Modelling lymph flow in conduit system of the lymph node

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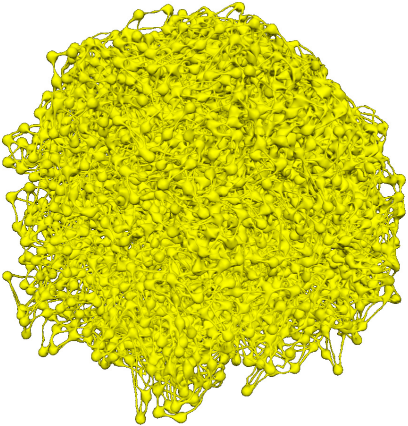
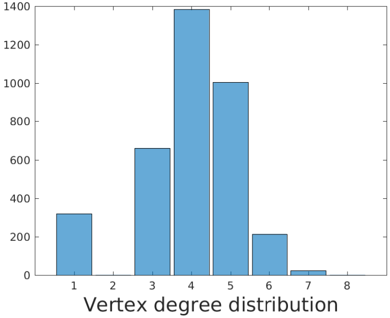
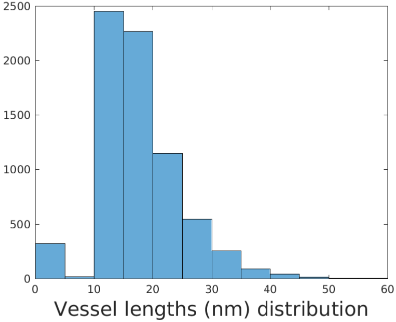
Conduit system



Conduit system graph statistics





Mathematical model

- denote set of vertices and edges $\mathbf{G} = (\mathbf{V}, \mathbf{E})$.
- for each $e_{ij} \in \mathbf{E}$ apply Poiseuille equation
- for each vertex $v_i \in V$ apply the mass conservation law

$$Q_{ij} = \frac{1}{R_{ij}} (P_i - P_j) \qquad \sum_{k_i : ik_i \in \mathbf{E}} Q_{ik_i} = 0 \tag{1}$$

Variables

$Q_{ij}[(\mu m)^3/s]$ – lymph flow $\qquad$ P_i[Pa] – hydravlic pressure

Parameters

$R_{ij} = \frac{8\mu l_{ij}}{\pi r_{ij}^4}$ – hydravlic resistance, $\mu = 0.0015$ Pa·s – lymph dynamic viscosity, $r_{ij} = 1\mu$m – radii, l_{ij} – channel lengths.

Boundary conditions

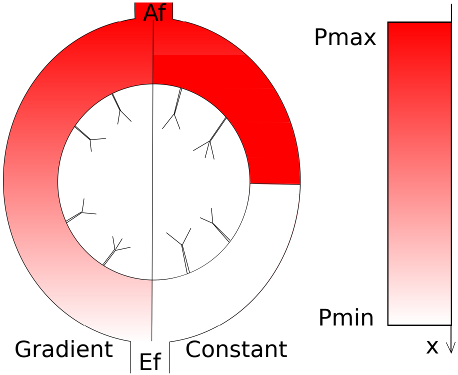


Figure : SCS pressure distribution for two types of pressure boundary conditions ((1) gradient and (2) constant).

Results: pressure distribution

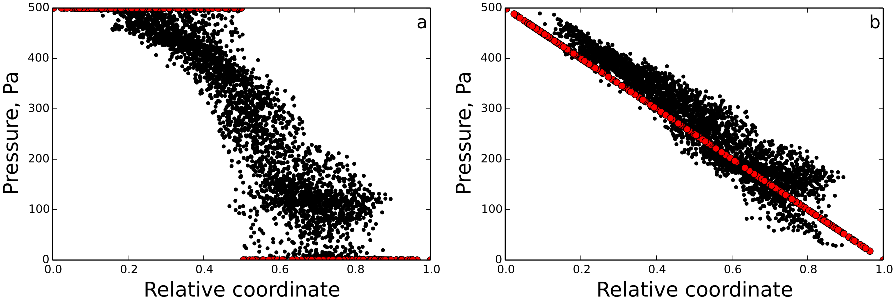


Figure : Pressure values depending on coordinate (**a**) for constant boundary conditions, (**b**) for gradient boundary conditions. Boundary points are highlighted in red.

