# Numerical modeling of transcranial ultrasound

Blood flow and vascular pathologies modeling workgroup (INM RAS)
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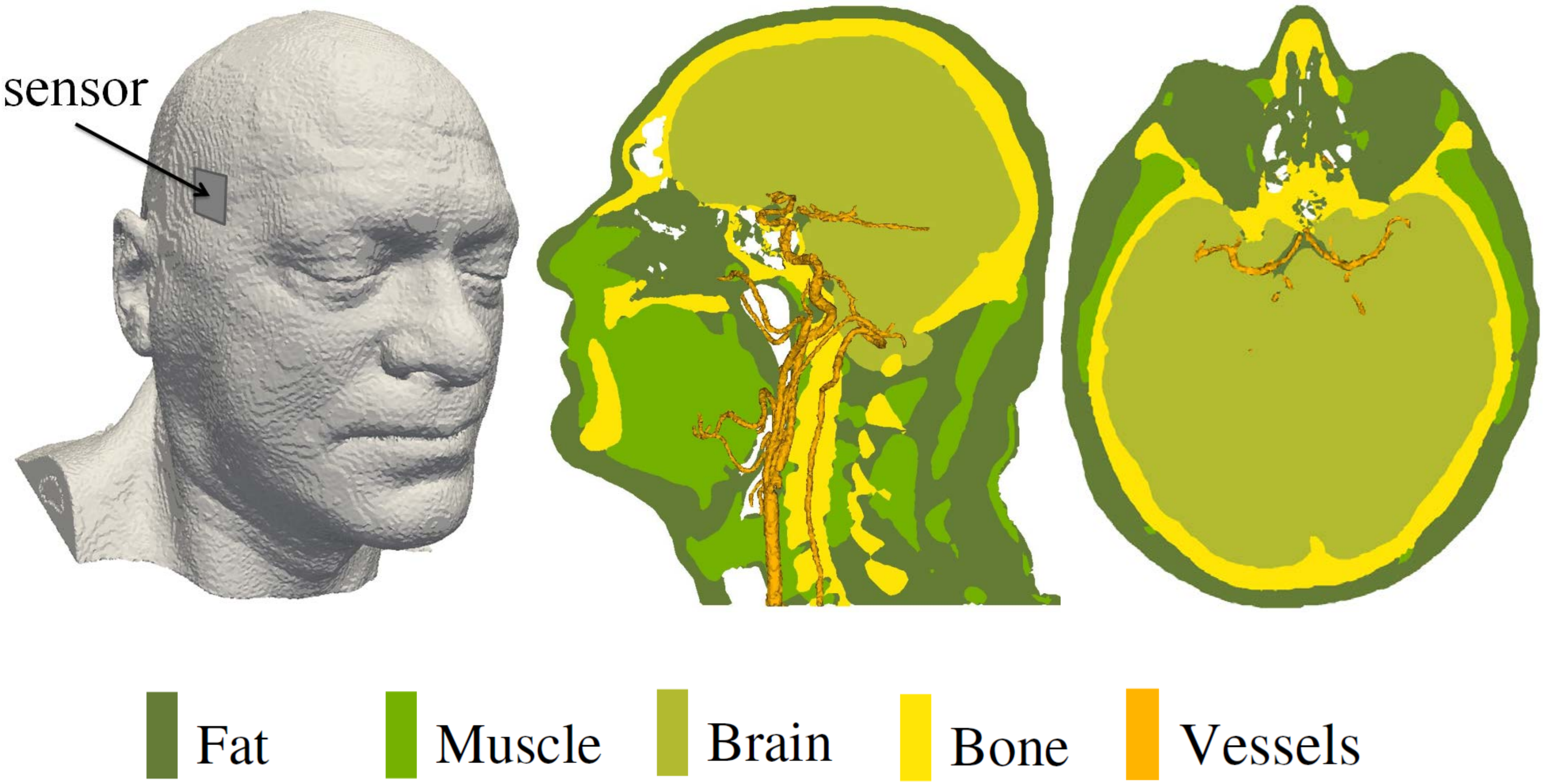
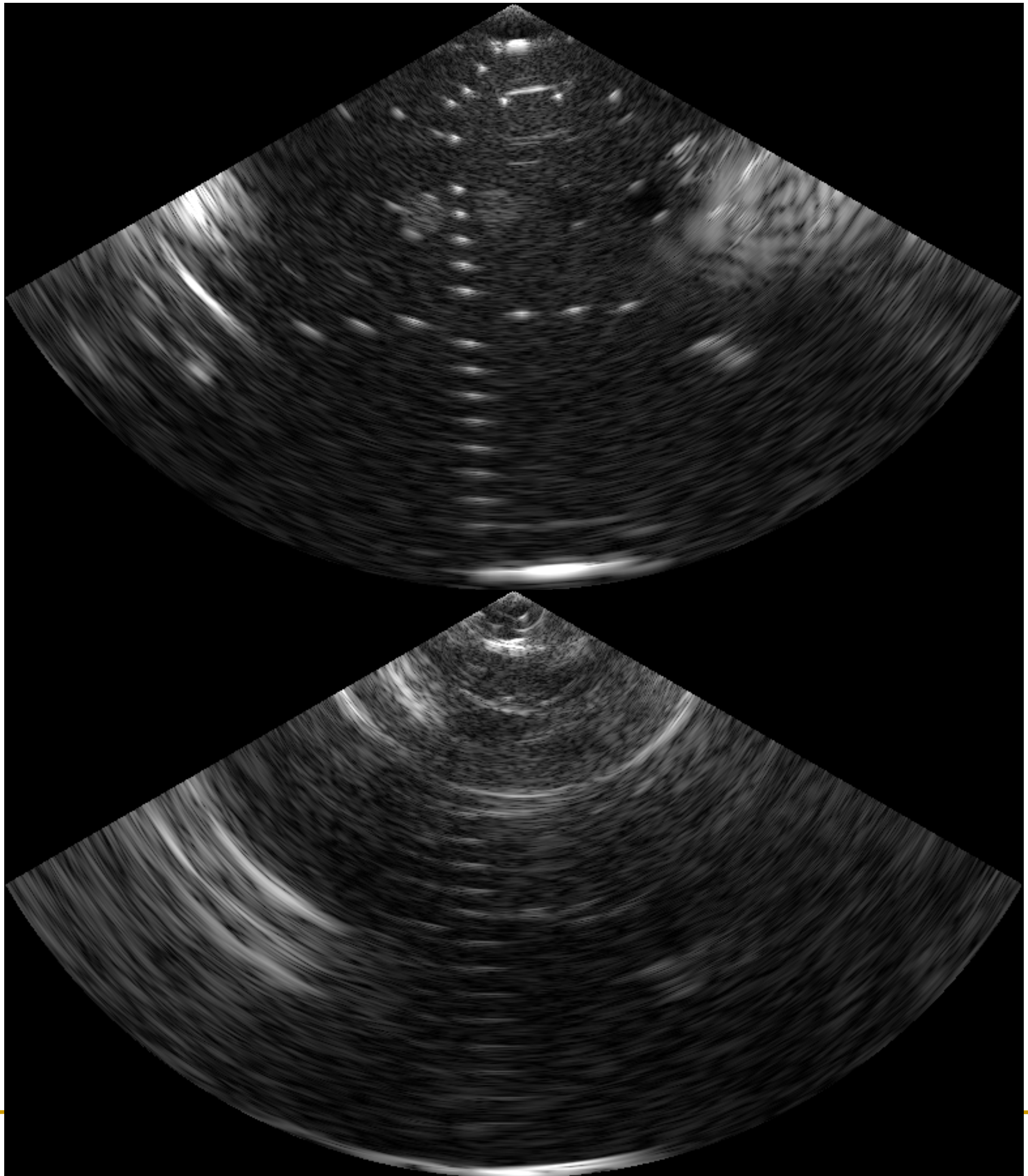
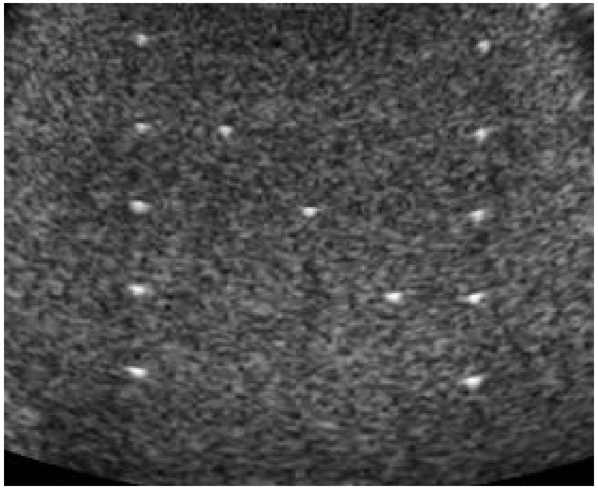
# Problem statement:
## transcranial ultrasound

