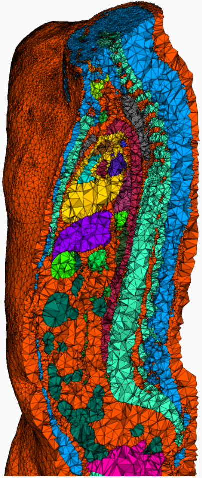
Numerical techniques for bioimpedance and ECG modelling

Alexander Danilov

Institute of Numerical Mathematics, Russian Academy of Sciences
Moscow Institute of Physics and Technology
Moscow, Russia

Bioimpedance modelling

Mathematical model



$$
\begin{aligned}
\mathrm{div}(\mathsf{C}\nabla U) = 0 \quad & \text{in} \quad \Omega \\
\mathsf{J}_n = \pm I_0 / S_\pm \quad & \text{on} \quad \Gamma_\pm \\
\mathsf{J}_n = 0 \quad & \text{on} \quad \partial\Omega \setminus \Gamma_\pm
\end{aligned}
$$

U – potential field
C – conductivity (admittivity) tensor
$\mathsf{E} = \nabla U$ – current intensity field
$\mathsf{J} = \mathsf{C}\,\mathsf{E}$ – current density field
I_0 – current injection
$S_\pm$ – electrode contact surfaces

ECG modelling

Bidomain problem

Domain Ω with boundary $\partial\Omega$

$$
\begin{aligned}
&\chi\left(C_m\frac{\partial v}{\partial t}+I_{\mathrm{ion}}(\mathbf{u},v)\right)-\nabla\cdot(\sigma_i\nabla(v+\phi_e))=I_i && \text{in } \Omega\\
&\nabla\cdot((\sigma_i+\sigma_e)\nabla\phi_e+\sigma_i\nabla v)=-I_{\mathrm{total}} && \text{in } \Omega\\
&\frac{\partial\mathbf{u}}{\partial t}=\mathbf{f}(\mathbf{u},v)
\end{aligned}
$$

ϕ_e – extracellular electrical potential
v – transmembrane voltage
C_m – membrane capacitance per unit area
χ – cell membrane surface to volume ratio
σ_i & σ_e – intra- & extracellular conductivity tensors
I_i – intracellular stimulus current
I_{total} = $I_i + I_e$ – total stimulus current
$\mathbf{u}$ – state variables
I_{ion} & $\mathbf{f}$ – cellular model

Bidomain problem

Boundary conditions

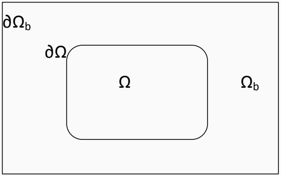
$$\begin{aligned} \mathbf{n} \cdot (\sigma_i \nabla (v + \phi_e)) &= 0 \quad \text{on } \partial\Omega \\ \mathbf{n} \cdot (\sigma_e \nabla \phi_e) &= 0 \quad \text{on } \partial\Omega \end{aligned}$$

"Bidomain with bath" problem

$$\nabla \cdot (\sigma_b \nabla \phi_e) = 0 \text{ in } \Omega_b$$

$$\begin{aligned} \mathbf{n} \cdot \sigma_e \nabla \phi_e = \mathbf{n} \cdot \sigma_b \nabla \phi_e \quad & \text{on } \partial\Omega \\ \mathbf{n} \cdot \sigma_b \nabla \phi_e = I_E^{(\mathrm{surf})} \quad & \text{on } \partial\Omega_b \setminus \partial\Omega \end{aligned}$$

$I_E^{(\mathrm{surf})}$ – external stimulus current



Monodomain problem

Assuming $\sigma_e = K\sigma_i$

$$\begin{aligned}
&\chi\left(C_m\frac{\partial v}{\partial t} + I_{\text{ion}}(\mathbf{u}, v)\right) - \nabla\cdot(\sigma\nabla v) = I \quad && \text{in } \Omega\\
&\frac{\partial \mathbf{u}}{\partial t} = \mathbf{f}(\mathbf{u}, v)\\
&\mathbf{n}\cdot(\sigma\nabla v) = 0 && \text{on } \partial\Omega
\end{aligned}$$

