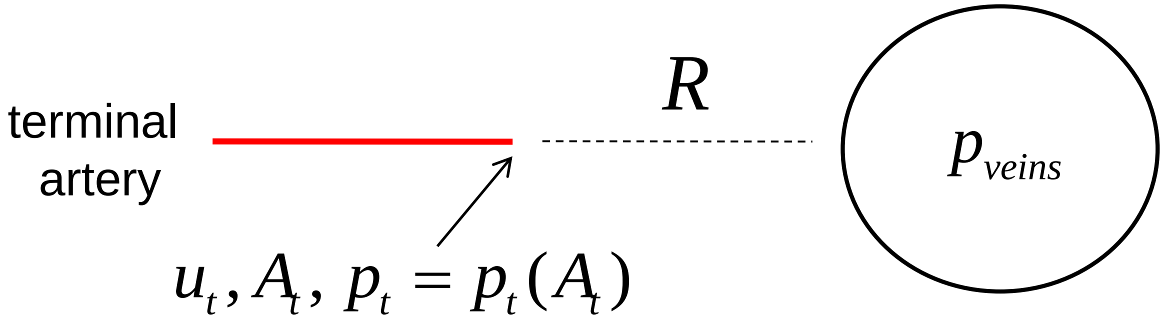
$$p_k - p_{veins} = R_k Q_k$$



$$p_k + \frac{\rho u_k^2}{2} = p_j + \frac{\rho u_j^2}{2}, \quad \forall j, k$$



Bifurcations. Terminal vessel



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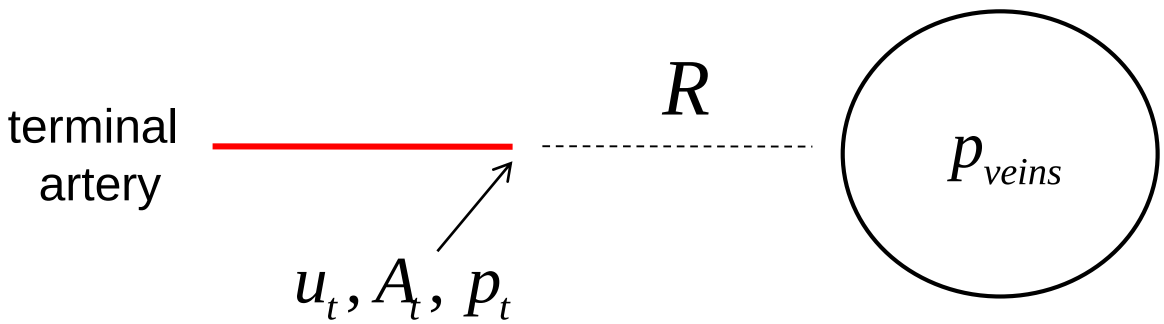
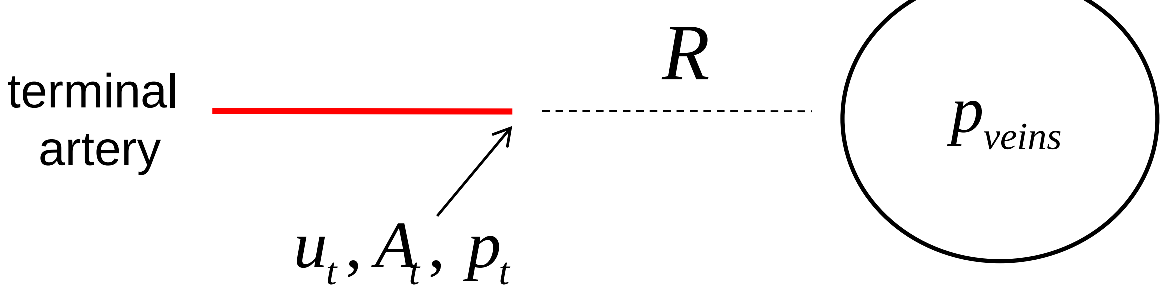
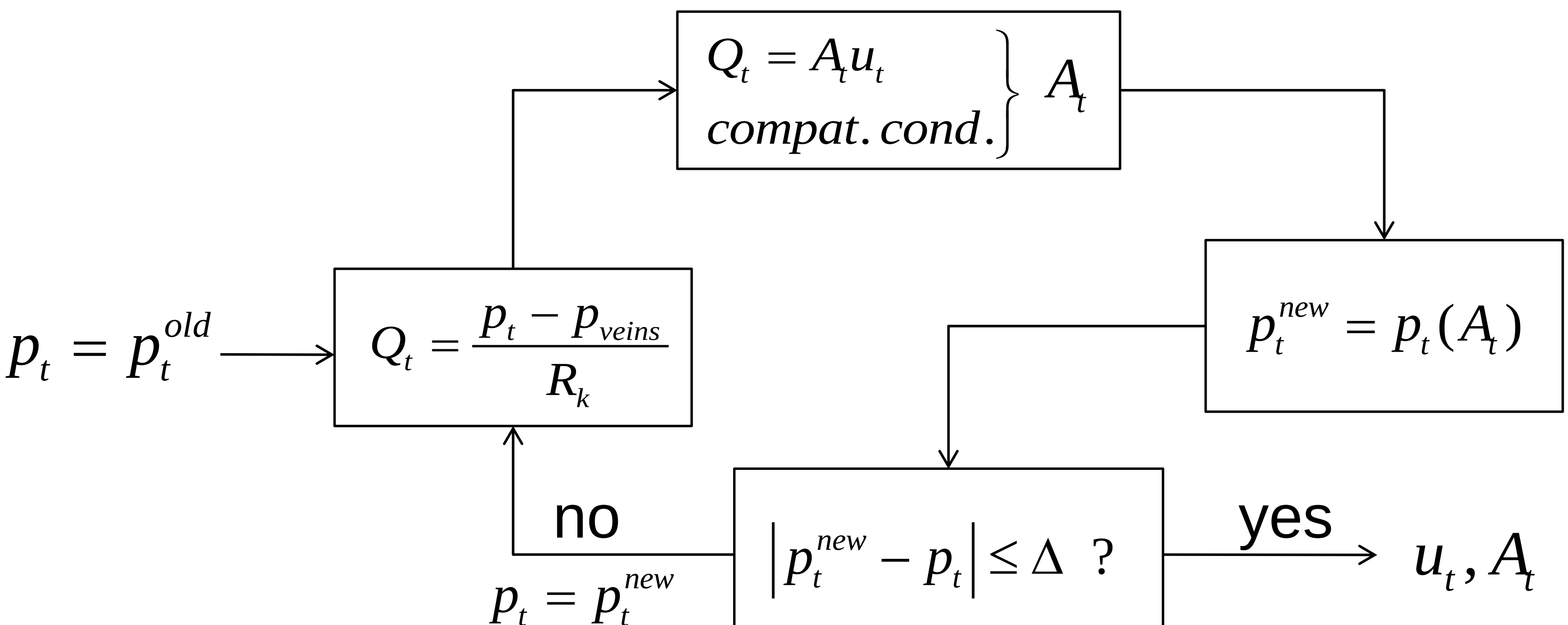


$$
\begin{cases}
p_t - p_{veins} = RQ_t, \\
p_t = p_t(A_t), \\
compat.\,cond. \rightarrow u_t = \alpha A_t + \beta.
\end{cases}
$$