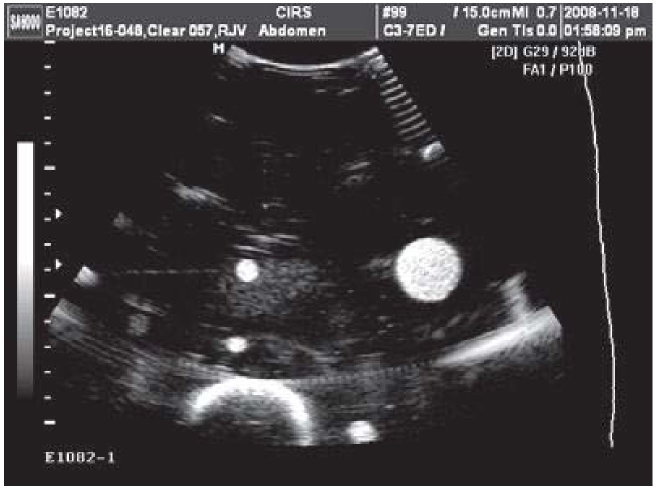


# Problem statement: aberrations



# Problem statement: phantoms



# Governing equations

$$\rho(\mathbf{x})\frac{\partial \mathbf{v}(\mathbf{x},t)}{\partial t} + \nabla p(\mathbf{x},t) = 0 \qquad \text{in } \Omega,$$

$$\frac{\partial p(\mathbf{x},t)}{\partial t} + \rho(\mathbf{x})c^2(\mathbf{x})\nabla\cdot\mathbf{v}(\mathbf{x},t) = -\alpha(\mathbf{x})c(\mathbf{x})p(\mathbf{x},t) \qquad \text{in } \Omega,$$

Acoustics equations system
Maxwell's viscosity model

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbb{A}_{x_1}\frac{\partial \mathbf{u}}{\partial x_1} + \mathbb{A}_{x_2}\frac{\partial \mathbf{u}}{\partial x_2} + \mathbb{A}_{x_3}\frac{\partial \mathbf{u}}{\partial x_3} = \mathbb{C}\mathbf{u}$$

$$\mathbf{u} = (v_1, v_2, v_3, p)^T$$

# Grid-characteristic method

3D: split by spatial coordinates

$$\frac{\partial \vec{u}}{\partial t} + A_x \frac{\partial \vec{u}}{\partial x} + A_y \frac{\partial \vec{u}}{\partial y} + A_z \frac{\partial \vec{u}}{\partial z} = \vec{f}$$

↓

$$\frac{\partial \vec{u}}{\partial t} + A_z \frac{\partial \vec{u}}{\partial z} = 0$$

$$\frac{\partial \vec{u}}{\partial t} + A_x \frac{\partial \vec{u}}{\partial x} = 0$$

$$\frac{\partial \vec{u}}{\partial t} + A_y \frac{\partial \vec{u}}{\partial y} = 0$$

$$\frac{\partial \vec{u}}{\partial t} = \vec{f}$$

$$\vec{u}_{n+1} = (F_z(A_z) + F_y(A_y) + F_x(A_x))\, \vec{u}_n$$

$$\vec{u}_{n+1} = F_z(A_z) F_y(A_y) F_x(A_x)\, \vec{u}_n$$

↓

$$\vec{u}_{n+1} = \frac{1}{6} \sum\nolimits_{i \neq j \neq k \neq i} F_i(A_i) F_j(A_j) F_k(A_k)\, \vec{u}_n$$

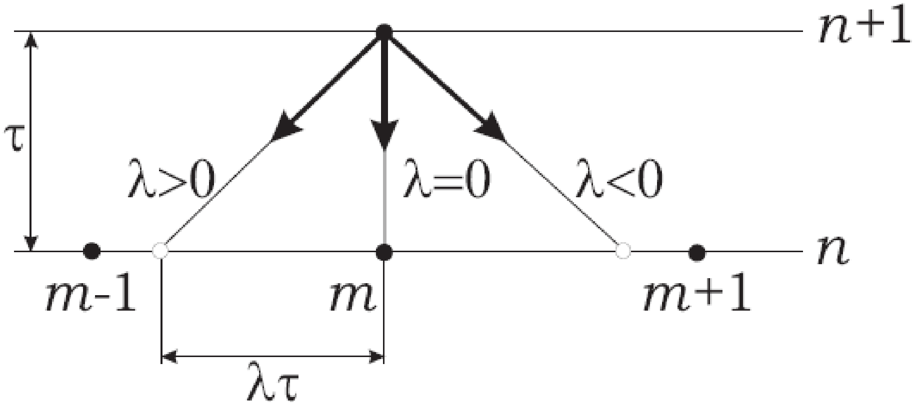
↓

$$\vec{u}_{n+1} = F_1(A_1) F_2(A_2) F_3(A_3) \vec{u}_n$$

# Grid-characteristic method

1D hyperbolic equations system (ξ, t)

$$\frac{\partial \vec{u}}{\partial t} + A_\xi \frac{\partial \vec{u}}{\partial \xi} = 0$$




$$A_\xi = \Omega^{-1}$$

$$\frac{\partial \vec{u}}{\partial t} + \mathbf{\Omega}^{-1}\mathbf{\Lambda}\mathbf{\Omega}\frac{\partial \vec{u}}{\partial \xi} = 0$$

$$\frac{\partial \vec{v}}{\partial t} + \mathbf{\Lambda}\frac{\partial \vec{v}}{\partial \xi} = 0 \quad (\vec{v} \equiv \mathbf{\Omega}\vec{u})$$