$\sigma = \dfrac{K}{1+K}\sigma_i$

I – stimulus current

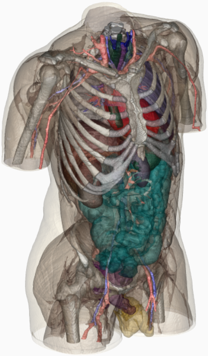
Segmentation and mesh generation

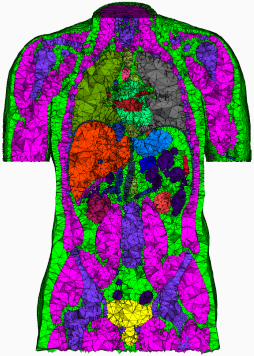
Technology overview

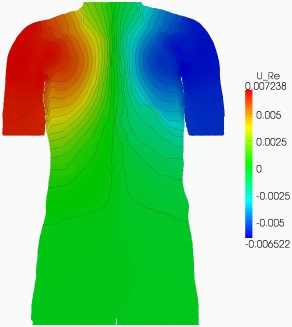
Segmentation

Meshing

FEM







ITK-SNAP

CGAL Mesh

Ani3D

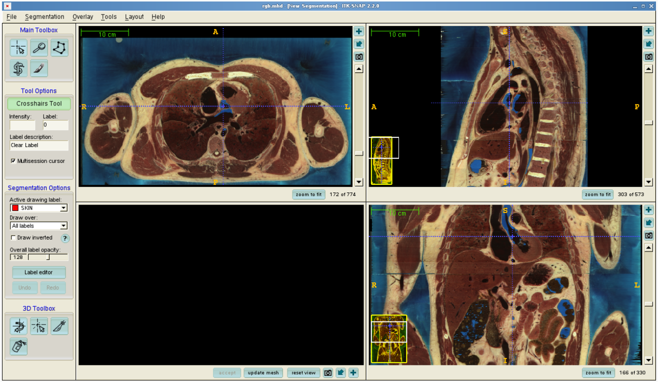
ParaView

A. A. Danilov, D. V. Nikolaev, S. G. Rudnev, V. Yu. Salamatova and Yu. V. Vassilevski, Modelling of bioimpedance measurements: unstructured mesh application to real human anatomy. *Russ. J. Numer. Anal. Math. Modelling*, 2012 **27** (5), 431–440

ITK-SNAP software

ITK-SNAP (`www.itksnap.org`)

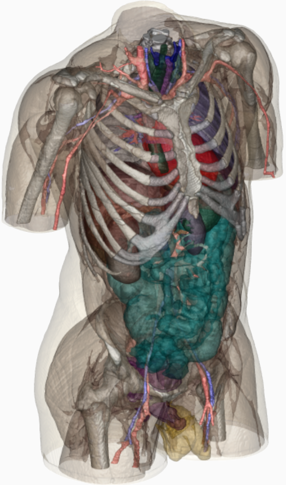
Free software for Visualization and Segmentation



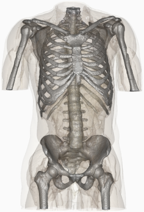
Visible Human Project, U.S. National Library of Medicine

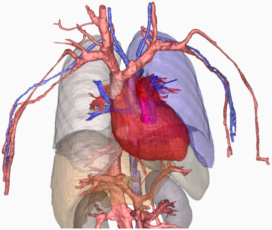
`www.nlm.nih.gov/research/visible`

High resolution segmented model of VHP torso



$567 \times 305 \times 843$ voxels
$1 \times 1 \times 1$ mm
26 organs and tissues

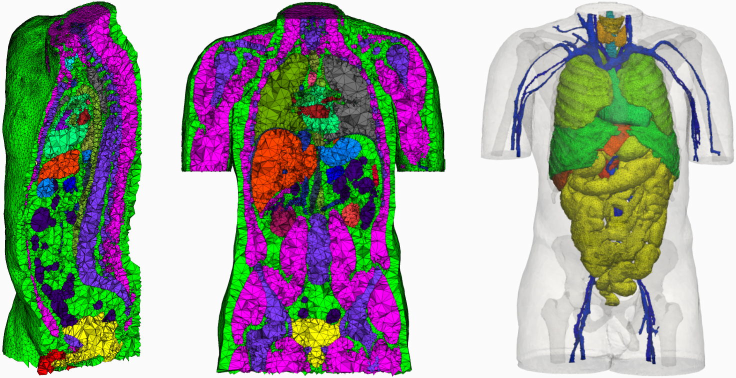




Total 146m voxels, 68m material voxels

Unstructured tetrahedral meshes

CGAL Mesh (`www.cgal.org`) – Delaunay mesh generation
Ani3D (`sf.net/p/ani3d`) – mesh cosmetics

