<u>"Mathematical models and numerical methods</u> <u>in biomathematics - VIII "</u> <u>Moscow, 31.10-03.11 2016</u>

#### Systems approach to immunology

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## Systems Immunology

- A field of **study of the interactions** between the components of the immune system, and **how these interactions give rise to the function and behavior** of that system
- The **application of systems theory** to the complexity of the immune interactions at all levels

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### **A Systems Approach to Immunology**

RONALD R. MOHLER, MEMBER, IEEE, CARLO BRUNI, AND ALBERTO GANDOLFI

Invited Paper

964

# What to look for in Immunology for a Systems-level understanding?

Kitano, Science, 2002, 1662-1664; Csete & Doyle, Science, 2002, 1664-1669





"...understanding cellular physiology and molecular details usually summarized by the term **'system biology'** or <u>'systematically, completely measured biology'</u> are eventually necessary to understand the strengths and weaknesses of immune defences."

*R. M. Zinkernagel. Immunity Against Infections & Vaccines: Credo 2004. Scand J Immunol. 2004, 60: 9–13* 

- 1. Identification of feedback mechanism for 'balance of growth and differentiation' hypothesis
- 2. Modelling the structure of lymph node

## Systems Immunology approach to specific questions

- Experiments + Measurements
- Big Data analyses + Mathematical Modeling +
  - Theoretical concepts and Hypotheses

### Theoretical Immunology

The main body of immunology is constituted by **non-mathematical theories** (empirically derived):

- 1. Clonal Selection Theory (Nobel Laureate, F.M. Burnet)
- 2. Network-type (e.g. idiotypic) Theory of Immune Regulation (Nobel Laureate, N. Jerne)
- 3. Spatiotemporal view of rules for immune responses regulation (Nobel Laureate R.M. Zinkernagel)
- 4. Balance of growth and differentiation; activation threshold tuning: conceptual framework of IR regulation Immune system responds to a rapid perturbation in its homeostasis); Individual lymphocytes respond to a rapid change in the level of stimulation, rather than a stimulation per se (Z. Grossman, W.E. Paul)
- Successful theories in immunology are still in the stage of growth & revision

### Regulation of clonal lymphocyte dynamics

- Antigen dose, time period during which it is available and its "geographical" distribution within this host influences immune responses (R.M. Zinkernagel: "Immunology and immunity against infection: general rules" JCAM 2005, 184:4-9)
- A program of antigen-independent clonal expansion and functional maturation: once triggered by a stimulatory APC, CD8<sup>+</sup> T lymphocytes primed in vivo or in vitro can proliferate through multiple rounds of division and differentiate in the absence of further stimulation

(M.J.B. van Stipdonk et al., Nature Immunology 2001 2: 423-428;

S.M. Kaech & Rafi Ahmed Nature Immunology 2001 2: 415-418)

#### Clonal expansion and contraction

#### Immune response ≡ Antigen-driven clonal dynamics Results from proliferation-differentiation-death of heterogeneous cell populations



From Bocharov et al., PNAS USA 2011, 108:3318-3323

The concept of "feedback-regulated balance of growth & differentiation" (Z. Grossman & W. Paul)

#### **1. Proliferation & differentiation**



#### 2. Feedback regulation



#### Precursor number profoundly affects "Factor of Expansion" (FE): *Low Precursor Frequency Shows More Proliferation*

#### The response of a single precursor in all the LNs could be detected

#### Antigen-stimulated CD4 T-cell expansion is inversely and log-linearly related to precursor number

Juan Quiel<sup>a</sup>, Stephane Caucheteux<sup>a</sup>, Arian Laurence<sup>b</sup>, Nevil J. Singh<sup>c</sup>, Gennady Bocharov<sup>d</sup>, Shlomo Z. Ben-Sasson<sup>a</sup>, Zvi Grossman<sup>a</sup>, and William E. Paul<sup>a,1</sup>

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#### Materials & Methods

#### Antigen-stimulated CD4 T-cell expansion is inversely and log-linearly related to precursor number

Juan Quiel<sup>a</sup>, Stephane Caucheteux<sup>a</sup>, Arian Laurence<sup>b</sup>, Nevil J. Singh<sup>c</sup>, Gennady Bocharov<sup>d</sup>, Shlomo Z. Ben-Sasson<sup>a</sup>, Zvi Grossman<sup>a</sup>, and William E. Paul<sup>a,1</sup>