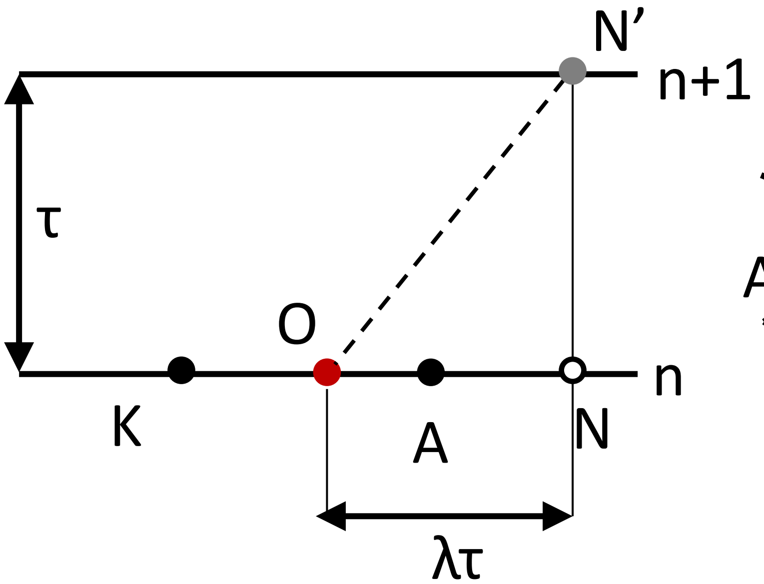
$$v^{n+1}(\xi) = v^n(\xi - \lambda\tau)$$

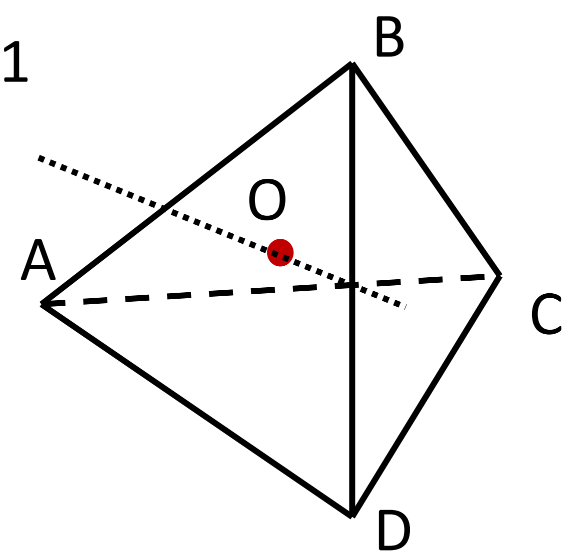
$$\vec{u}_{n+1} = F_\xi(A_\xi)\,\vec{u}_n$$

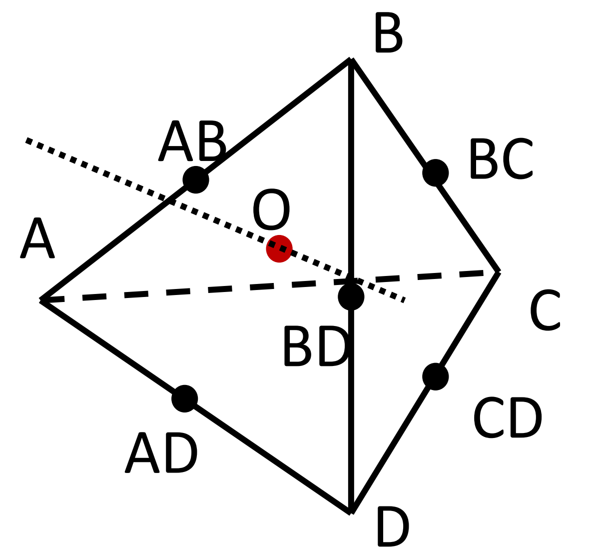
# GCM on unstructured grid

Tetrahedral interpolation of a Riemann’s invariant :

- linear – first order of approximation;
- quadratic – second order of approximation;
- scheme hybridization depending on a solution “smoothness”.