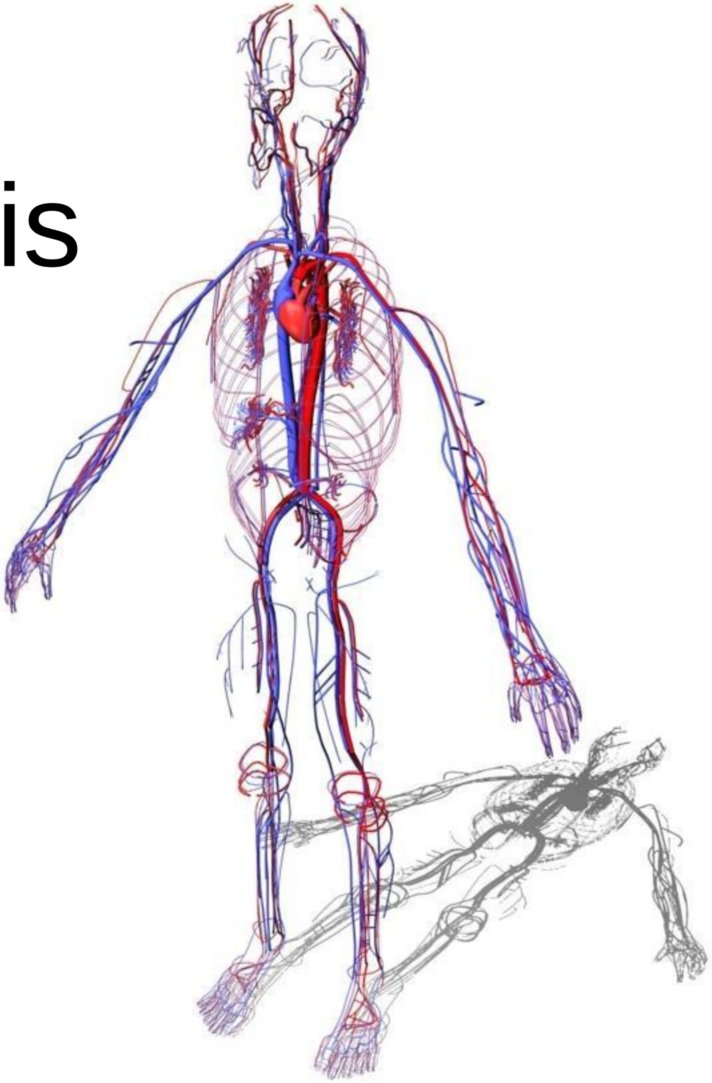
Bifurcations. Terminal vessel



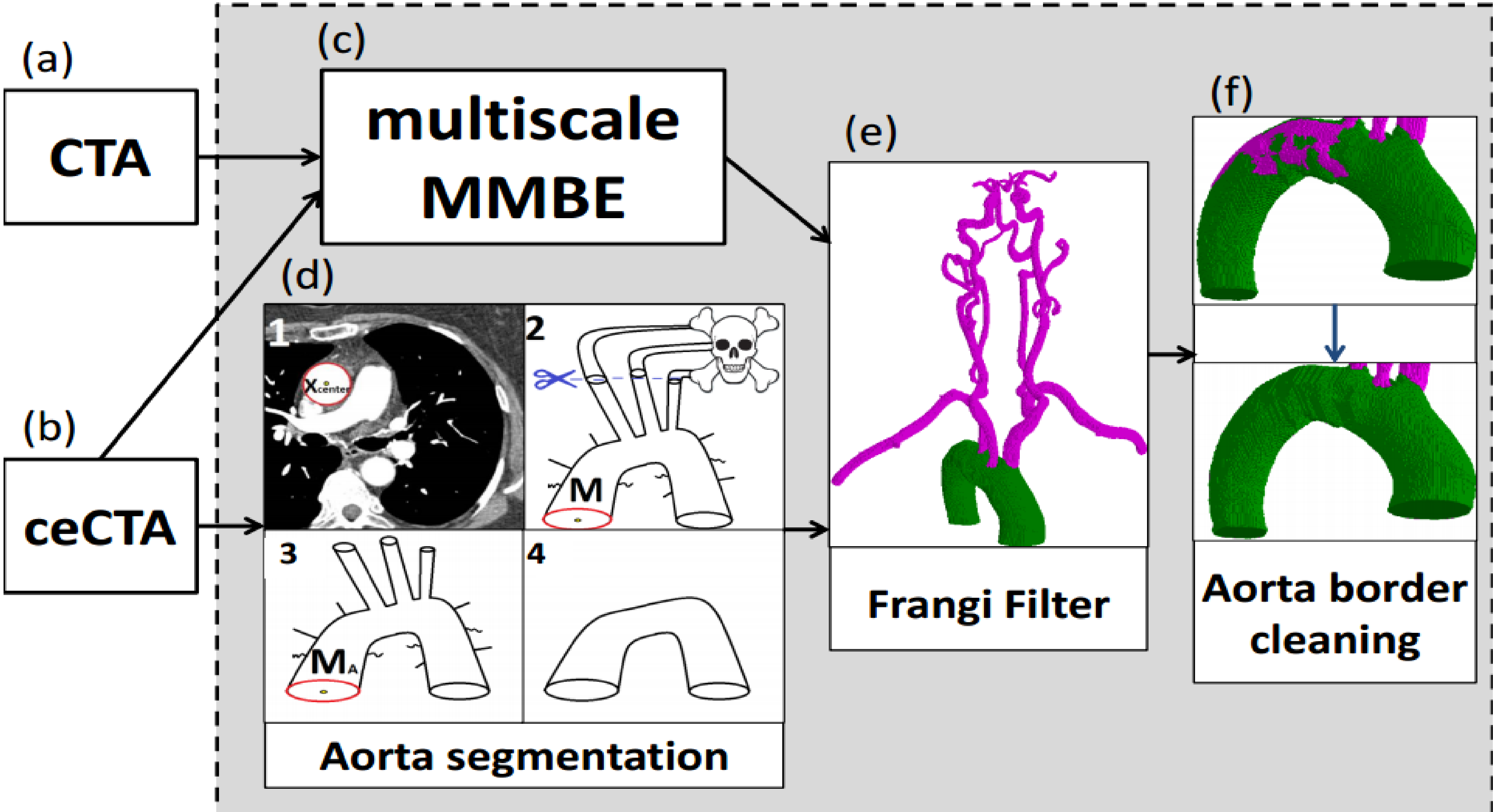
$$\begin{cases} p_t - p_{veins} = RQ_t, \\ p_t = p_t(A_t), \\ compat.\,cond. \to u_t = \alpha A_t + \beta. \end{cases}$$