Lymph flow in disrupted FRC network

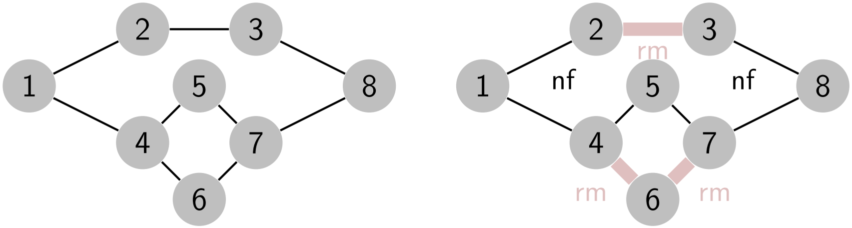


Figure : Scheme of edges deletion from graph

Table : System degradation for constant boundary conditions

Damage (%)	Nodes	Edges	n-f edges	Inputs	Outputs	relative out-flow	relative sum. flow
0	3609	7160	0	197	123	1.0	1.0
10	3573	6444	25	174	113	0.83	0.85
20	3495	5728	89	146	102	0.66	0.66
30	3375	5012	185	128	95	0.41	0.32
40	3062	4296	2082	103	92	0.22	0.13
50	2663	3580	2355	64	92	0.03	0.01
60	2156	2864	2383	14	86	0.01	0.005
70	1615	2148	2060	9	76	0.0	0.0

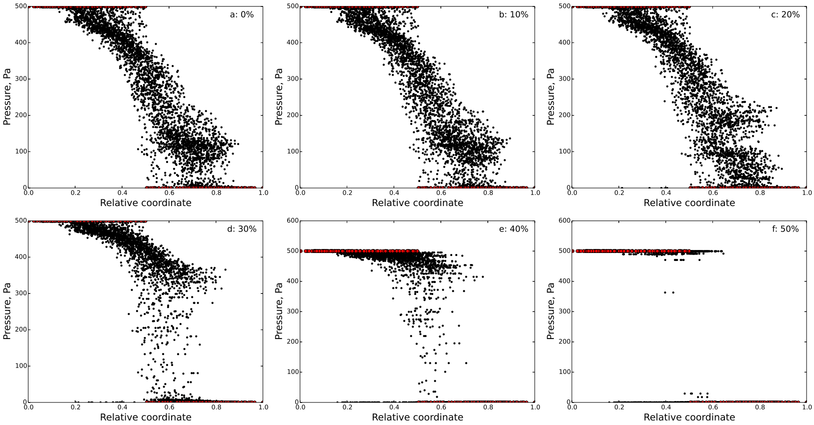
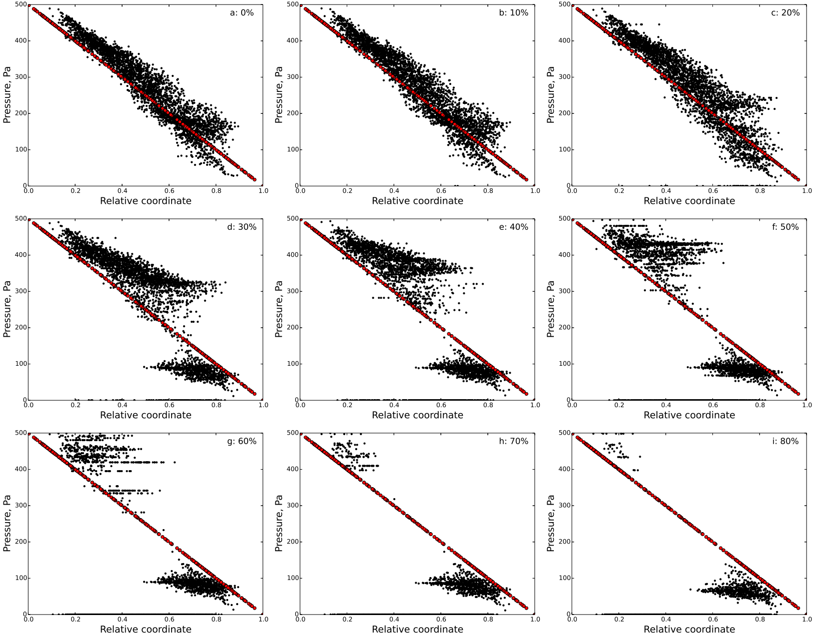


Figure : Pressure gradient for (**a**)–(**f**) 0% – 50% edges removed (constant boundary conditions). Boundary points are highlighted in red.

Table : System degradation for gradient boundary conditions

Damage (%)	Nodes	Edges	n-f edges	Inputs	Outputs	relative out-flow	relative sum. flow
0	3609	7160	0	156	164	1.0	1.0
10	3573	6444	27	143	144	0.82	0.85
20	3495	5728	81	129	120	0.66	0.65
30	3375	5012	226	112	112	0.53	0.38
40	3062	4296	331	104	97	0.43	0.26
50	2663	3580	458	89	92	0.35	0.17
60	2156	2864	590	63	80	0.27	0.12
70	1615	2148	497	46	64	0.21	0.099
80	1063	1432	256	28	48	0.135	0.061
90	507	716	67	16	20	0.06	0.026


a: 0%
b: 10%
c: 20%
d: 30%
e: 40%
f: 50%
g: 60%
h: 70%
i: 80%
Pressure, Pa
Relative coordinate
0
100
200
300
400
500
0.0
0.2
0.4
0.6
0.8
1.0

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Thank you for your attention!