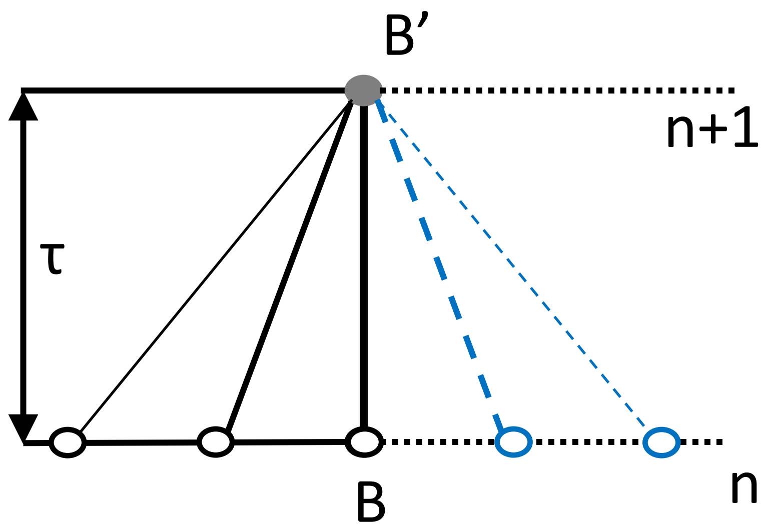


# Borders and contacts

External surface




- External force

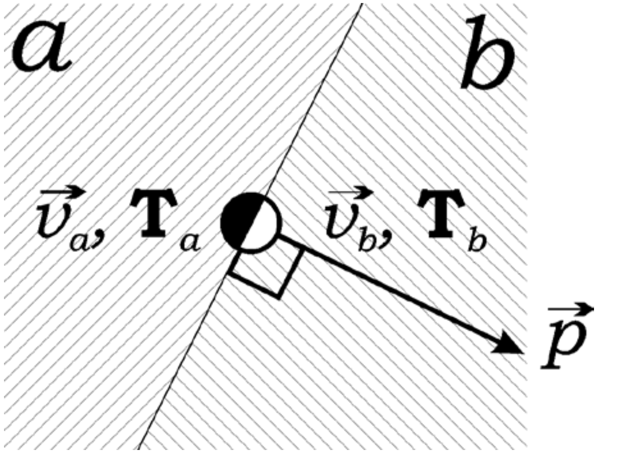
$$\mathbf{T}\vec{p} = \vec{f}$$

- External velocity

$$\vec{v} = \vec{V}$$

- Mixed conditions
- Absorbing border

Surface between media




**A – real node**

**B – virtual node**

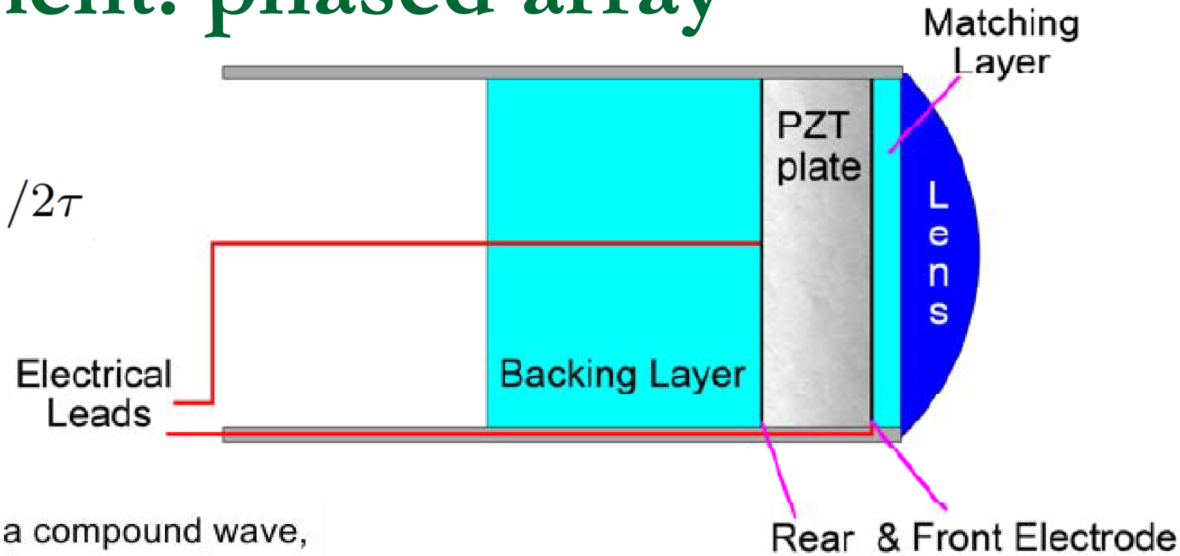
- Adhesion

$$\vec{v}_a = \vec{v}_b = \vec{V}, \quad \vec{f}_a = -\vec{f}_b$$

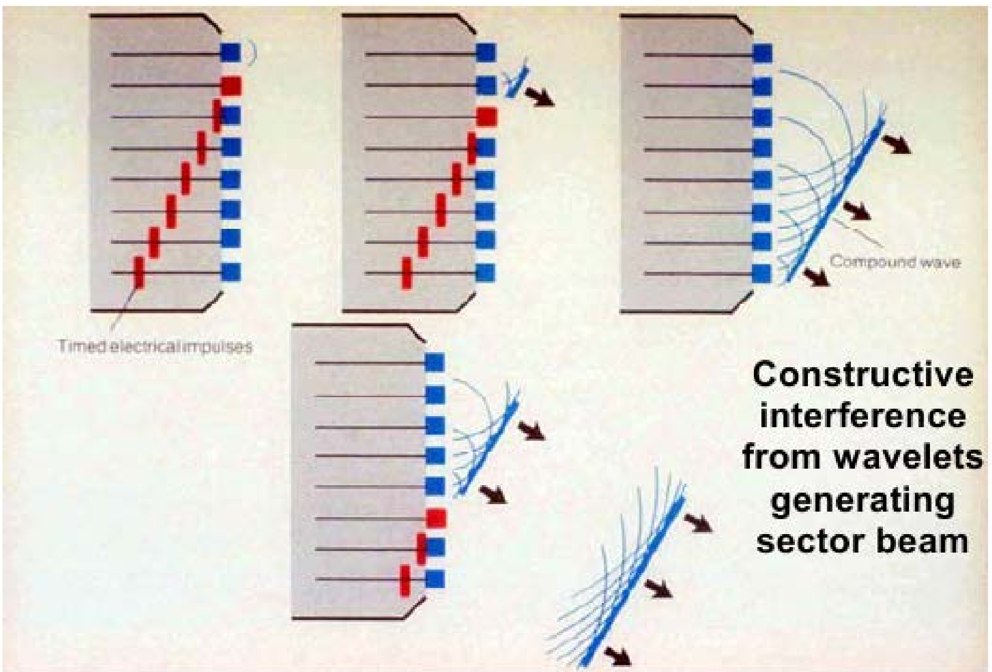
- Sliding
- Friction
- Destructible adhesion

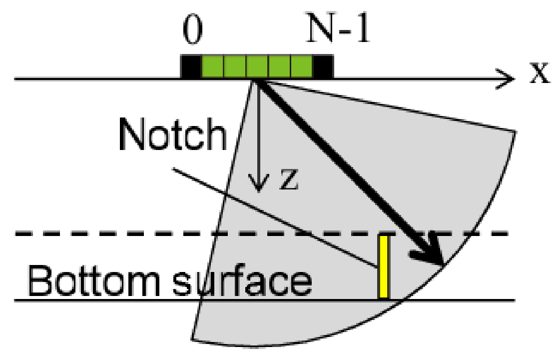
# Problem statement: phased array

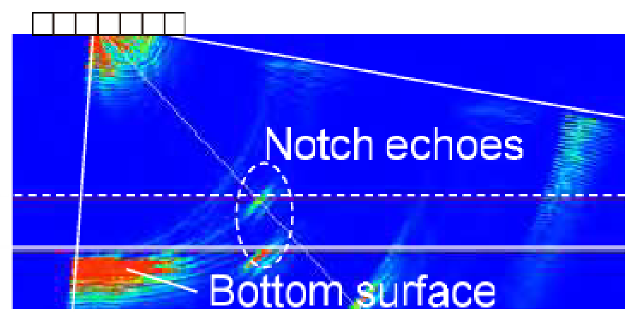
$$p^* = p_0 \exp^{i2\pi\nu t} \exp^{-t^2/2\tau}$$