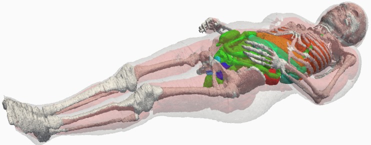


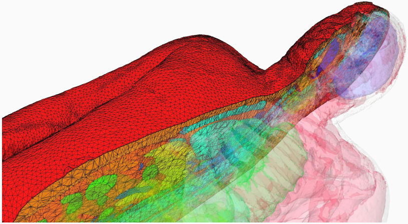
413 508 vertices, 2 315 329 tetraedra, 84 430 boundary faces

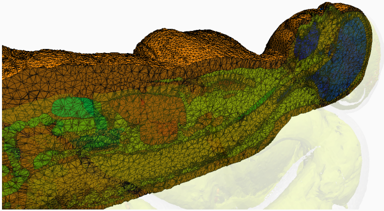
Full body male and female models

VHP-Man

VHP-Woman

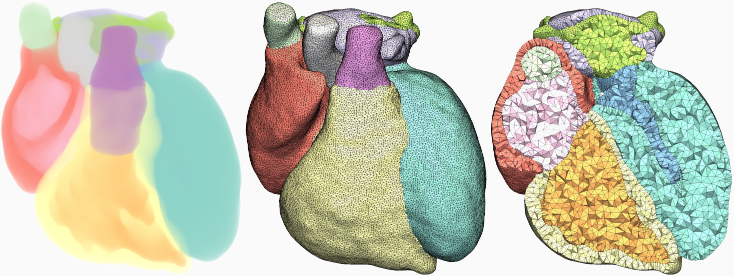






3m tetrahedra   effective resolution: $1 \times 1 \times 1$ mm   30 tissues

Heart models

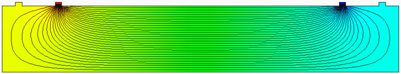


3D model of heart, atria and ventricles
Visible Human Project data

Solution postprocessing: Bioimpedance

Bioimpedance: sensitivity field

current lines for current-carrying electrodes – $\mathbf{J}_{\mathrm{cc}}$



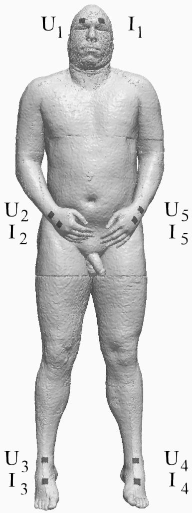
current lines of reciprocal lead field for pick-up electrodes – $\mathbf{J}_{\mathrm{reci}}$

sensitivity function

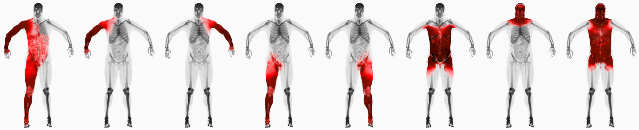
$S = \mathbf{J}_{\mathrm{reci}} \cdot \mathbf{J}_{\mathrm{cc}}$

$Z_t = \int_V S(x,y,z)\rho(x,y,z)dv$

Ten-electrode configuration



- Conventional scheme (I_2-I_3, U_2-U_3)
- Hands (I_2-I_1, U_2-U_3) and (I_5-I_1, U_5-U_4)
- Legs (I_3-I_2, U_3-U_4) and (I_4-I_5, U_4-U_3)
- Torso (I_5-I_3, U_2-U_4) and (I_5-I_4, U_2-U_3)
- Head (I_1-I_2, U_1-U_5)
- Head+Torso (I_1-I_3, U_1-U_4)



A. A. Danilov, V. K. Kramarenko, D. V. Nikolaev, S. G. Rudnev, V. Yu. Salamatova, A. V. Smirnov and Yu. V. Vassilevski, Sensitivity field distributions for segmental bioelectrical impedance analysis based on real human anatomy. *J. Phys.: Conf. Ser.* (2013) **434**, 012001, doi: 10.1088/1742-6596/434/1/012001.

