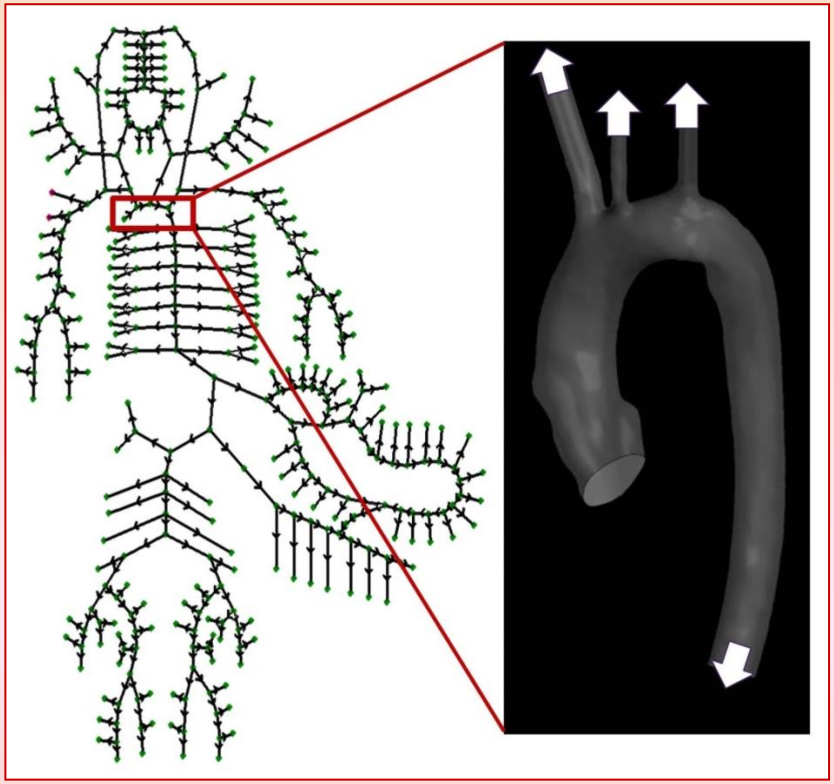
VIII Conference on Mathematical Models and Numerical Methods in Biomathematics

Coupling of 1D and 3D blood flow models

Tatiana Dobroserdova (INM RAS, MIPT)

Introduction



3D Model of Fluid Flow

- Navier-Stokes equations:

$$\frac{\partial \mathbf{u}}{\partial t} - \nu \Delta \mathbf{u} + (\mathbf{u} \cdot \nabla)\mathbf{u} + \nabla p = \text{f in } \Omega \times [0, T],$$

$$div\ \mathbf{u} = 0 \text{ in } \Omega \times [0, T]$$

- Boundary conditions:

$$\mathbf{u} = \mathbf{g} \text{ or } -\nu \frac{\partial \mathbf{u}}{\partial \mathbf{n}} + p\mathbf{n} = \mathbf{h} \text{ on } \Gamma_{in} \times [0, T],$$

$$\mathbf{u} = \mathbf{0} \text{ on } \Gamma_W \times [0, T],$$

$$-\nu \frac{\partial \mathbf{u}}{\partial \mathbf{n}} + p\mathbf{n} = \mathbf{h} \text{ on } \Gamma_{out} \times [0, T]$$

System of equations for 1D model

1. Mass conservation law

$$\frac{\partial S}{\partial t} + \frac{\partial (uS)}{\partial x} = 0$$

2. Momentum conservation law

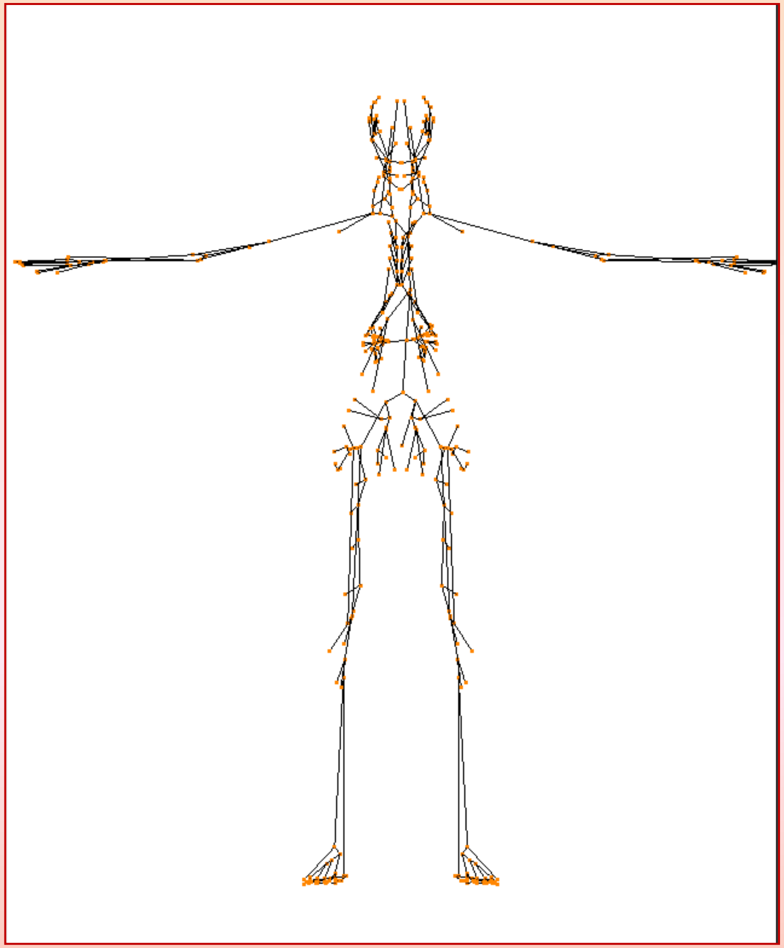
$$\frac{\partial u}{\partial t} + \frac{\partial}{\partial x}\left(\frac{u^2}{2} + \frac{p}{\rho}\right) = -16\mu u \frac{\eta(S)}{Sd^2},$$

$$\eta(S) = \begin{cases} 2, S > S_0 \\ \frac{S}{S_0} + \frac{S_0}{S}, S \le S_0 \end{cases}$$

3. State equation

$$p = \rho c^2 f(S),$$

$$f(S) = \begin{cases} \exp\left(\frac{S}{S_0} - 1\right) - 1, S > S_0 \\ \ln\left(\frac{S}{S_0}\right), S \le S_0 \end{cases}$$