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- <u>Mice</u>: 5C.C7 TCR transgenic, 5C.C7 *lpr/lpr*, 5C.C7 TNF- $\alpha$ R KO, 5C.C7 CTLA-4 KO, 5C.C7 IL-2 KO, 5C.C7 IFN- $\gamma$  KO, B10.A, C57BL/6, OT-II TCR transgenic C57BL/6, DO11.10 TCR tg BALB/c, BALB/c
- Adoptive Cell Transfer and immunization:
- 1. 5C.C7 or 5C.C7 CD45.1+ lymph node cells were mixed with polyclonal B10.A lymph node cells to give a total of  $10^6$  transferred cells. Cells were allowed to home for 24 hrs. Animals were immunized s.c. by a single dose of  $100 \mu g$  of cytochrome C protein or  $10 \mu g$  of pigeon cytochrome peptide in PBS. (LPS 25  $\mu g$ /animal was used as an adjuvant when indicated).
- 2. Similarly, DO11.10 tg, OT-II tg, Marylin tg, and A1(M) tg cells were transferred and recipients were immunized with 500  $\mu$ g of OVA, 10  $\mu$ g of HY peptide using 25  $\mu$ g of LPS as an adjuvant.
- 3. Cytokines  $(10 \mu g)$  were delivered via an s.c. miniosmotic pump
- <u>In vivo proliferation and division rate:</u> Recipients were pulsed with 1 mg of BrdU for a period of 6 hr. Lymph nodes were processed using the APC BrdU Flow Kit
- Enumeration of Cells Using Real-time PCR: uses the primers and probe sets directed against a short segment of the CDR3 region of the 5C.C7 β chain.
   <u>Resolution</u>: 1 TCR tg T cell in 100,000 non-tg T cells

#### **Experimental observations**

#### Antigen-stimulated CD4 T-cell expansion is inversely and log-linearly related to precursor number

Juan Quiel<sup>a</sup>, Stephane Caucheteux<sup>a</sup>, Arian Laurence<sup>b</sup>, Nevil J. Singh<sup>c</sup>, Gennady Bocharov<sup>d</sup>, Shlomo Z. Ben-Sasson<sup>a</sup>, Zvi Grossman<sup>a</sup>, and William E. Paul<sup>a,1</sup>

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- No evidence for competition for antigen or Dendritic Cell (DC) increased lymphocyte killing or shortage of cytokines
- Factor of Expansion (FE) dependence on PN is not significantly affected by antigen amount, DC number, the exogenous IL-2, IL-7, IL-15, Fas, TNF- $\alpha$  receptor, CTLA-4
- No evidence of differential Foxp3<sup>+</sup> T reg involvement
- FE of small numbers of transgenic precursors is not significantly affected by concomitant responses of large numbers of cells specific for different antigens
- Some type of local feedback control could explain results.

- To check the validity of 'feedback-regulated balance of growth and differentiation' concept
- To identify a parsimonious model of the kinetics of *physiological* regulation of antigen-driven CD4<sup>+</sup> T-cell expansion

(precursor number from 3 to 30,000 in the LNs; estimates of physiological frequencies range from 20 to 3,000 per animal)

# 'Global' data set to available for the identifying the regulation scheme using mathematical model



### Maximum likelihood parameter estimation

$$\frac{d}{dt}\mathbf{y}(t) = \mathbf{f}(\mathbf{y}(t), \mathbf{y}(t-\tau), \mathbf{p}), \quad t \in [t_0, T], \quad \tau > 0$$
$$\mathbf{y} \in R^{n_{\mathbf{Y}}}, \quad \mathbf{p} \in R^{n_{\mathbf{p}}}$$
$$\mathbf{y}(t) = \varphi(t), t \in [t_0 - \tau, t_0].$$

•Observation data

$$\{t_j, \mathbf{y}_j\}_{j=1}^{n_{obs}}$$

•Likelihood function

$$\mathbf{y}_{j} \sim \mathcal{N}(\mathbf{y}(t_{j}), \Sigma_{j})$$
$$\mathcal{H}(\mathbf{y}_{j}; \mathbf{p}) = \frac{1}{\sqrt{(2\pi)^{n_{y}} \det \Sigma_{j}}} \exp\left\{-\frac{1}{2}[\mathbf{y}(t_{j}) - \mathbf{y}_{j}]^{\mathrm{T}} \Sigma_{j}^{-1}[\mathbf{y}(t_{j}) - \mathbf{y}_{j}]\right\}$$
$$\mathcal{L}(\mathbf{p}) = \prod_{j=1}^{n_{obs}} \mathcal{H}(\mathbf{y}_{j}; \mathbf{p})$$
$$\mathbf{p}^{*} = \arg \max_{\mathbf{p} \in D} \mathcal{L}(\mathbf{p})$$