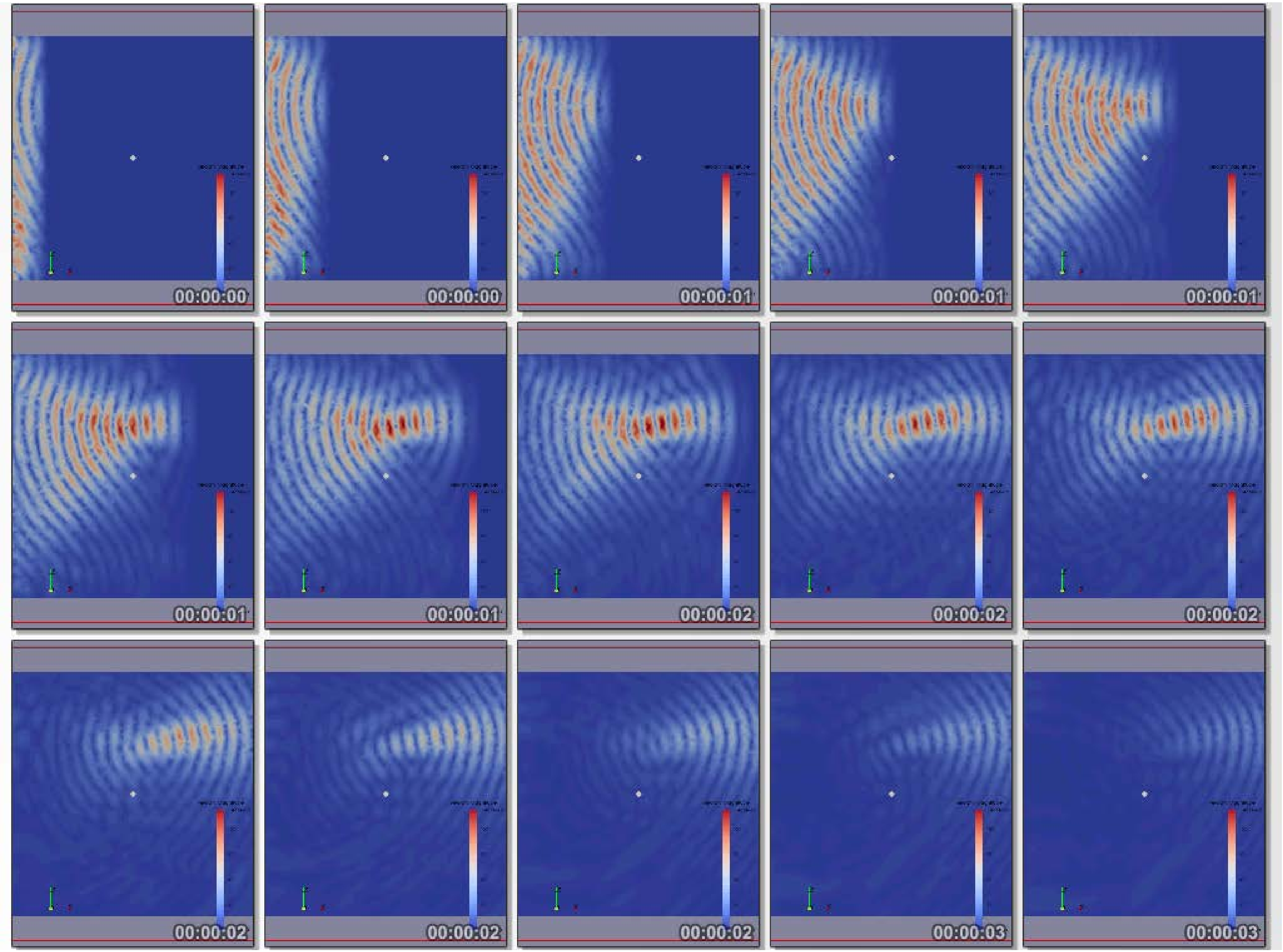

Each waveform merges to form a compound wave, generating a sector beam.