Boundary conditions

1. Mass balance condition:

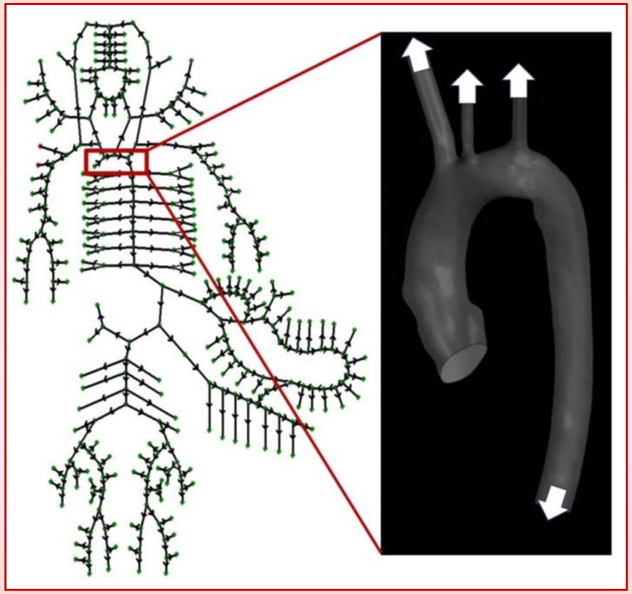
$$\sum_{k=k_1,\ldots,k_M} \alpha_k^m Q_k = 0, \alpha_k^m = \pm 1, Q_k = u_k S_k$$

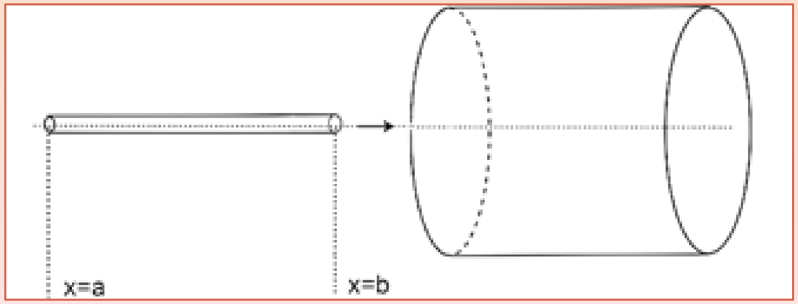
2. Poiseuille's pressure drop conditions:

$$p_k(t, x_k) - p_m^{node}(t) = \alpha_k R_k^m Q_k, x_k = 0, L_k$$

3. Compatibility conditions

Hard coupling of 1D and 3D blood flow models

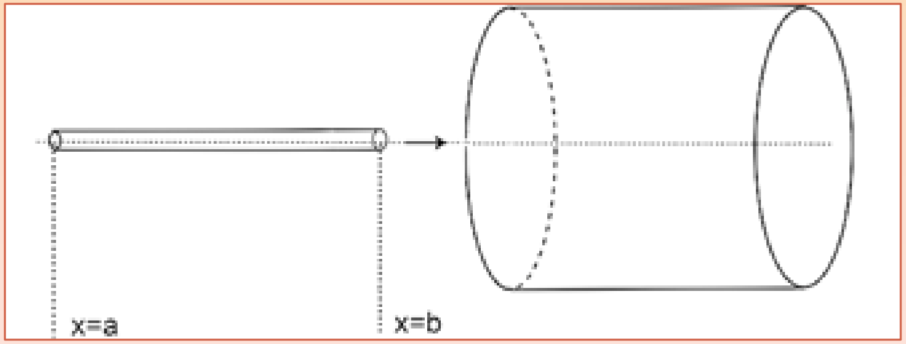


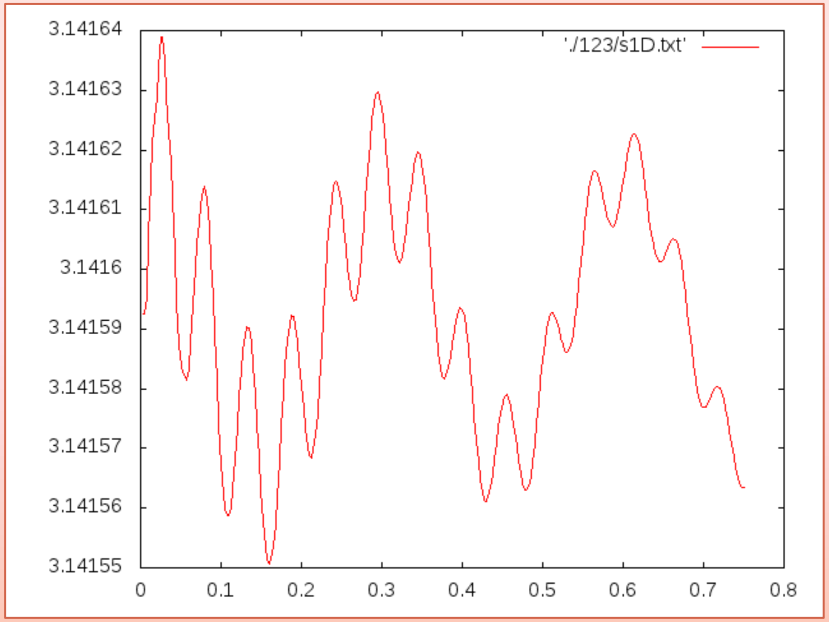


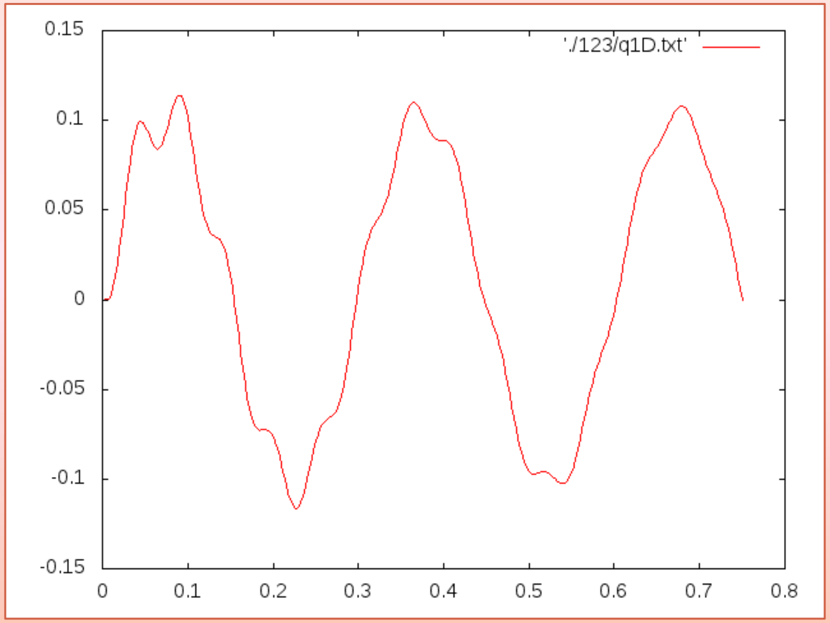
- $\int_{\Gamma_{in}} \mathbf{u} \bullet \mathbf{n}\, ds = \bar{u}_b\, S_b$
  $\left(-\nu \frac{\partial \mathbf{u}}{\partial \mathbf{n}} + p\mathbf{n}\right) = p_b \mathbf{n}$

1D: Compatibility condition.