#### Data on Changes in Proliferation Rates

- •Immunization of recipients of 300 and 30,000 cells
- •Treatment with BrdU and killing them 6 h later
- •The proportion of BrdU+ and BrdU- cells among CD45.1 cells evaluated via FACS analysis



#### Nested mathematical models to describe the feedback regulation of T cell expansion and contraction



## Basic Model of DC4 T cell dynamics

$$\frac{dX_1}{dt}=p_1\cdot X_1-(\alpha_1+\alpha_{12}\cdot f_{x1x2}\cdot Z_2)\cdot X_1,$$

$$\frac{dX_2}{dt} = p_2 \cdot X_2 + (\alpha_1 + \alpha_{12} \cdot f_{x1x2} \cdot Z_2) \cdot X_1 - (\alpha_2 + \alpha_{22} \cdot Z_2) \cdot X_2,$$

$$\frac{dZ_1}{dt} = (\alpha_2 + \alpha_{22} \cdot Z_2) \cdot X_2 - \beta_1 \cdot Z_1,$$

$$\frac{dZ_2}{dt} = \beta_1 \cdot Z_1 - \delta \cdot Z_2.$$





## Extension of the model for assimilation of the BrdU data



## Extension of the model for the analysis of CFSE labeling data



#### Inverse problem -> data assimilation

$$\frac{dX_{1}^{0}}{dt} = -(p_{1}+d) \cdot f_{2} \cdot X_{1}^{0} - (a_{1}+a_{2}) \cdot f_{1,0} \cdot (Z_{1}^{0}+Z_{2}^{0}) \cdot X_{1}^{0}}{-d \cdot f_{2} \cdot X_{1}^{0}}$$

$$\frac{dX_{1}^{0}}{dt} = -(p_{1}+d) \cdot f_{2} \cdot X_{1}^{0} - (a_{1}+a_{2}) \cdot f_{1,0} \cdot (Z_{1}^{0}+Z_{2}^{0}) \cdot X_{1}^{0}}{-d \cdot f_{2} \cdot X_{1}^{0}}$$

$$\frac{dX_{1}^{0}}{dt} = -(p_{1}+d) \cdot f_{2} \cdot X_{1}^{0} - (a_{1}+a_{2}) \cdot f_{1,0} \cdot (Z_{1}^{0}+Z_{2}^{0}) \cdot X_{1}^{0}}{-d \cdot f_{2} \cdot X_{1}^{0}}$$

$$\frac{dX_{1}^{0}}{dt} = -(p_{2}+d) \cdot f_{2} \cdot X_{1}^{0} - (a_{1}+a_{2}) \cdot f_{1,0} \cdot (Z_{1}^{0}+Z_{2}^{0}) \cdot X_{1}^{0}}{-d \cdot f_{2} \cdot X_{1}^{0}}$$

$$\frac{dX_{1}^{0}}{dt} = 2 \cdot (p_{1}+d) \cdot f_{2} \cdot X_{1}^{0} - (a_{1}+a_{2}) \cdot f_{1,0} \cdot (Z_{1}^{0}+Z_{2}^{0}) \cdot X_{1}^{0}}{-(a_{1}+a_{2}) \cdot f_{1,0} \cdot (Z_{2}^{0}+Z_{2}^{0}) \cdot X_{1}^{0} - d \cdot f_{2} \cdot X_{1}^{0}}$$

$$\frac{dX_{1}^{0}}{dt} = 2 \cdot (p_{1}+d) \cdot f_{2} \cdot X_{1}^{0} - (a_{1}+a_{2}) \cdot f_{1,0} \cdot (Z_{2}^{0}+Z_{2}^{0}) \cdot X_{1}^{0}}{-(a_{1}+a_{2}) \cdot f_{1,0} \cdot (Z_{2}^{0}+Z_{2}^{0}) \cdot X_{1}^{0}}{-(a_{1}+a_{2}) \cdot f_{1,0} \cdot (Z_{2}^{0}+Z_{2}^{0}) \cdot X_{1}^{0} - d \cdot f_{2} \cdot X_{1}^{0}}{-(a_{1}+a_{2}) \cdot f_{1,0} \cdot (Z_{2}^{0}+Z_{2}^{0}) \cdot X_{1}^{0}}{-(a_{1}+a_{2}) \cdot f_{1,0} \cdot (Z_{2}^{0}+Z_{2}^{0}) \cdot X_{1}^{0} - d \cdot f_{2} \cdot X_{1}^{0} - (a_{1}+a_{2}) \cdot f_{1,0} \cdot X_{$$