Volume impedance density

Sensitivity field

S

Volume impedance density

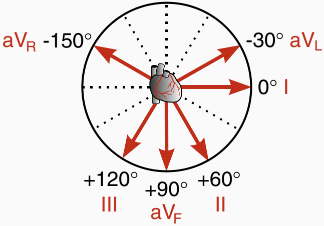
$S \cdot \rho$

Solution postprocessing: ECG signals

Vector model

$\mathbf{q}_{\mathrm{heart}}$ – electrical cardiac vector

$$\mathbf{q}_{\mathrm{heart}} = \int_{\Omega} \sigma \nabla v \, \mathrm{d}V$$



$\mathbf{p}$ – lead projection vector

s – lead signal

$$s = \mathbf{q}_{\mathrm{heart}} \cdot \mathbf{p}$$

Kotikanyadanam M., Göktepe S., Kuhl E. *Computational modeling of electrocardiograms: A finite element approach toward cardiac excitation* // Int. J. Numer. Meth. Biomed. Engng., 2010, 26: 524–533

Equations

Ω_0 – human body around heart

Γ_{ext} – external body surface

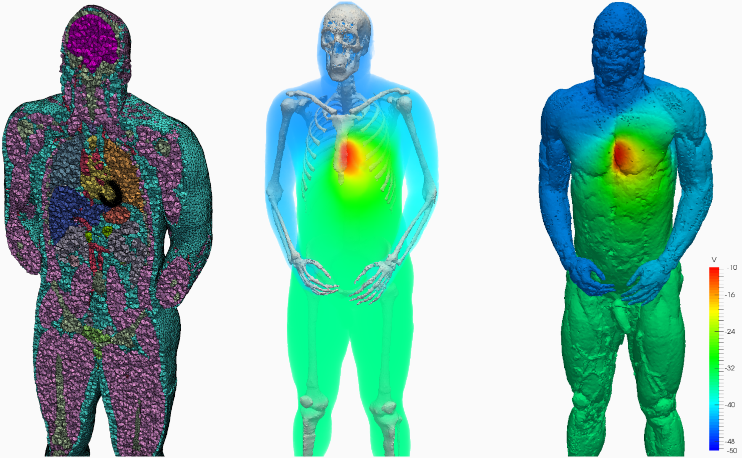
Γ_H – heart-body interface

$$
\begin{aligned}
\nabla \cdot (\sigma_0 \nabla \phi_0) = 0 \quad & \text{in } \Omega_0 \\
\mathbf{n} \cdot \sigma_0 \nabla \phi_0 = 0 \quad & \text{on } \Gamma_{\mathrm{ext}} \\
\phi_0 = \phi_e \quad & \text{on } \Gamma_H
\end{aligned}
$$

ϕ_0 – electrical potential

σ_0 – conductivity tensor (heterogeneous)

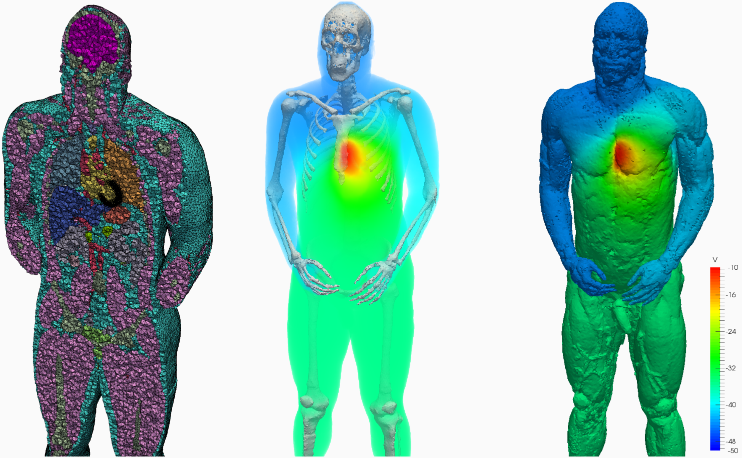
Full human body



VHP model, mesh generated using CGAL Mesh and Ani3D, AniFEM solution, boundary conditions computed using Chaste