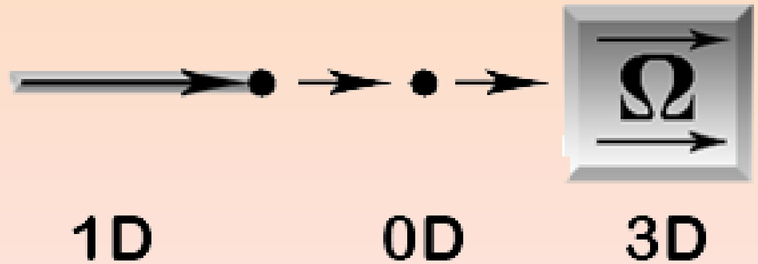
1D-3D blood flow model



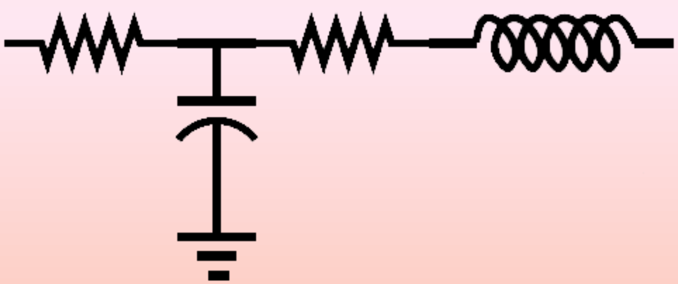




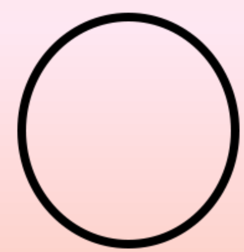
Soft coupling of 1D and 3D blood flow models



Electrical circuit



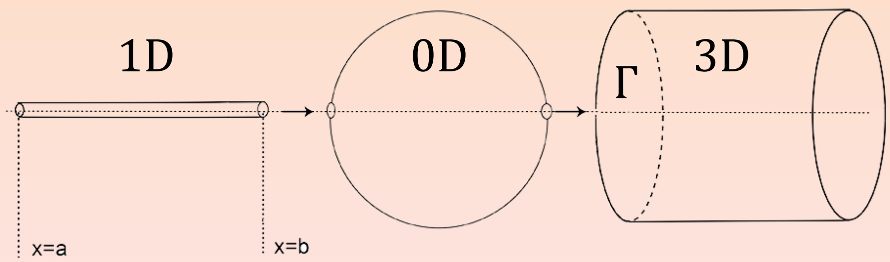
Elastic sphere



A 3D/1D geometrical multiscale model of cerebral vasculature
Tiziano Passerini, Maria Rita de Luca, Luca Formaggia, Alfio Quarteroni, Alessandro Veneziani

T.Dobroserdova, M.Olshanskii, S.Simakov. Multiscale coupling of compliant and rigid walls blood flow models. Int. J. Numer. Meth. Fluids, doi: 10.1002/fld.4241 (2016).

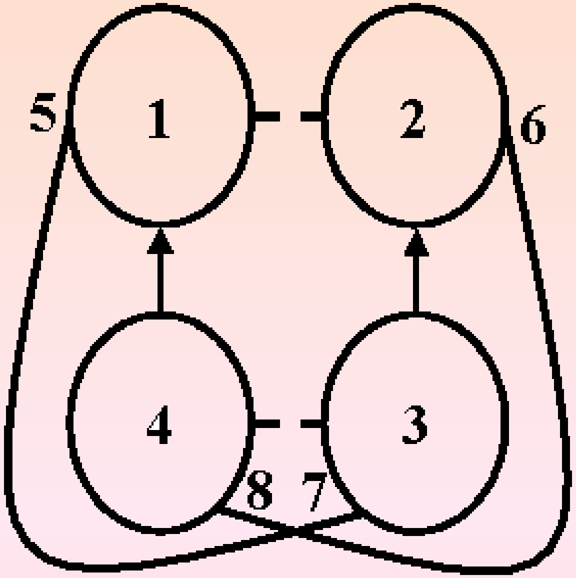
0D elastic sphere



$$I\frac{d^2V}{dt^2}+R_0\frac{dV}{dt}+\frac{V-V_0}{C}=p_{0D}$$

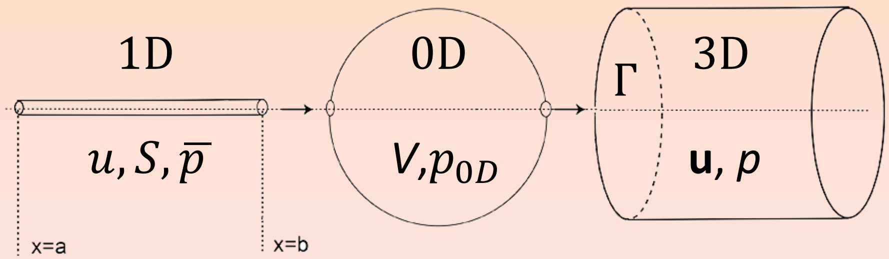
$$C\sim\frac{V}{c^2}$$

Heart model



S.S. Simakov, A.S. Kholodov. Computational study of oxygen concentration in human blood under the low-frequency disturbances, 2007