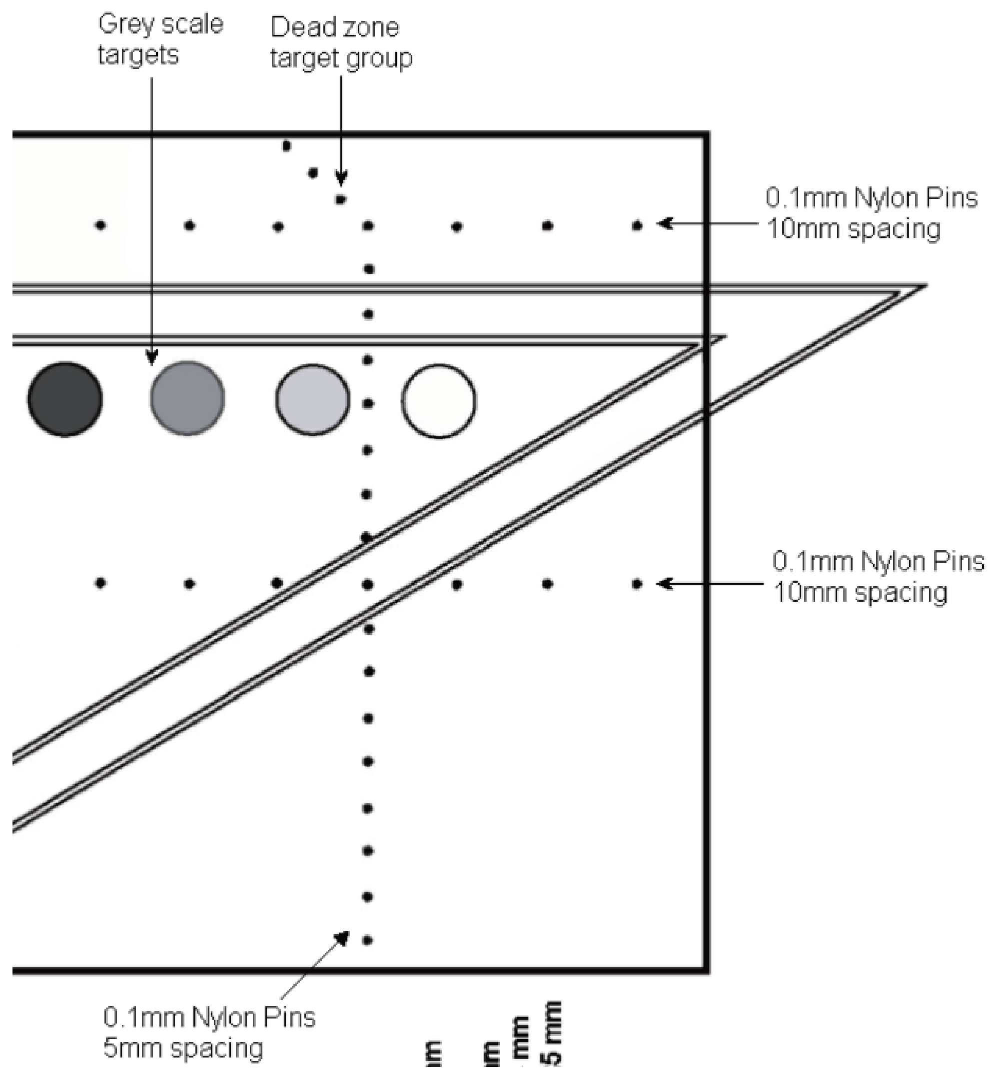



# Problem statement: phased array



# Calculation results: phantom Gammex 1430 LE.



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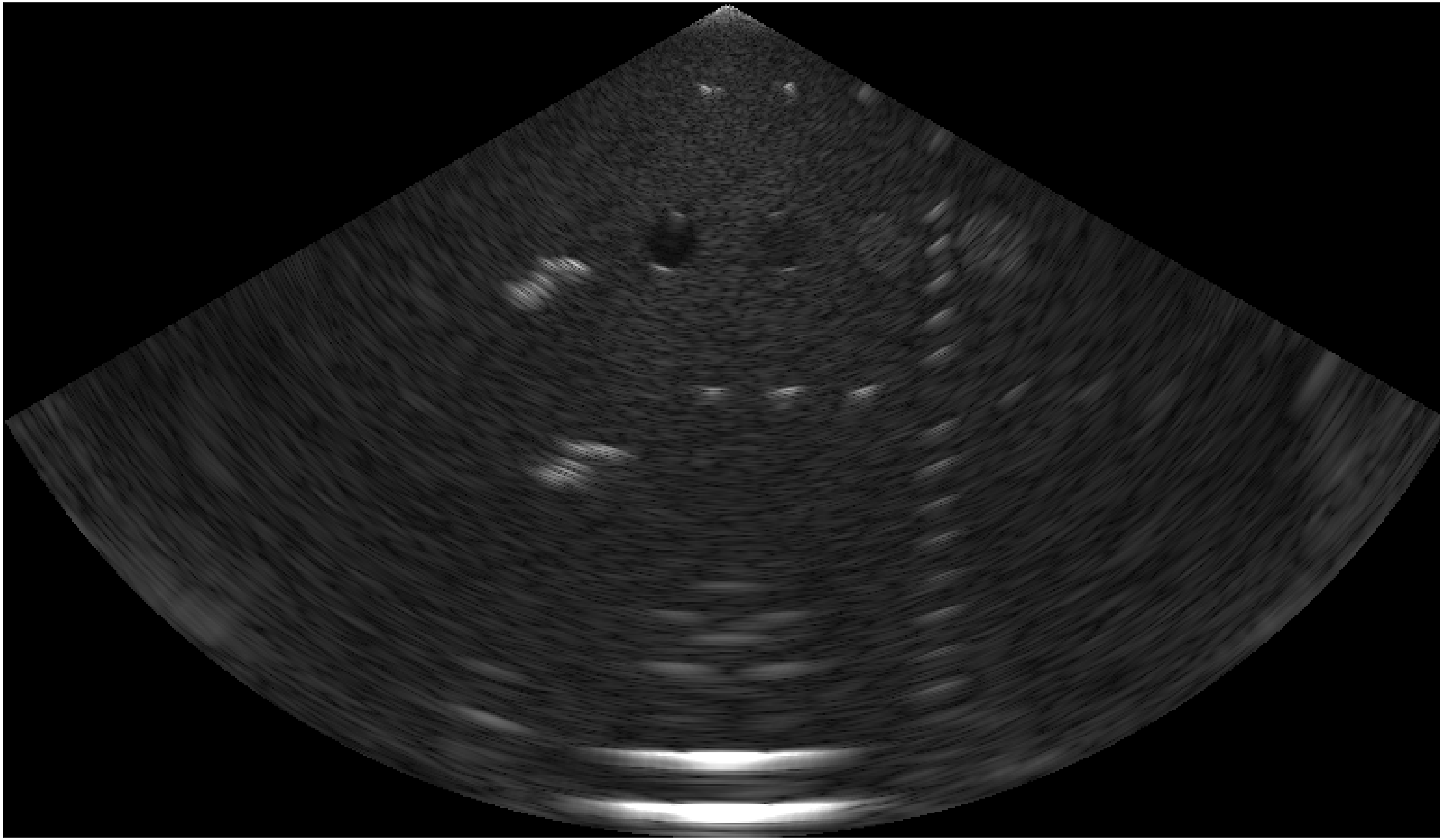

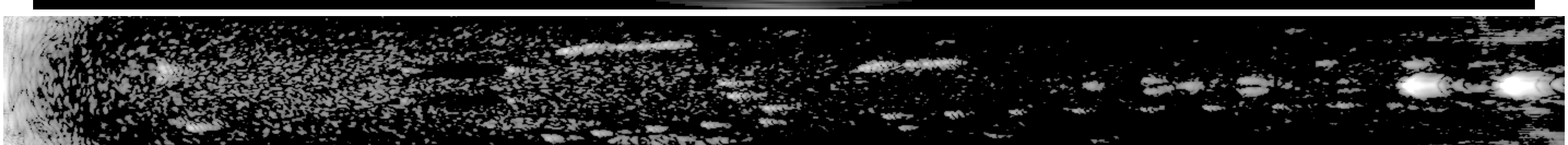

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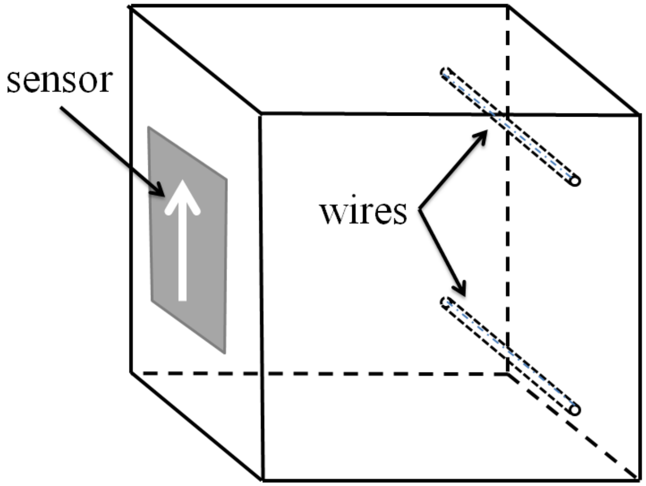