Lead signal $s = \phi_h \cdot \mathbf{p}_h$, ϕ_h – cardiac potential, $\mathbf{p}_h$ – precomputed

Full human body



VHP model, mesh generated using CGAL Mesh and Ani3D, AniFEM solution, boundary conditions computed using Chaste

Lead signal $s = \phi_\mathsf{h} \cdot \mathsf{p}_\mathsf{h}$, ϕ_h – cardiac potential, p_h – precomputed

Conclusions

Work status

Work in progress:

1. ECG: benchmarks
2. ECG: real anatomy

Future plans:

1. ECG: sensitivity analysis
2. Bioimpedance: modelling UI

Conclusions

1. Developed numerical methods for bioimpedance and ECG modelling
2. Proposed efficient tehnique for bidomain ECG signals calculation
3. Preliminary results of ECG modelling are presented

- VHP – `www.nlm.nih.gov/research/visible`
- ITK-SNAP – `www.itksnap.org`
- CGAL Mesh – `www.cgal.org`
- Ani3D – `sf.net/p/ani3d`
- Chaste – `www.cs.ox.ac.uk/chaste`

Thank you!

Bidomain numerical scheme

Convergence analysis by A. Chernyshenko

P_1 FEM on tetrahedral meshes (Ani3D)

Table 1: Bidomain

#d.o.f.	L^2-norm	rate
2801	1.097e-1	
20417	3.834e-2	1.58
155905	1.210e-2	1.70

Table 2: Bidomain with bath

#d.o.f.	L^2-norm	rate
8279	1.755e-1	
59912	6.124e-2	1.56
462811	1.933e-2	1.71

Benchmark solutions

P.Pathmanathan, R.A.Gray, *Verification of computational models of cardiac electro-physiology* // IJNMBE 2014 30:525–544

BIA Numerical scheme

Bioelectrical conductivity

Typical conductivity parameters @ 50kHz (S/m)

Blood	0.7	+	$0.02 \cdot j$
Muscles	0.36	+	$0.035 \cdot j$
Fat	0.0435	+	$0.001 \cdot j$
Bones	0.021	+	$0.001 \cdot j$
Skin	0.03	+	$0.06 \cdot j$
Heart	0.19	+	$0.045 \cdot j$
Lungs	0.27	+	$0.025 \cdot j$

Gabriel S., Lau R.W., Gabriel C. The dielectric properties of biological tissue: III. Parametric models for the dielectric spectrum of tissues. // Phys.Med.Biol. 1996. V.41(11). P.2271-2293.

Numerical scheme

$$\mathrm{div}(\mathsf{C}\nabla U) = 0$$

$$\mathsf{C} = \mathsf{C}_R + j \cdot \mathsf{C}_I, \quad U = U_R + j \cdot U_I,$$

$$\begin{cases} \mathrm{div}(\mathsf{C}_R \nabla U_R) - \mathrm{div}(\mathsf{C}_I \nabla U_I) = 0 \\ \mathrm{div}(\mathsf{C}_R \nabla U_I) + \mathrm{div}(\mathsf{C}_I \nabla U_R) = 0 \end{cases}$$

$$\begin{pmatrix} A_R & -A_I \\ A_I & A_R \end{pmatrix} \begin{pmatrix} x_R \\ x_I \end{pmatrix} = \begin{pmatrix} b_R \\ b_I \end{pmatrix}$$

P1 FEM (AniFEM, Ani3D package, `sf.net/projects/ani3d`)

Numerical scheme

$$\mathrm{div}(\mathsf{C}\nabla U) = 0$$

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P1 FEM (AniFEM, Ani3D package, `sf.net/projects/ani3d`)

Convergence analysis

Series of hierarchically refined meshes

N_V	N_T	Memory, Mb	N_{it}	Time, s	L_2-norm
2032	9359	7.16	13	0.02	1.24E-03
14221	74872	37.3	23	0.18	9.31E-04
106509	598976	299.1	58	3.70	5.07E-04
824777	4791808	2437.5	127	68.55	1.53E-04
6492497	38334464	20015.3	353	2634.15	–