1D-0D-3D coupling



$$\frac{dV}{dt} = Q_{1D} - Q_{3D}$$

$$\bar{p} - p_{0D} = R_{1D0D} Q_{1D}$$

$$p_{0D} - p_{3D} = R_{0D3D} Q_{3D}$$

Parameter estimation

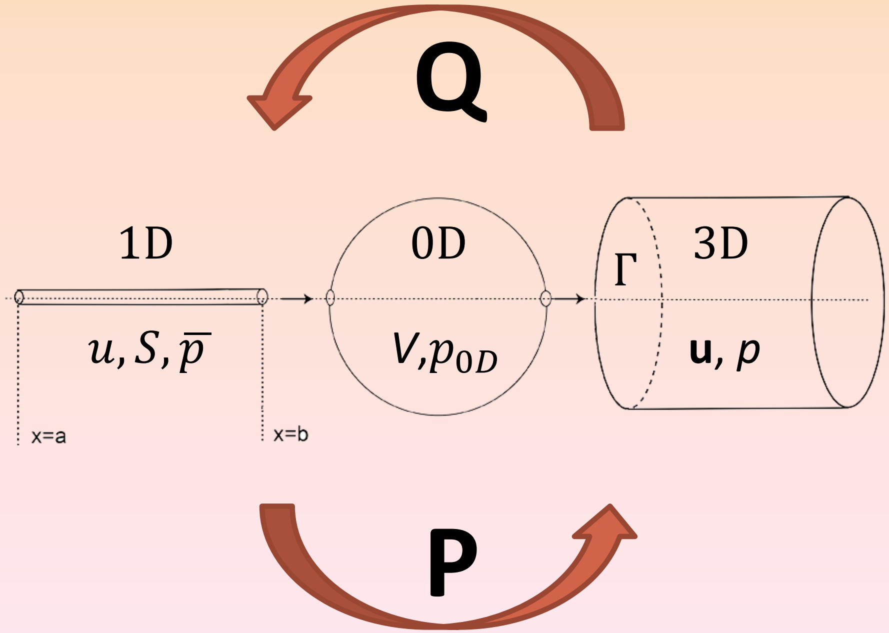
$$\boxed{R_{\mathrm{1D0D}} + R_{\mathrm{0D3D}} = \frac{\rho c}{\hat{S}}}$$

$$I\frac{d^2V}{dt^2} + R_0\frac{dV}{dt} + \frac{V - V_0}{C} = p_{0D} \xrightarrow{I = 0,\ R_0 = 0} C = \frac{\Delta V}{\Delta p_{0D}}$$

$$\boxed{C = \frac{\rho l \hat{S}}{c^2}}$$

T.Dobroserdova, M.Olshanskii, S.Simakov. Multiscale coupling of compliant and rigid walls blood flow models. Int. J. Numer. Meth. Fluids, doi:10.1002/fld.4241 (2016).

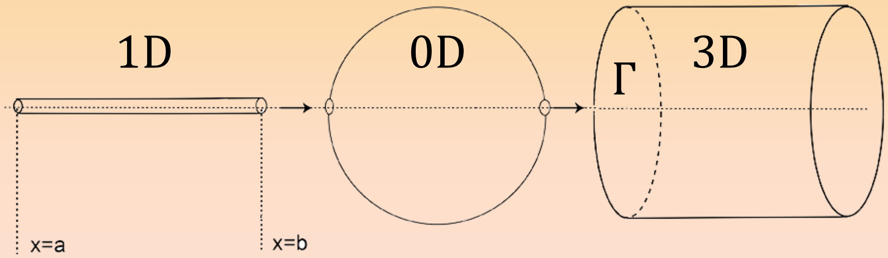
Numerical Iterative Algorithm



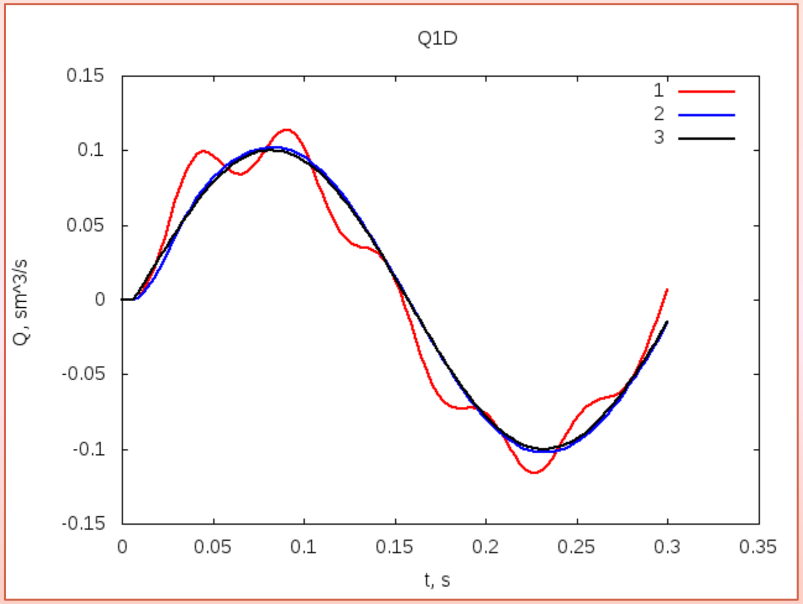
Soft coupling: C, R_{0D3D}, R_{3D0D}, R_{0D}
Hard coupling: $C = R_{0D3D} = R_{3D0D} = R_{0D} = 0$

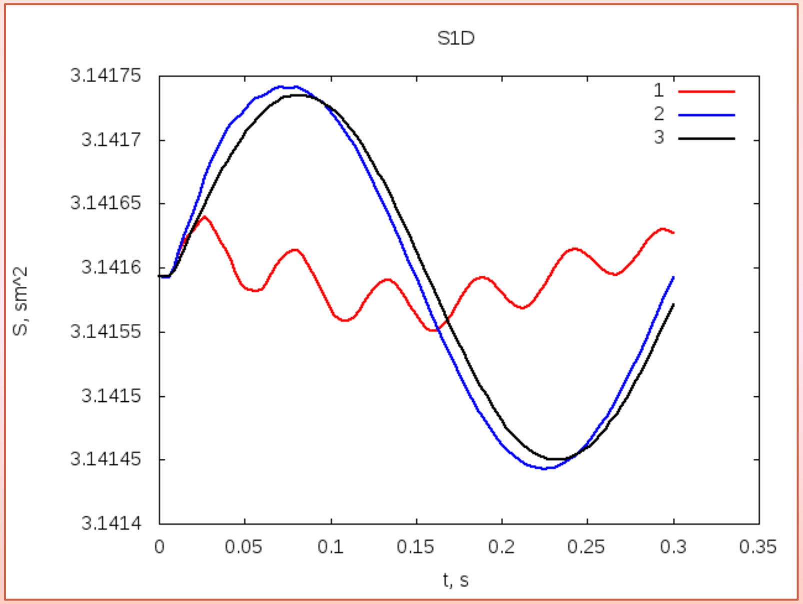
T.Dobroserdova, M.Olshanskii, S.Simakov. Multiscale coupling of compliant and rigid walls blood flow models. Int. J. Numer. Meth. Fluids, doi:10.1002/fld.4241 (2016).