Carotid artery stenosis



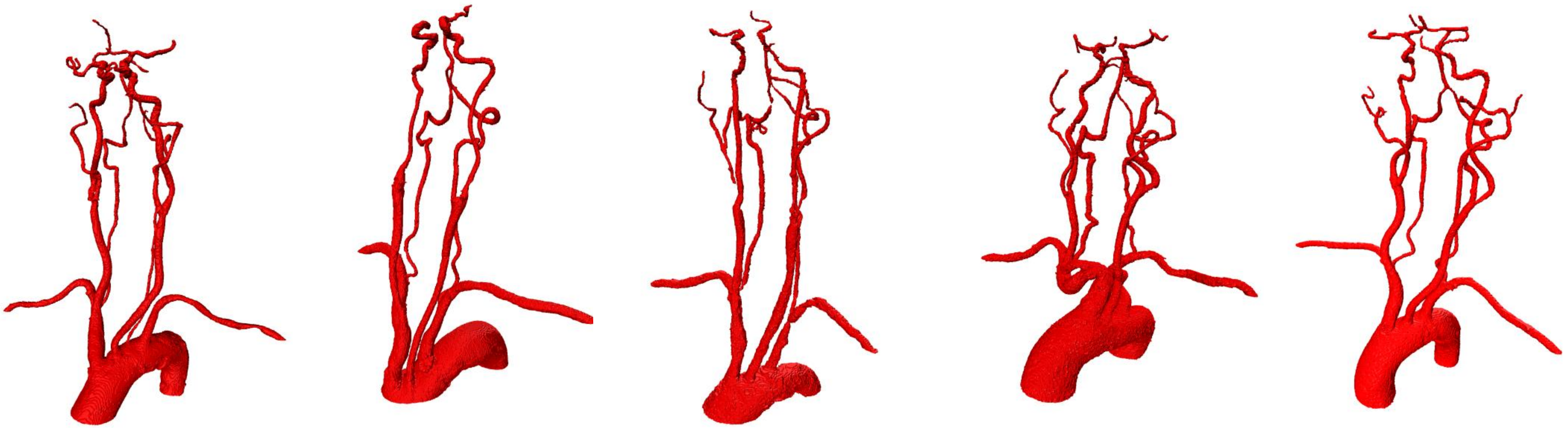
Geometry extraction

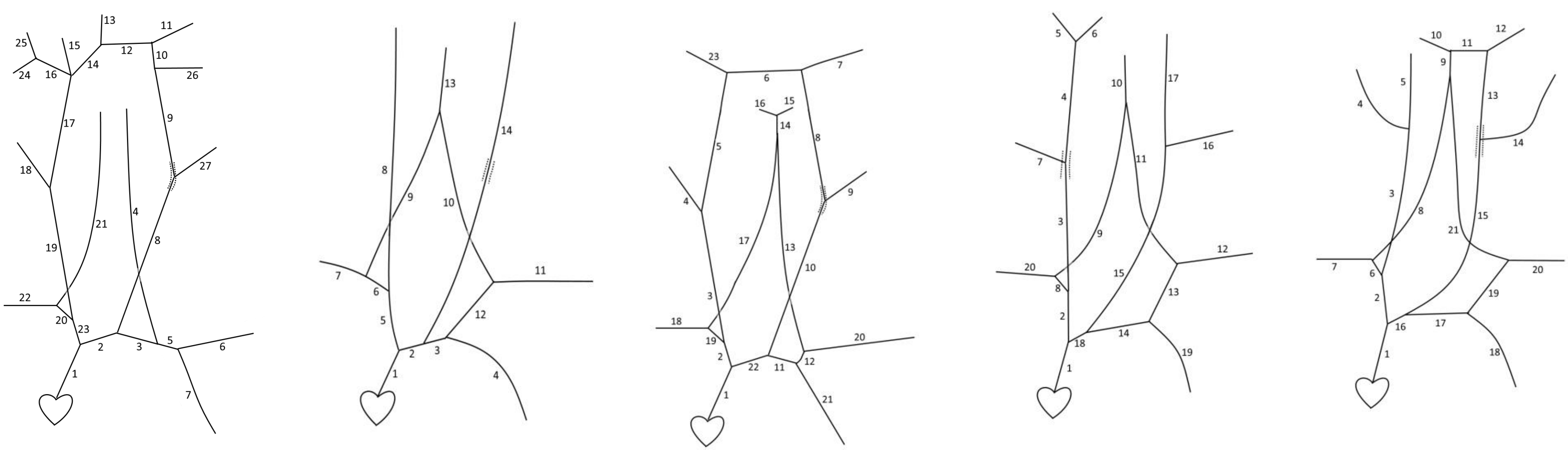


Wednesday (2 Nov),13:20
R.Pryamonosov (INM, Moscow). Patient-oriented graph network reconstruction of coronary arteries