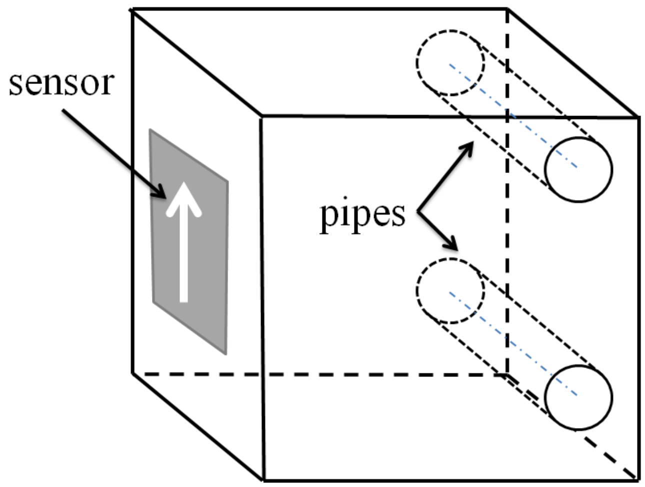


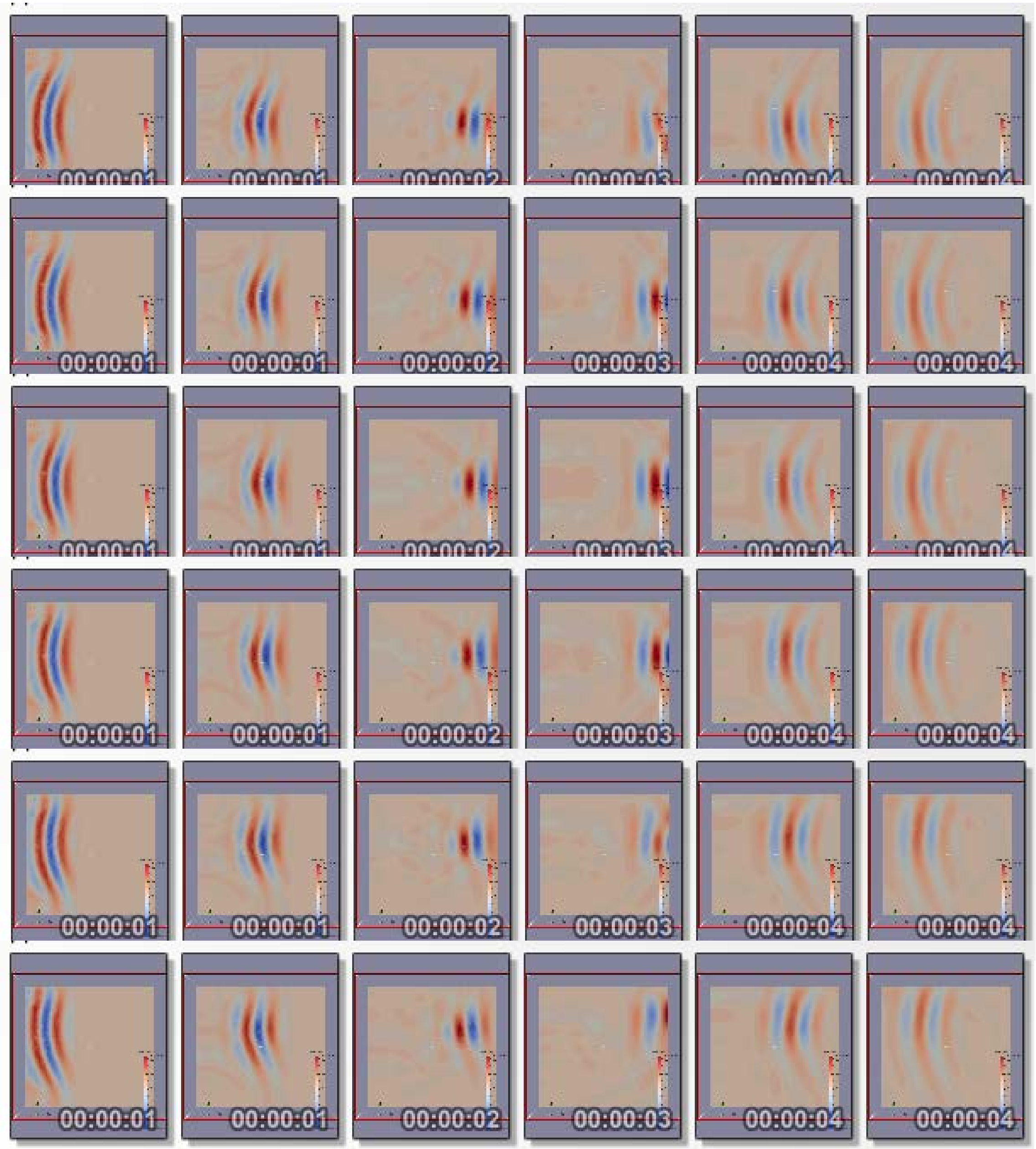

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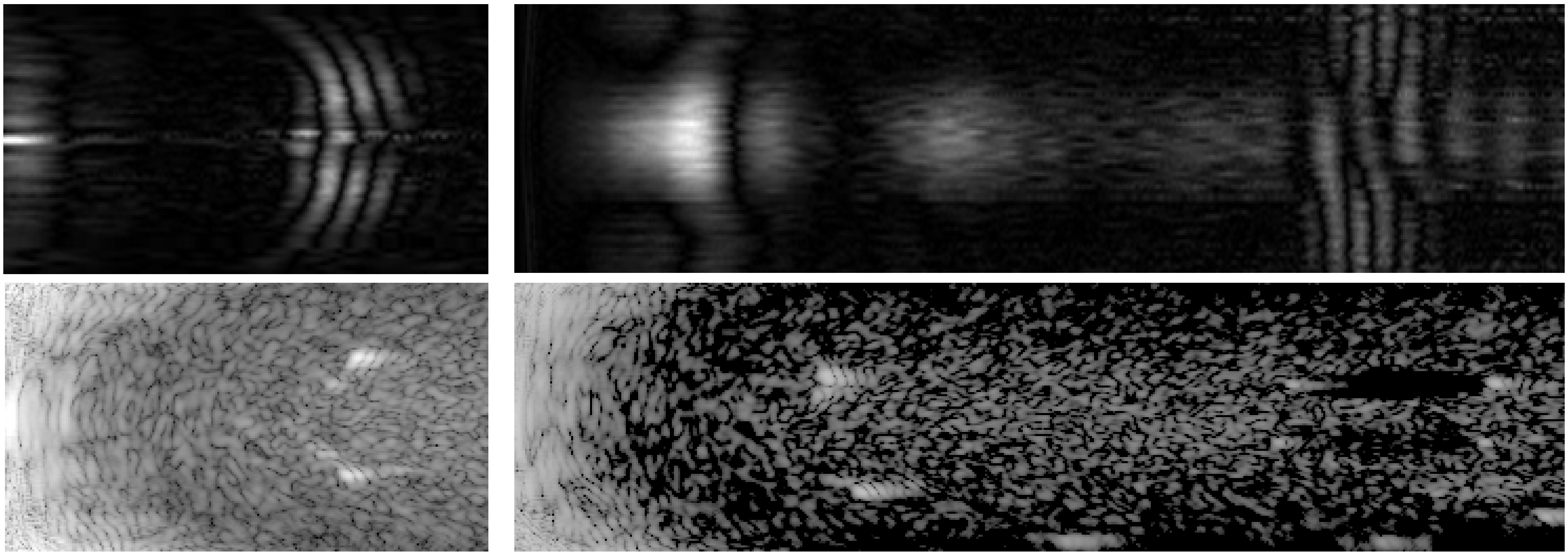
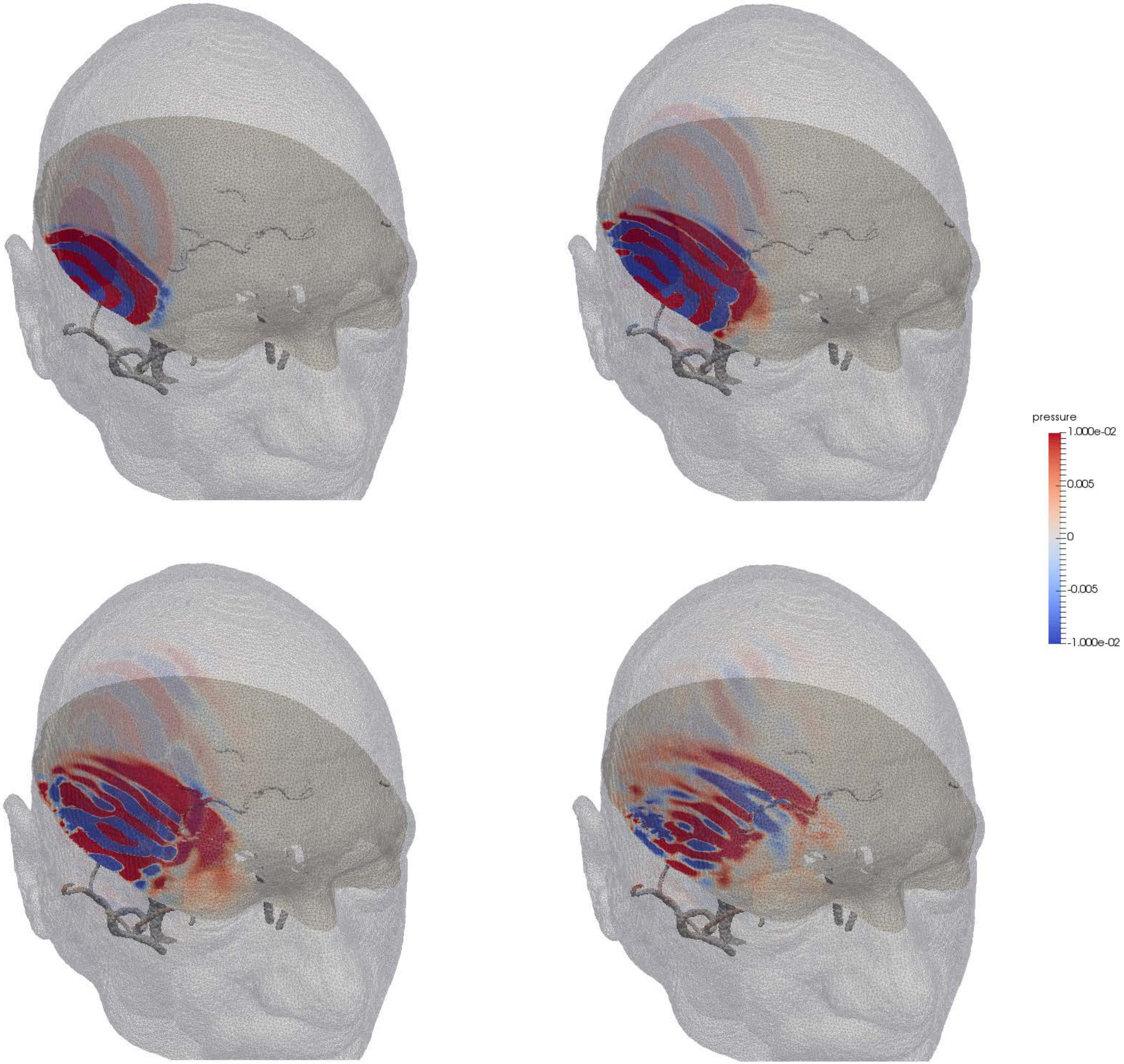
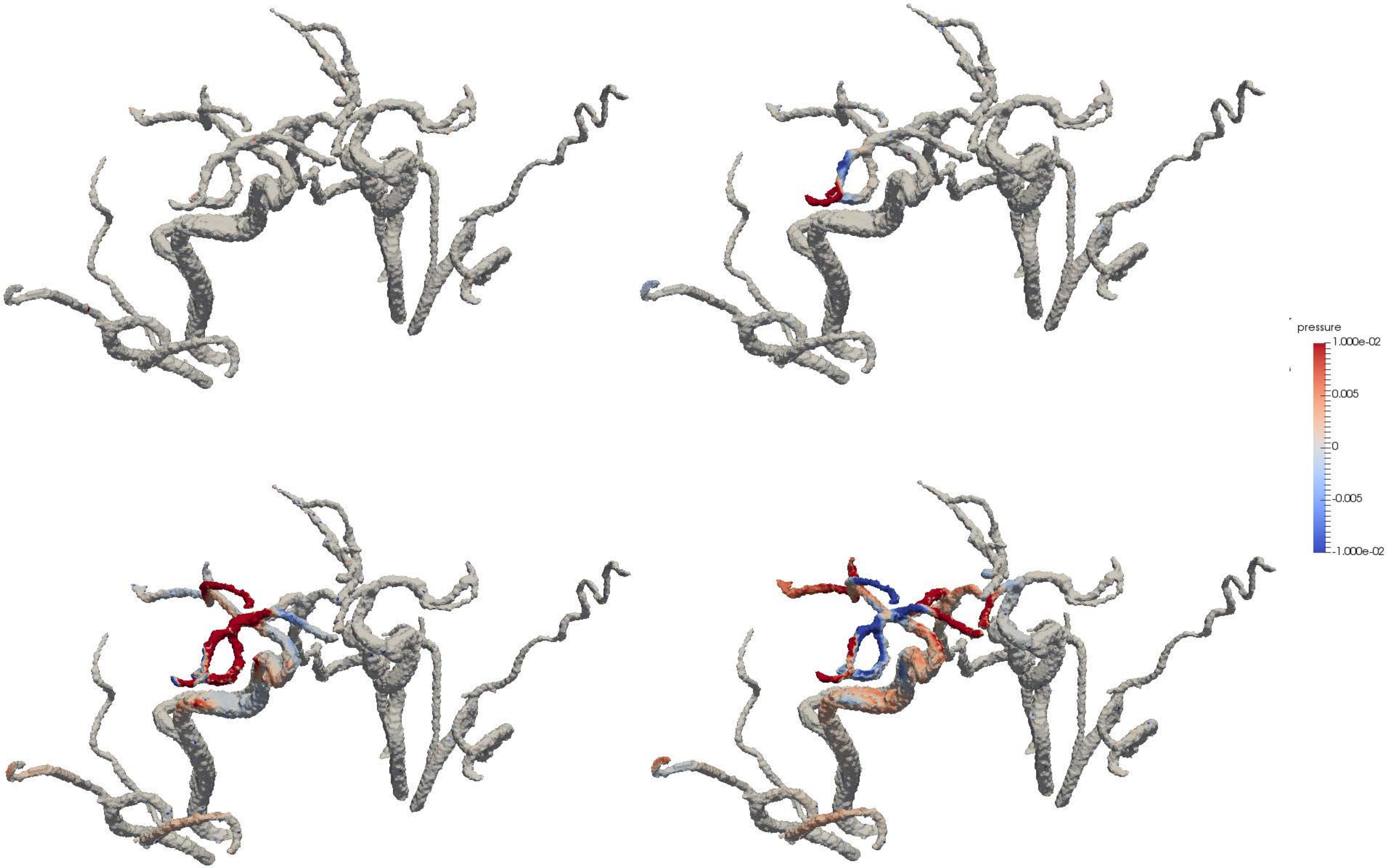


| Type | Experiment | Calculation |
|---|---|---|
| Wires | 15 +- 4 | 14 +- 8 |
| Pipes | 19 +- 4 | 20 +- 8 |

# Calculation results: acoustic waves in skull.



# Calculation results: acoustic waves on vessels.



# Conclusion

- The grid-characteristic method for mechanics of solids was adapted for 3D ultrasound problems.
- A model of human craniocerebral area was created.
- A set of calculations was performed for the ultrasound of phantom and human craniocerebral area.
- A comparison with experiments was conducted:
    - calculated A-scans are in a good agreement with the experiment (raw data from the sensor);
    - calculated B-scans don't have a good agreement and require further research on signal processing.

# Thank you!

# Problem statement: materials

| Tissue | $\rho$, kg/m$^3$ | $c$, m/s | $\alpha$, dB/cm |
|---|---|---|---|
| Phantom bulk material | 1000 | 1540 | 4.02 |
| Water | 1000 | 1540 | - |
| Nylon | 1140 | 2290 | - |
| Fat | 916 | 1435 | 3.82 |
| Muscle | 1041 | 1595 | 6.21 |
| Brain | 1030 | 1550 | 4.34 |
| Bone | 1904 | 2031 | 25.75 |
| Vessels | 1066 | 1616 | 4.18 |