Test with sinusoidal waveform

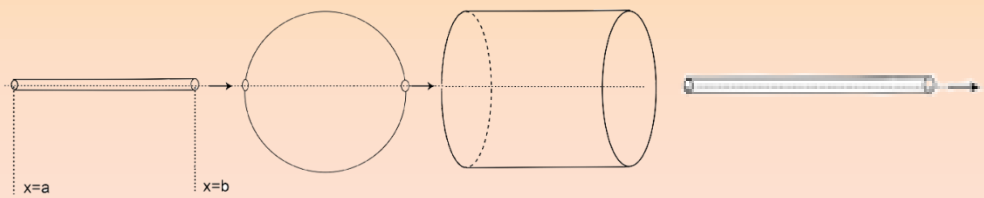


c = 700 cm/s

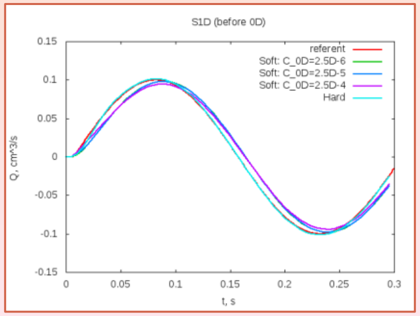




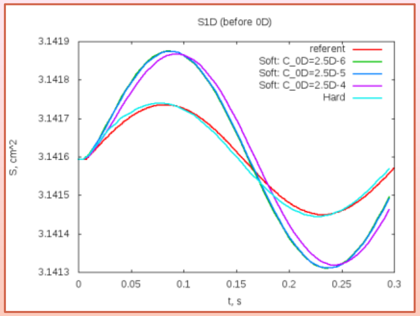
1D-0D-3D-1D blood flow simulation



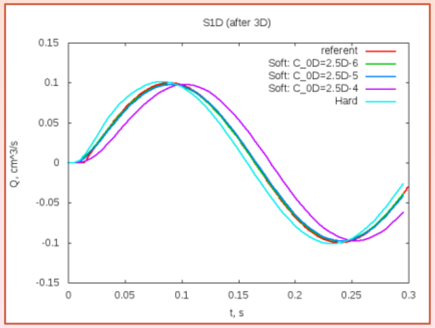
Flux



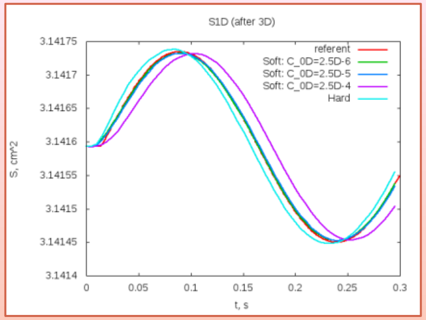
Cross Section



Flux



Cross Section



1D-0D-3D-1D blood flow simulation

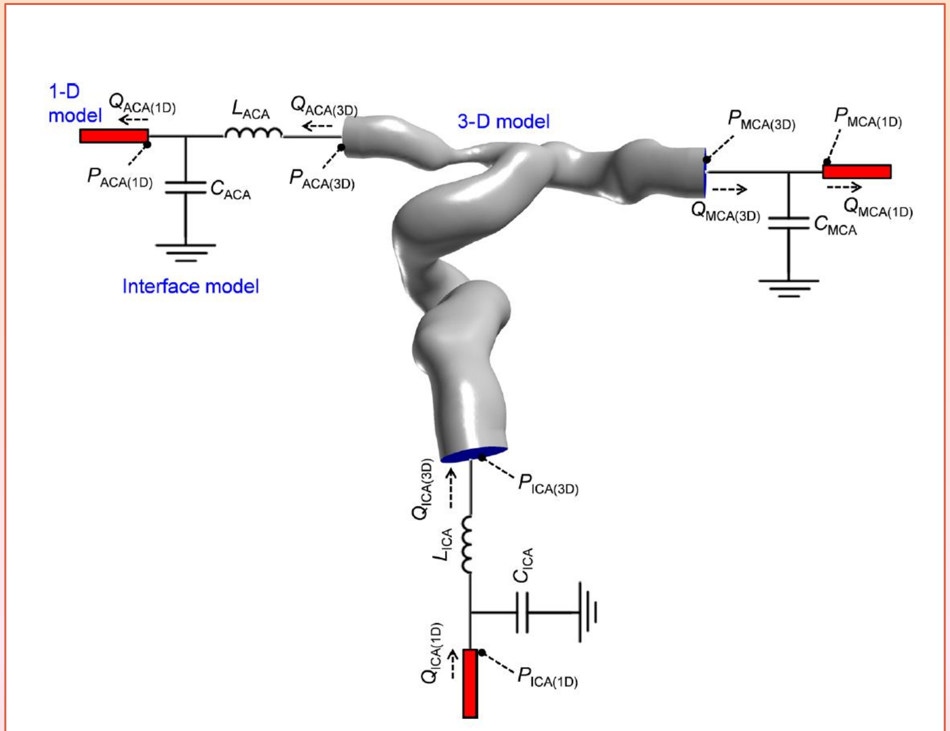


Fig. 2 Illustration of the coupling between the 1D model and the 3D model through a 0D interface model

$$P_{MCA(3D)} = 0$$

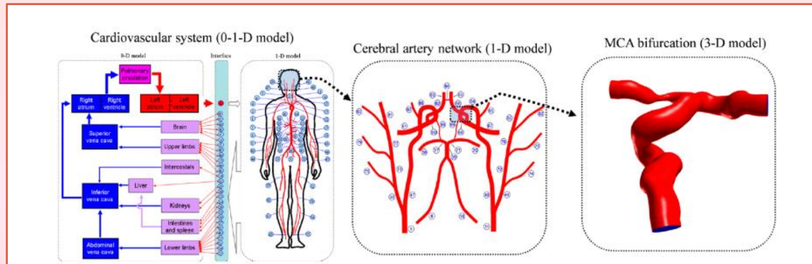
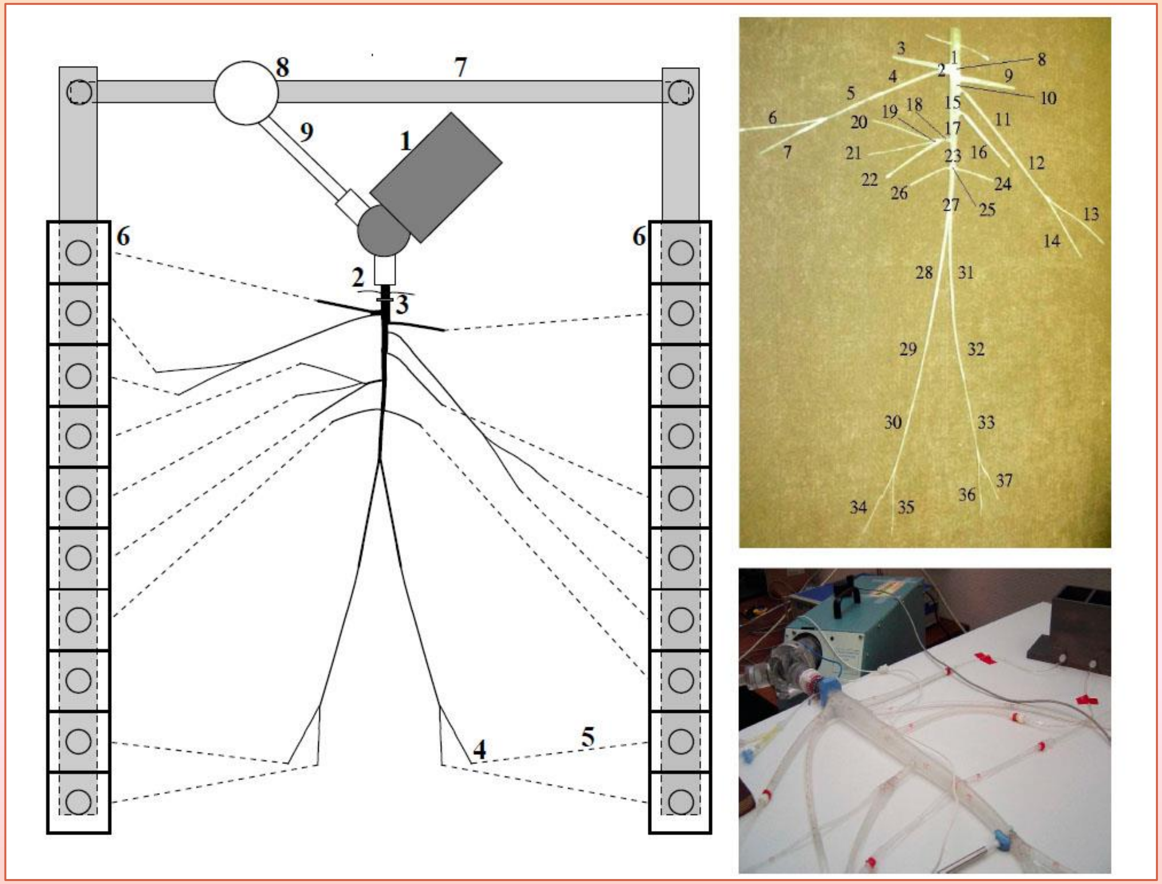