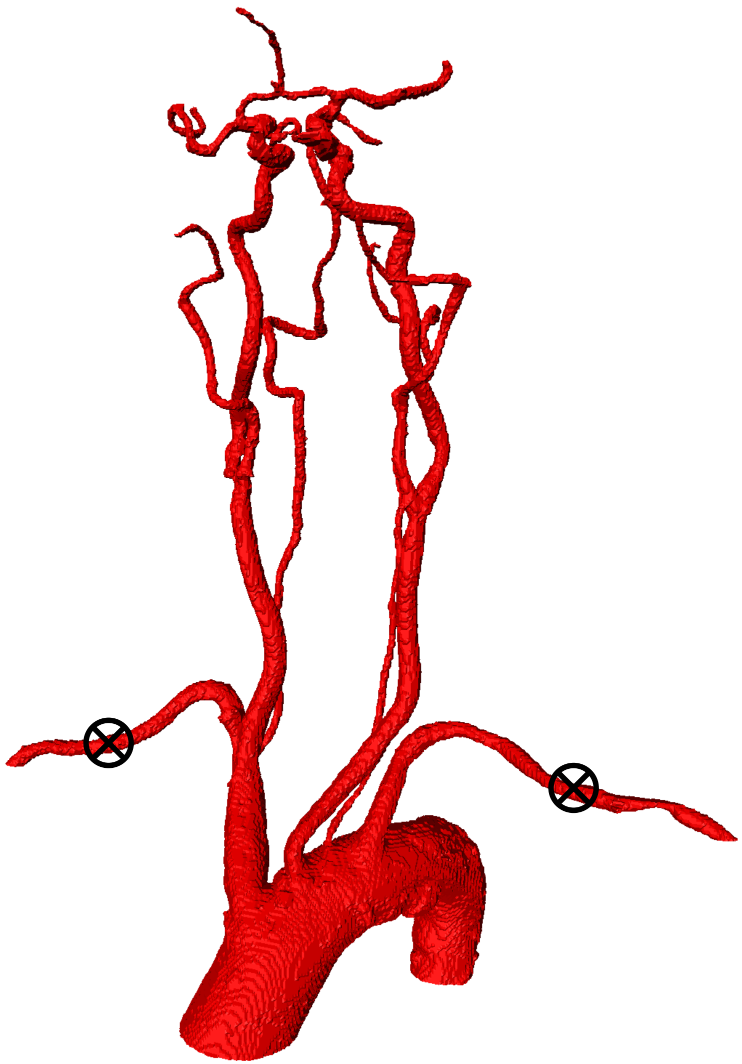
Vessels structure

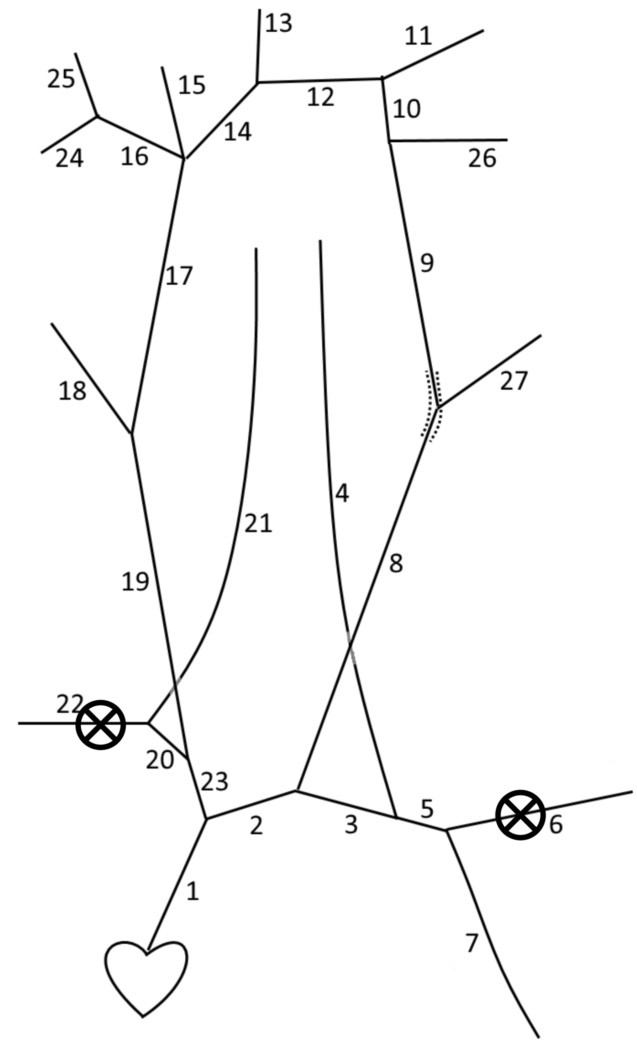
5 patients





Ultrasound velocity measurements





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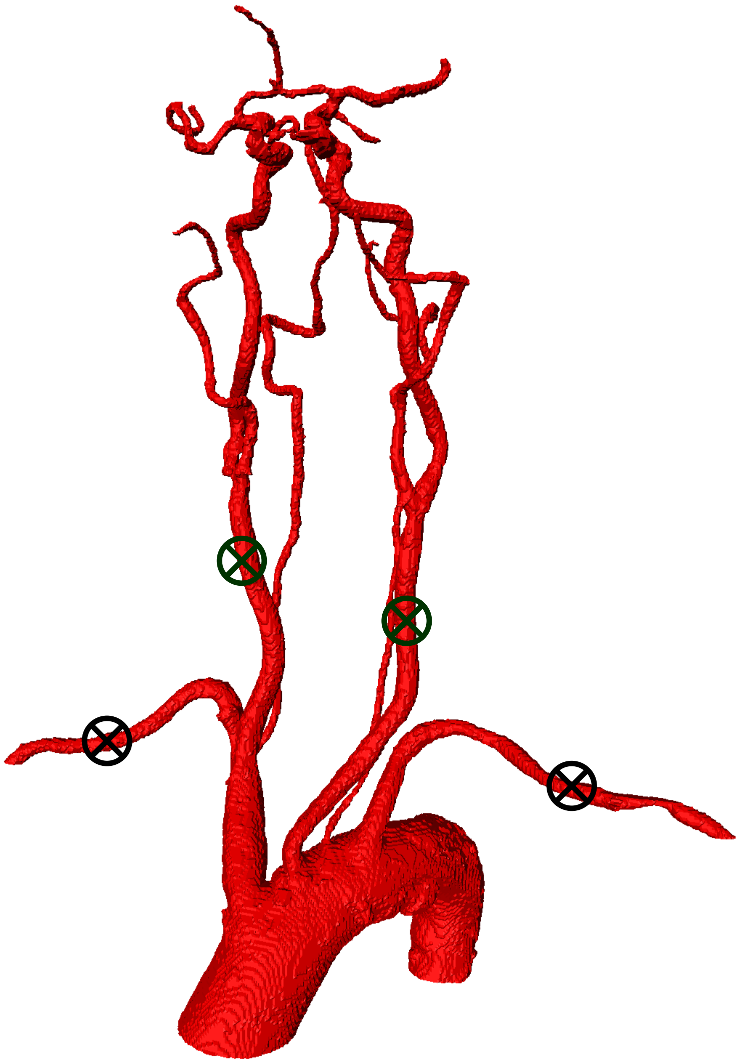