Asymptotically second order convergence

Fast ECG signals calculation

Numerical scheme

Linear system is generated using FEM

$$\mathsf{A}\mathsf{x} = \mathsf{b}$$

- x – solution vector in Ω_0 (grid points, length $= n$)
- A – symmetric positive definite matrix $n \times n$
- b – right hand side, length $= n$, $\mathsf{b} = \mathsf{B}\mathsf{g}_d$
- g_d – vector of Dirichlet boundary values, length $= m$
- B – RHS operator in FEM model, matrix $n \times m$

$$\mathsf{g}_d = \mathsf{G}\phi_e^h$$

- ϕ_e^h – ϕ_e solution vector in Ω (grid points, length $= N$)
- G – interpolation operator, matrix $m \times N$

$$\mathsf{A}\mathsf{x} = \mathsf{B}\mathsf{G}\phi_e^h$$

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Numerical scheme

Linear system is generated using FEM

$$\mathsf{Ax} = \mathsf{b}$$

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- G – interpolation operator, matrix $m \times N$

$$\mathsf{Ax} = \mathsf{BG}\phi_e^h$$

Numerical scheme

Lead signals $\mathbf{s}$ are computed using ϕ_0 values in points $c_1, \dots, c_k$.

$$\mathbf{s} = \mathbf{S}\mathbf{c}_s$$

$\mathbf{s}$ – lead signal vector

$\mathbf{c}_s$ – ϕ_0 vector, length $= k$.

$\mathbf{S}$ – computational matrix

Vector $\mathbf{c}_s$ is interpolated from vector x

$$\mathbf{c}_s = \mathbf{C}\mathbf{x}_s$$

$\mathbf{x}_s$ – subvector of $\mathbf{x}$, length $= K$, $K \leq 4k$

$\mathbf{C}$ – interpolation operator, matrix $k \times K$

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$$\mathbf{s} = \mathbf{S}\mathbf{C}\mathbf{x}_s$$

Numerical scheme

Effective computation of partial solution x_s

$$\mathsf{x} = \mathsf{A}^{-1}\mathsf{b}, \quad \mathsf{x}_s = \mathsf{M}_s\mathsf{b}$$

M_s – K rows from matrix A^{-1}, size $K \times n$

Row i of matrix A^{-1} is constructed from linear system solution

$$\mathsf{A}^\top \mathsf{m}_i = \mathsf{e}_i$$

e_i – basis vector (all zeros, but one at i-th position)

Matrix M_s is computed using K solutions of initial linear system, since $\mathsf{A} = \mathsf{A}^\top$

$$\mathsf{s} = \mathsf{SCM}_s\mathsf{BG}\phi_e^h = \mathsf{Z}\phi_e^h$$

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$$\mathsf{s} = \mathsf{SCM}_s\mathsf{BG}\phi_e^h = \mathsf{Z}\phi_e^h$$

Numerical scheme

Effective computation of partial solution x_s

$$\mathsf{x} = \mathsf{A}^{-1}\mathsf{b}, \quad \mathsf{x}_s = \mathsf{M}_s\mathsf{b}$$

M_s – K rows from matrix A^{-1}, size $K \times n$

Row i of matrix A^{-1} is constructed from linear system solution

$$\mathsf{A}^\top \mathsf{m}_i = \mathsf{e}_i$$

e_i – basis vector (all zeros, but one at i-th position)

Matrix M_s is computed using K solutions of initial linear system, since $\mathsf{A} = \mathsf{A}^\top$