Fig. 1 Schematic description of the multiscale modeling system, where a 3D model of the MCA bifurcation is embedded in a 0-1-D multiscale model of the entire cardiovascular system

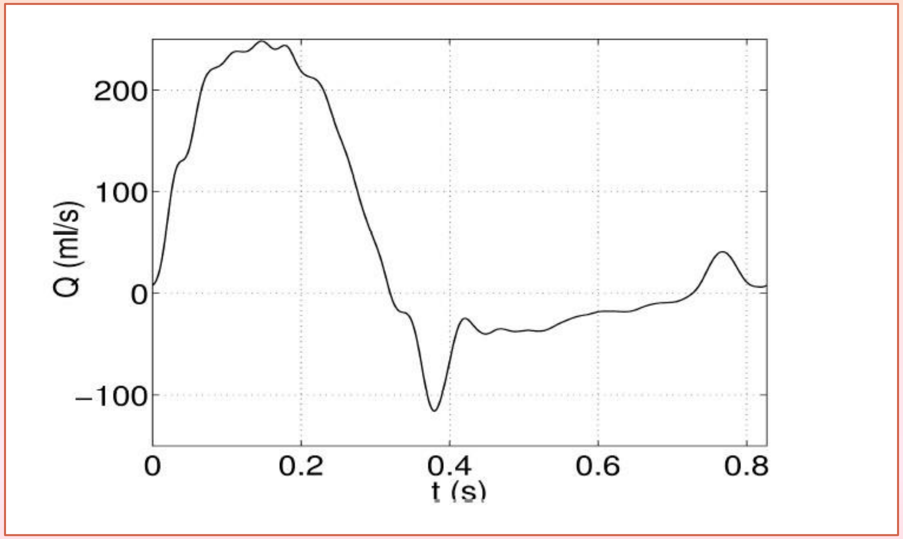
Liang F, Oshima M, Huang H, Liu H, Takagi S. Numerical Study of Cerebroarterial Hemodynamic Changes Following Carotid Artery Operation: A Comparison Between Multiscale Modeling and Stand-Alone Three-Dimensional Modeling. Journal of Biomechanical Engineering. 137. PMID 26343584 DOI: 10.1115/1.4031457

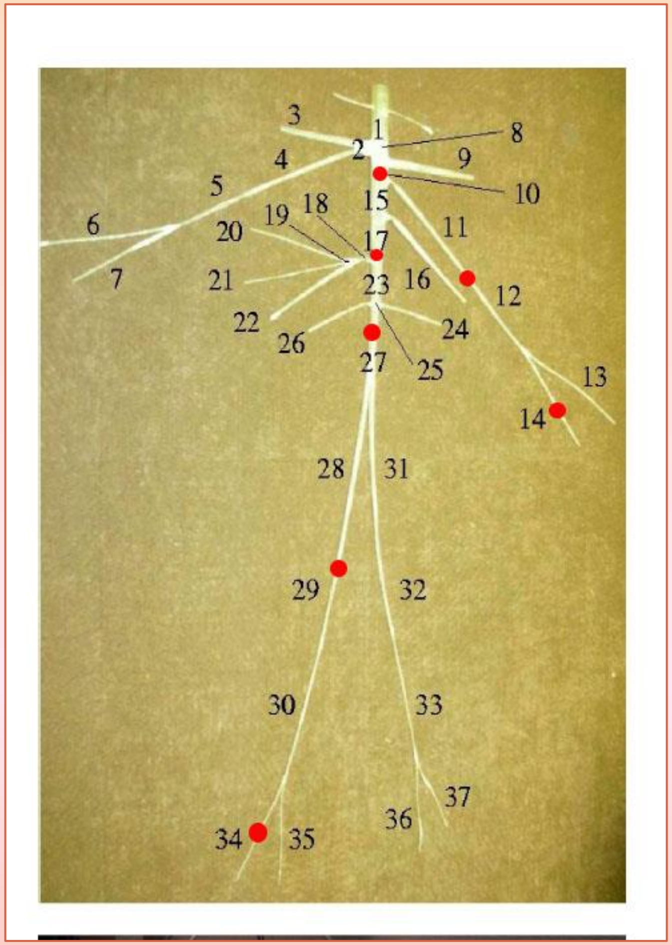
Hydraulic model



Alastruey, **J.**, **Khir**, A.W., **Matthys**, K.S., **Segers**, **P.**, **Sherwin**, S.J.,**Verdonck**, P.R., **Parker**, K.H. and **Peiró**, **J**. (2011, August). **Pulse wave propagation** in a **model human arterial network**: **Assessment** of **1-D visco-elastic simulations against** in **vitro measurements**.