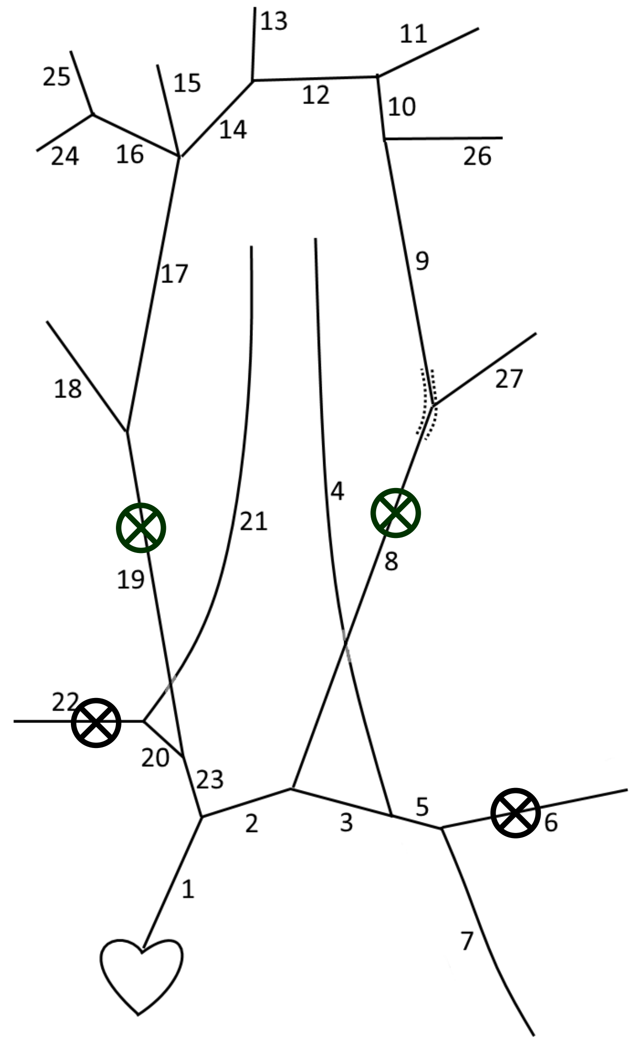

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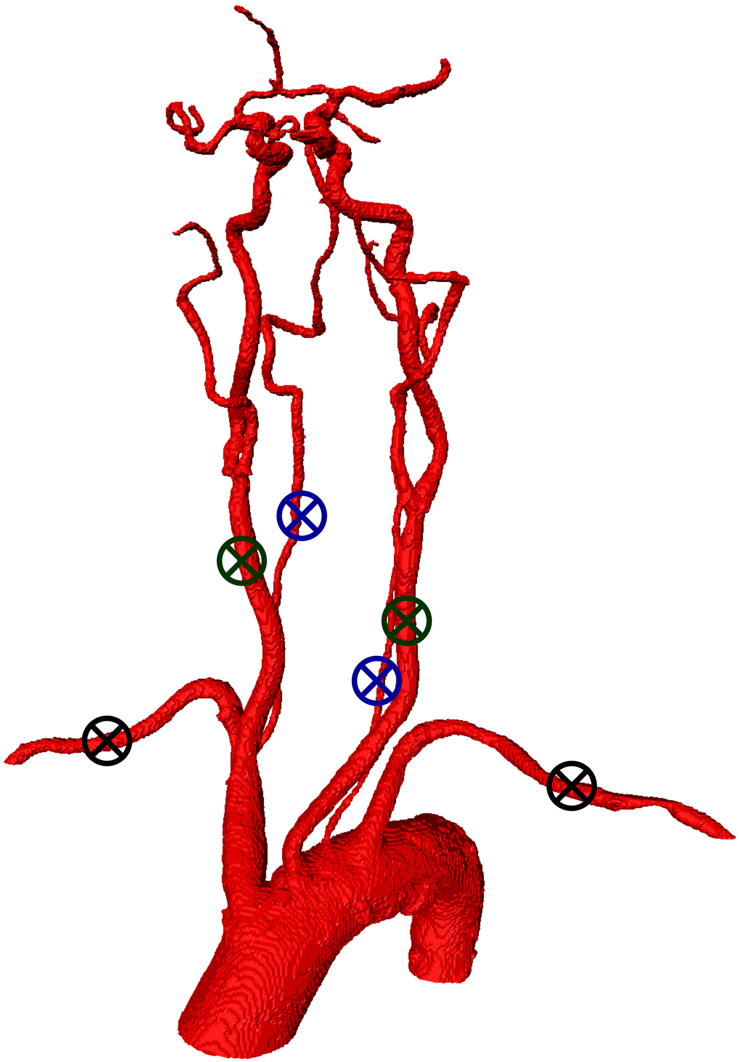