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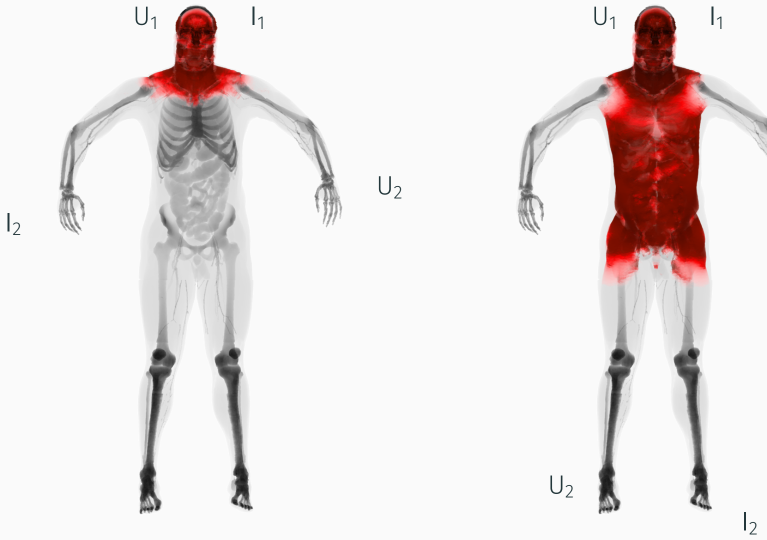
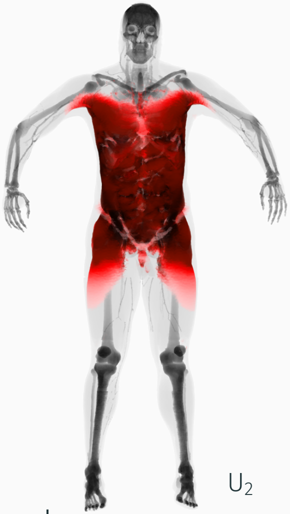
Sensitivity fields

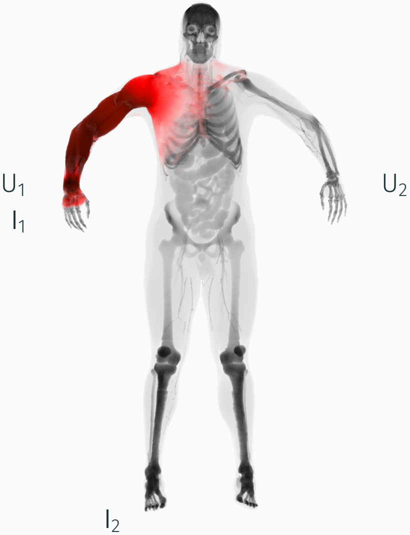
Ten-electrode configuration

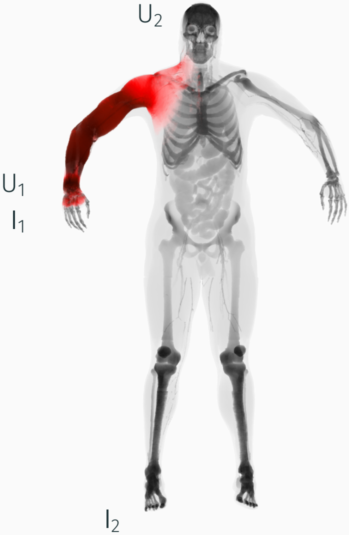


Ten-electrode configuration

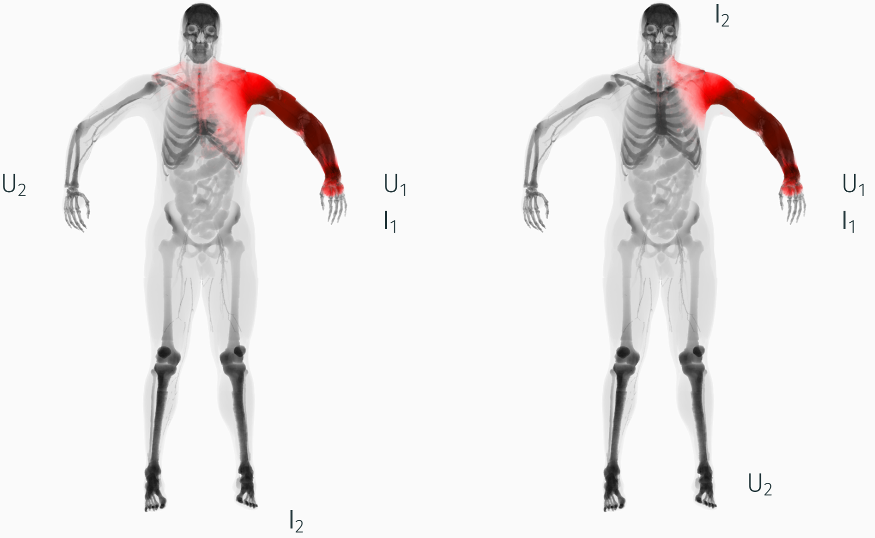


Ten-electrode configuration





Ten-electrode configuration



Ten-electrode configuration