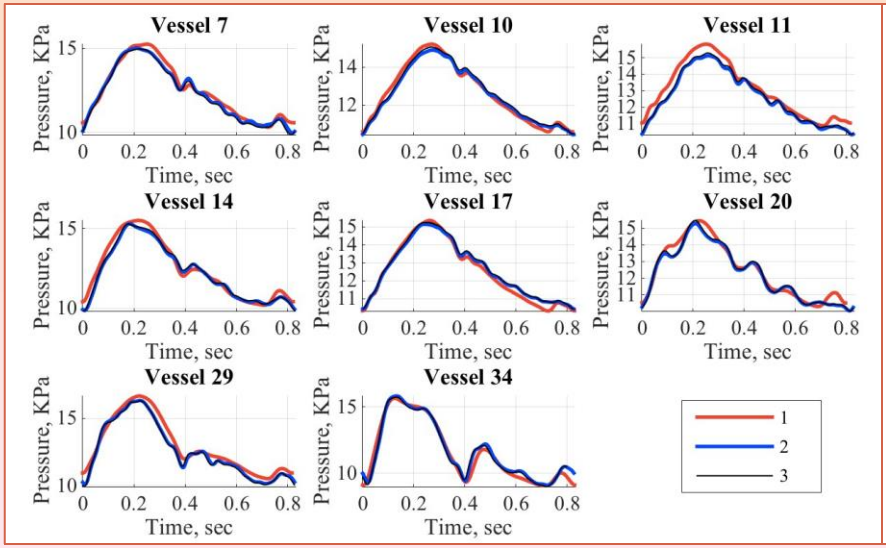
Hydraulic model



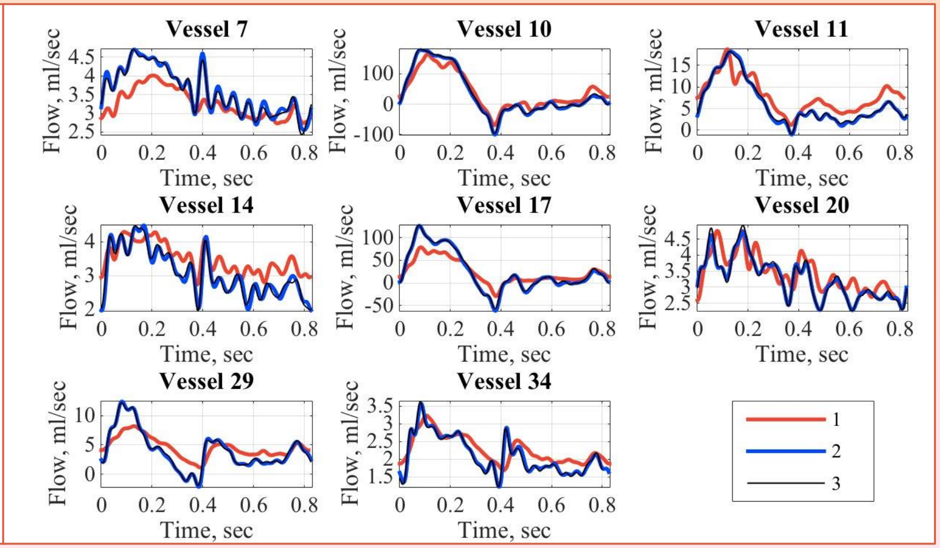


Numerical results vs Experimental measurements

Flux

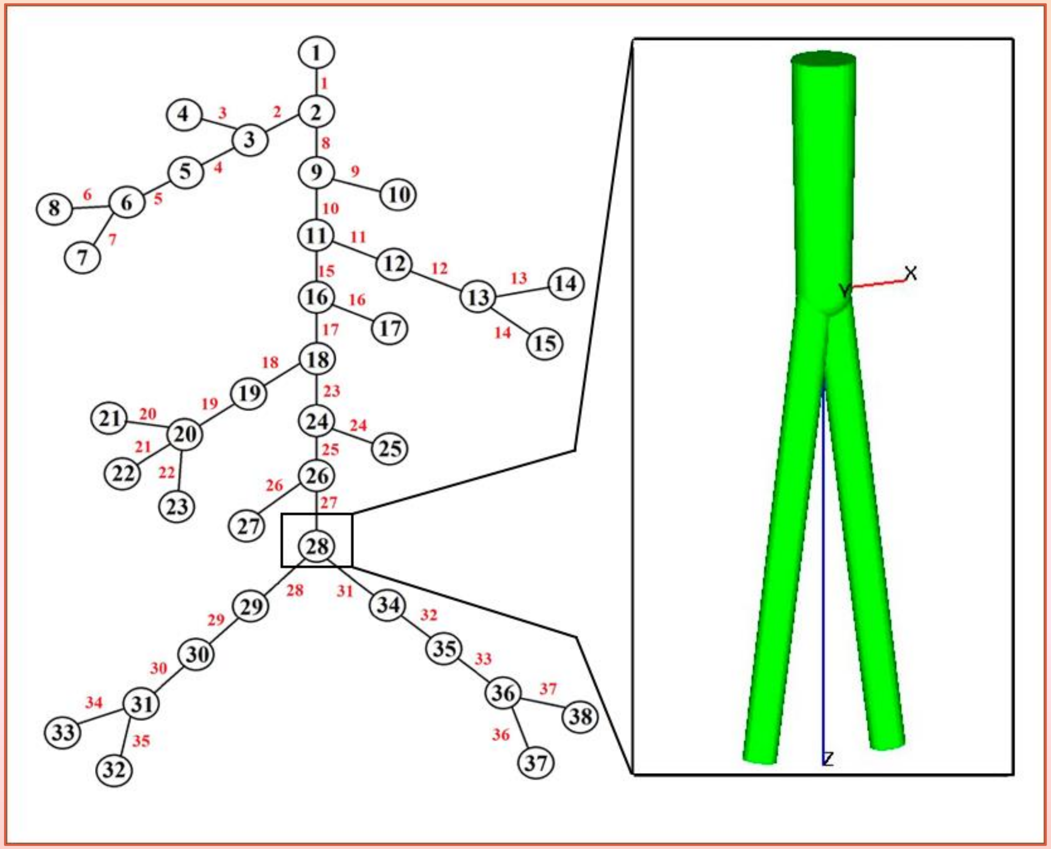


Pressure



Red line (1): experimental measurements;
Black line (2): results of numerical simulation with described model;
Blue line (3): results of numerical simulation from the paper