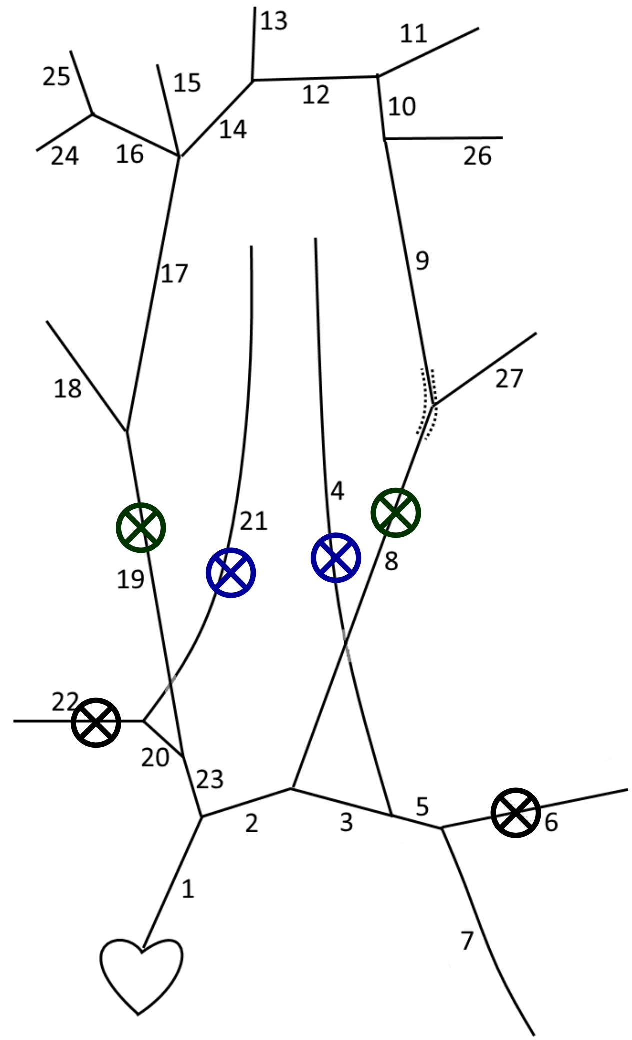

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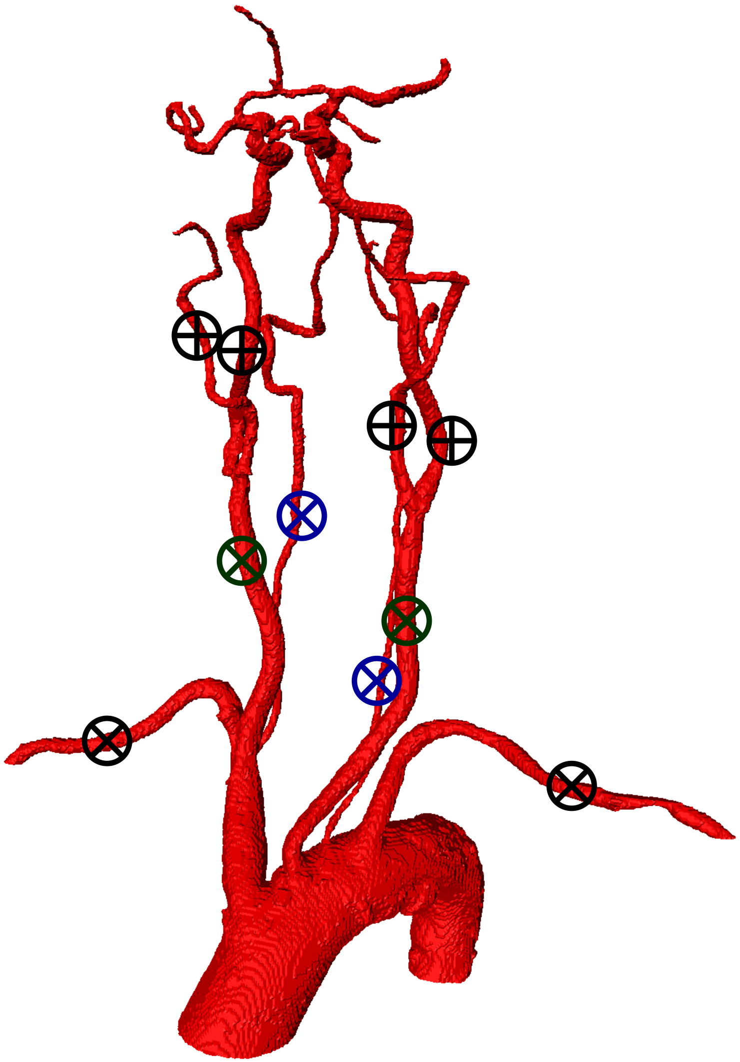
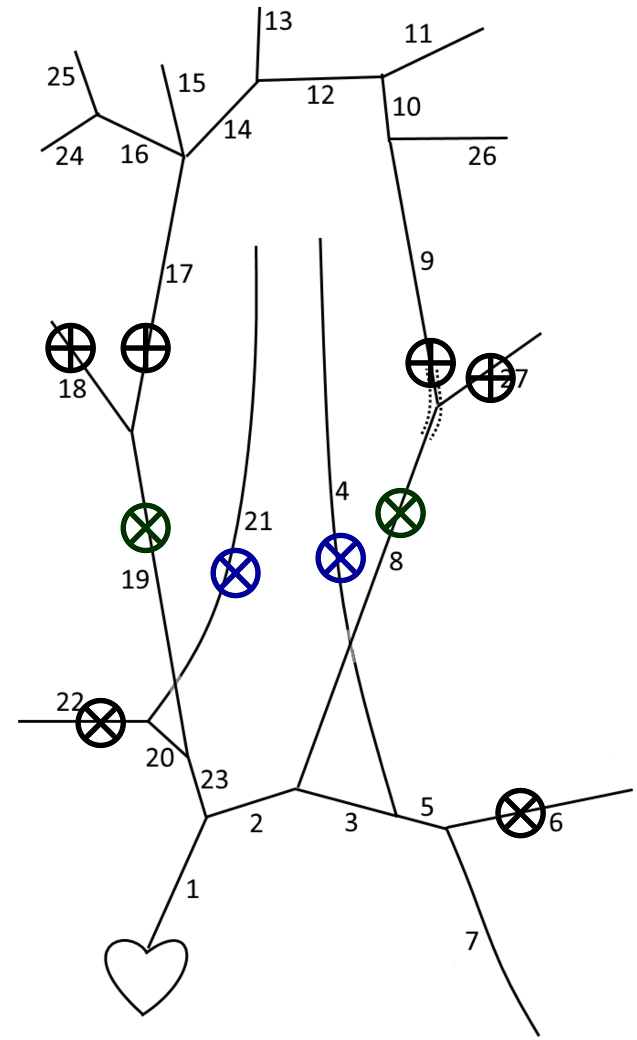
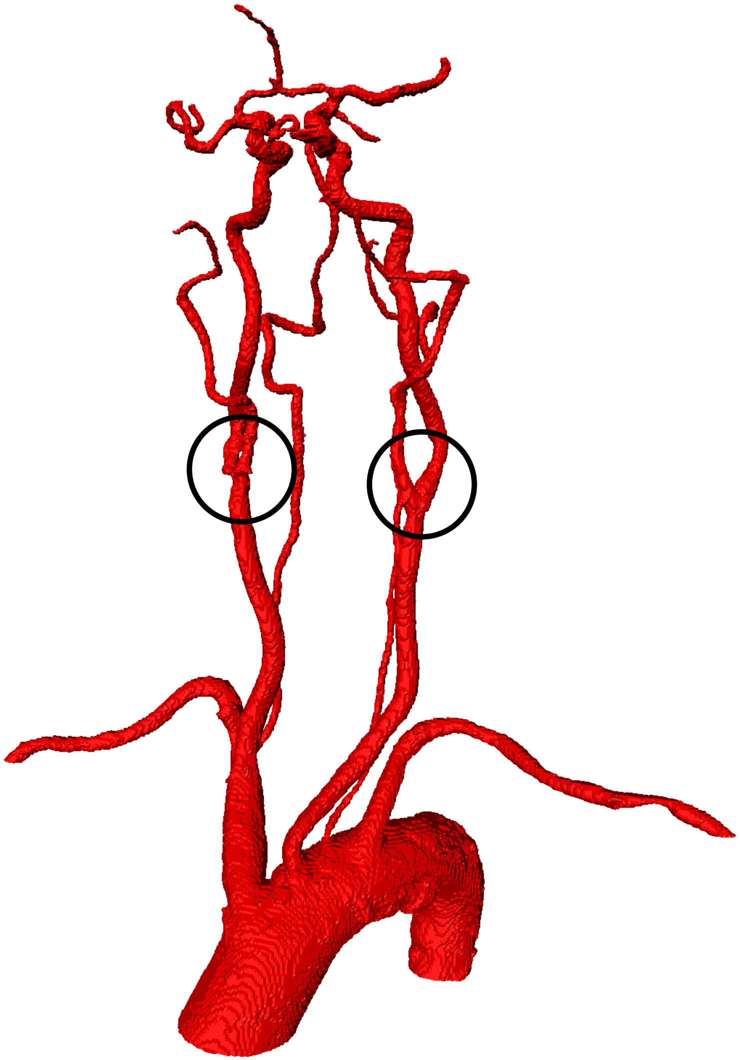
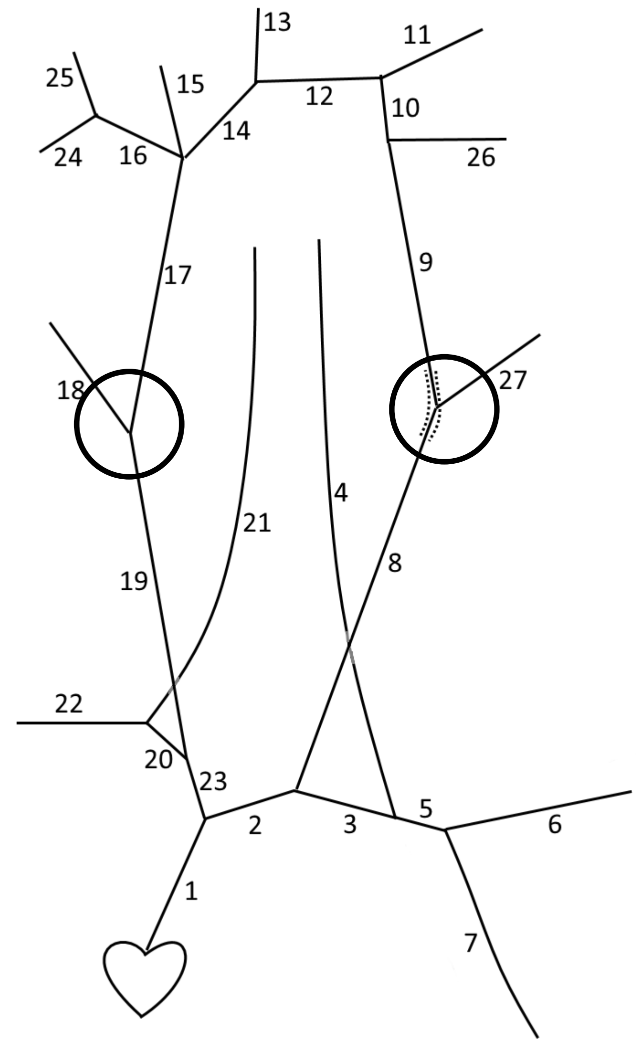