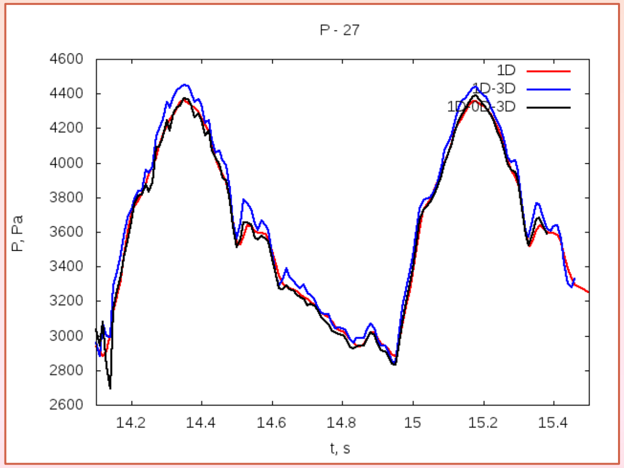
1D-3D blood flow model

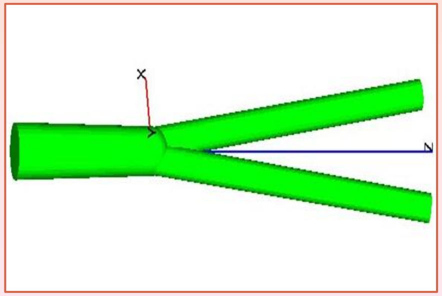


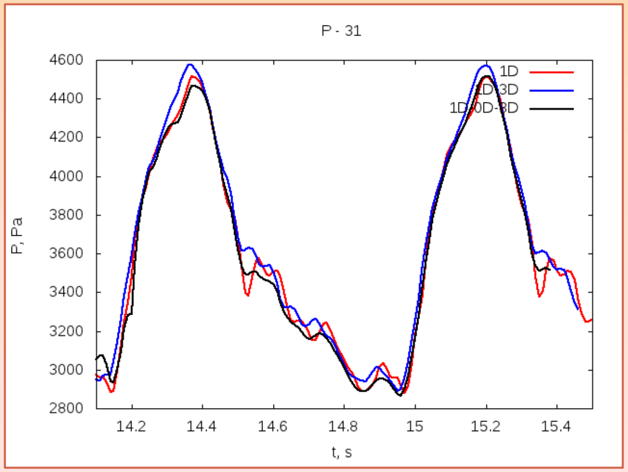
u_{max}= 25 см/с – maximal velocity
h = 0.14 – tetrahedral size
ν = 0.025 – viscosity

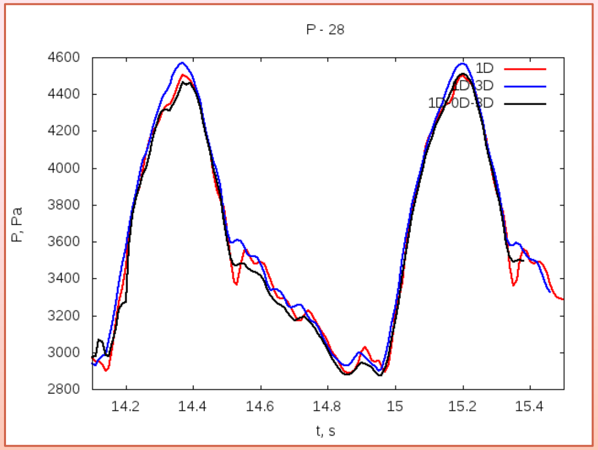
$$Pe = \frac{u_{max} h}{\nu} \sim 140$$

Pressure on 1D-3D interfaces

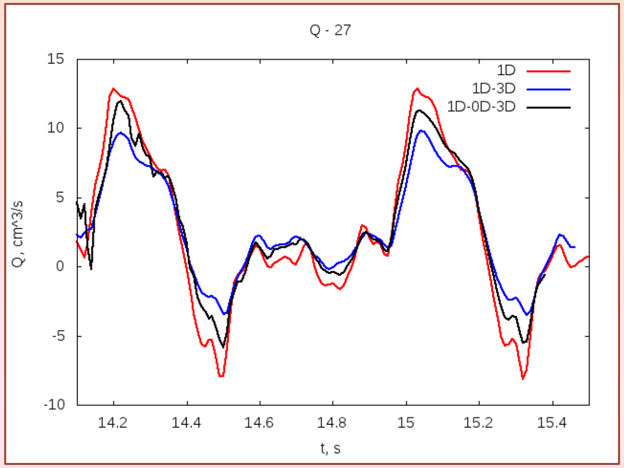


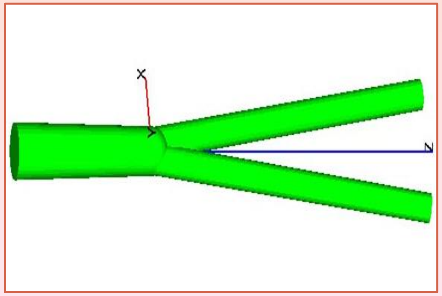


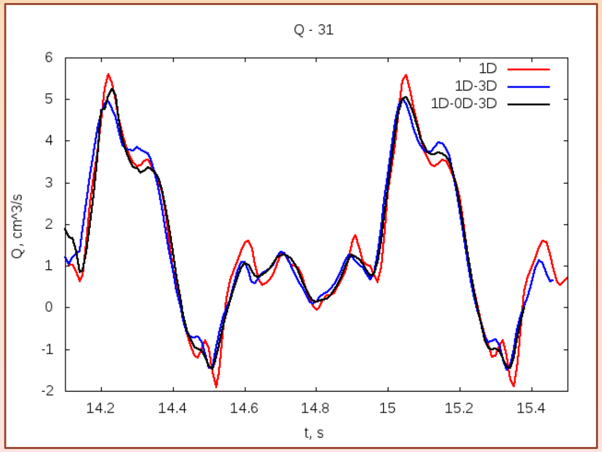


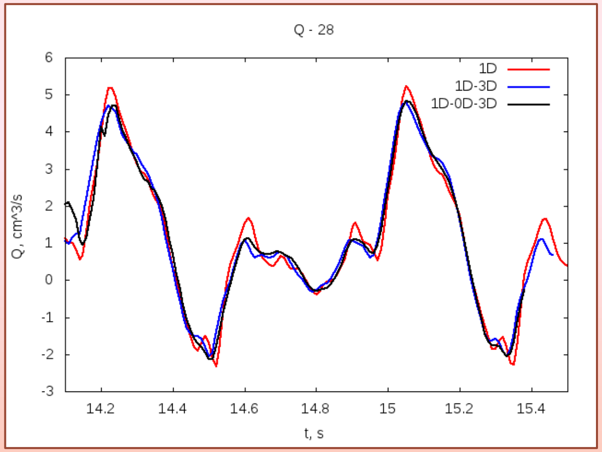


Fluid flux on 1D-3D interfaces









Conclusions

- Downstream boundary conditions are very important for multiscale blood flow modelling
- Elastic sphere used for 1D-3D coupling helps to compensate lack of elasticity in the 3D domain. Numerical 1D-3D model with soft coupling gives more accurate solution comparing with the referent 1D solution than the same model with hard coupling.

Thank you for your attention!