Stenosis





Results. Velocity measurements

CT scans

Geometry

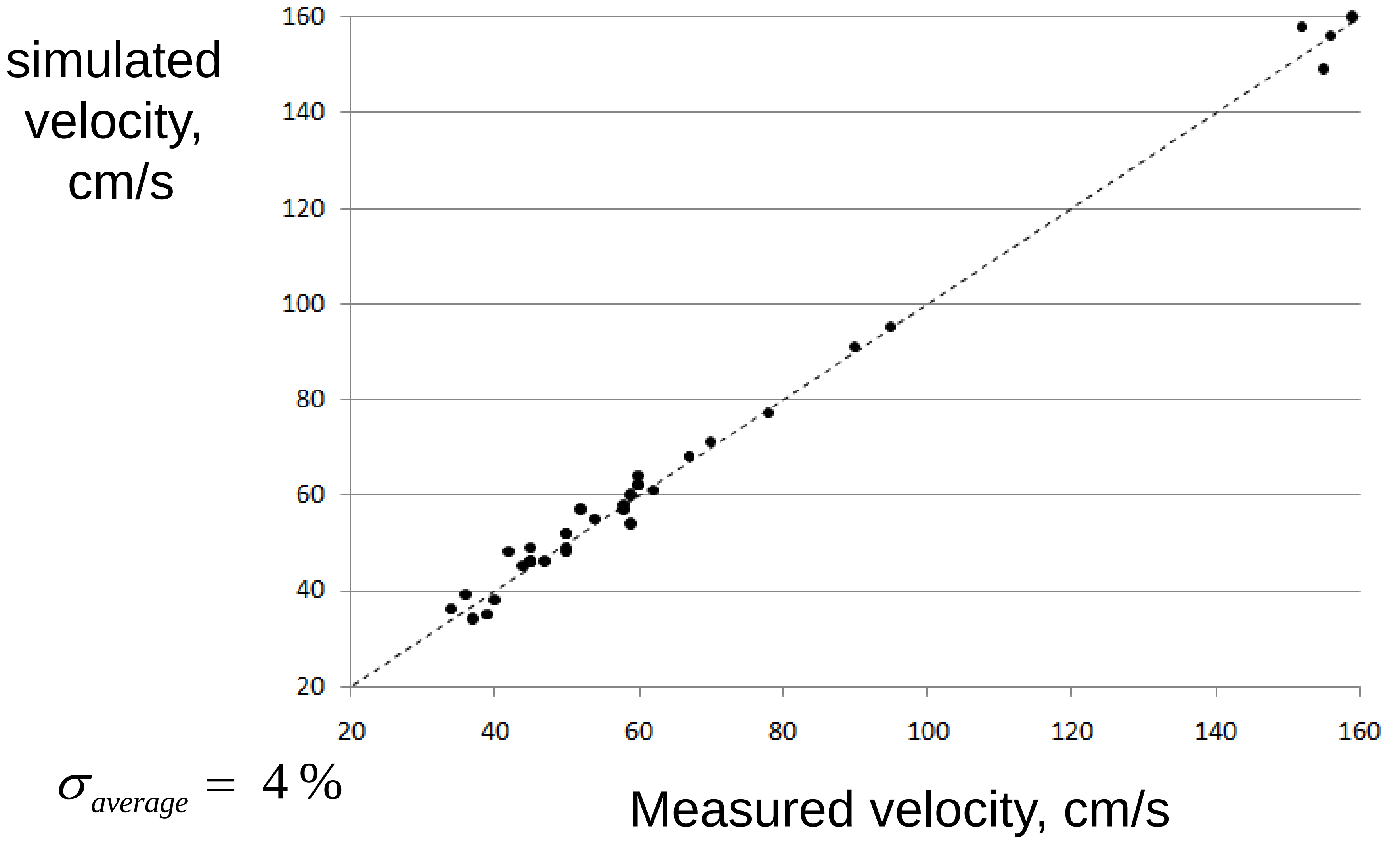
Velocities after treatment

Velocities before treatment

$$c_k, R_k, p_{veins}, Q_{heart}(t)$$

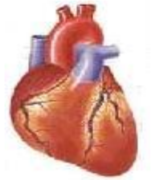
Patient data
Clinical studies

Velocity before treatment



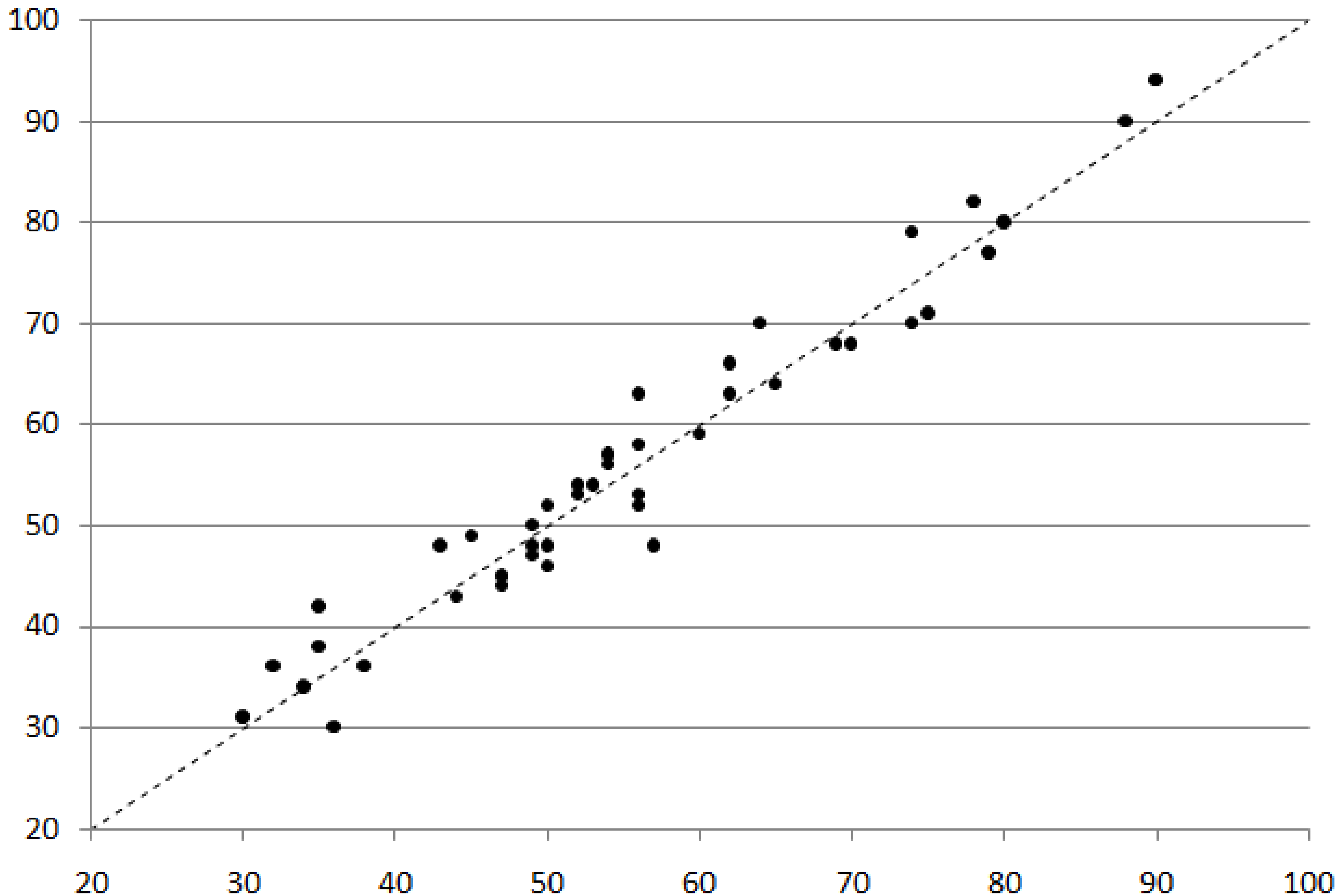
$$\sigma_{average} = 4\,\%$$

$$\sigma_{\max} = 16\,\%$$



Velocity after treatment

simulated velocity, cm/s

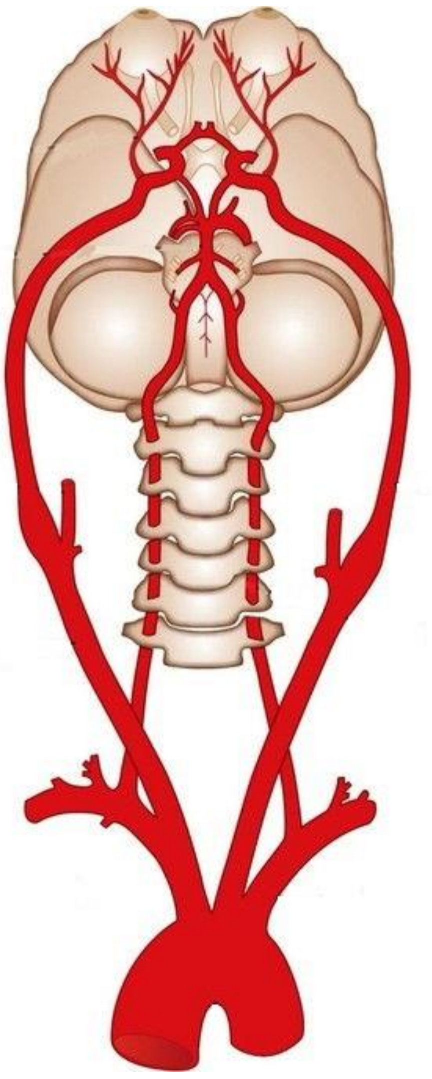


Measured velocity, cm/s

$$\sigma_{average} = 6\,\%$$

$$\sigma_{\max} = 20\,\%$$

Conclusion



Model:

- Individual structure
- blood flow pattern
- parameters
- 10 cases -> 5 cases