#### Assimilation of the data on BrdU labeling & validation of the model by CFSE dilution data



# The general parametric structure of the mathematical model of feedback regulation





Parameter	Notation (units)	Best-fit estimate	95% confidence interval
Net proliferation rate of CD4 T cells at initial stage $X_1$	p <sub>1</sub> (d <sup>-1</sup> )	0.81	[0.80*, 1.12]
Net proliferation rate of CD4 T cells at transit stage $X_2$	p <sub>2</sub> (d <sup>-1</sup> )	2.0*	[1.0, 2.0*]
Death rate of proliferating CD4 T cells	<i>d</i> (d <sup>-1</sup> )	0.95	[0.45, 1.0]
Maximal effect parameter of cell crowding on proliferation rate	γ	$5.0  imes 10^{-4}$	$[0, \infty)$
Threshold cell number for the onset of the effect of cell crowding on proliferation rate	c (cell)	$2.8 \times 10^5$	(0, ∞)
Constitutive maturation rate of proliferating cells	$\alpha_1$ (d <sup>-1</sup> )	0.0038	[0, 0.23]
Rate constant of the feedback-regulated maturation of proliferating cells	$\alpha_{12}$ (d <sup>-1</sup> ·cell <sup>-1</sup> )	0.06	[10 <sup>-4</sup> , >10 <sup>10</sup> ]
Constitutive differentiation rate of proliferating cells into nonproliferating stage	α <sub>2</sub> (d <sup>-1</sup> )	0.001*	[10 <sup>-3</sup> , >1.0]*
Rate constant of the feedback-regulated differentiation of proliferating cells into nonproliferating cells	$\alpha_{22} (d^{-1} \cdot cell^{-1})$	0.023	$[3 \times 10^{-4}, >10^{10}]$
Constitutive terminal differentiation rate of nonproliferating cells	β <sub>1</sub> (d <sup>-1</sup> )	0.5	[10 <sup>-3</sup> , >1.0]*
Rate constant of the feedback-regulated terminal differentiation of nonproliferating cells	$\beta_{12}$ (d <sup>-1</sup> ·cell <sup>-1</sup> )	$9.0 \times 10^{1}$	[0, ∞)
Threshold nonproliferating cell number for the onset of the saturation in the maturation rate of proliferating cells	$\theta_{x1}$ (cell)	$2.4 \times 10^{1}$	[<10 <sup>−10</sup> , ∞)
Threshold nonproliferating cell number for the onset of the saturation in the differentiation rate of proliferating cells	$\theta_{x2}$ (cell)	$3.5 \times 10^2$	[<10 <sup>-10</sup> , ∞)
Threshold nonproliferating cell number for the onset of the saturation in the maturation rate of nonproliferating cells	$\theta_z$ (cell)	$8.9  imes 10^{-2}$	(0, ∞)
Death/migration rate of terminally differentiated cells	δ (d <sup>-1</sup> )	0.59	[0.21, 1.25]

\*Lower or upper bound constrained estimation.

#### The parsimonious version of the general mathematical model: its complexity is optimal with respect to the information content of the data set



#### Table S2. Parameters of the parsimonious model with their best-fit estimates and 95% confidence intervals

Parameter	Notation (units)	Best-fit estimate	95% confidence interval
Net proliferation rate of CD4 T cells at initial stage $X_1$	p <sub>1</sub> (d <sup>-1</sup> )	0.80*	[0.80*, 1.02]
Net proliferation rate of CD4 T cells at transit stage $X_2$	p <sub>2</sub> (d <sup>-1</sup> )	2.0*	[1.17, 2.0*]
Death rate of proliferating CD4 T cells	d (d <sup>-1</sup> )	0.99	[0.41, 1.0*]
Constitutive maturation rate of proliferating cells	α <sub>1</sub> (d <sup>-1</sup> )	0.0059	[0.0018, 0.104]
Rate constant of the feedback-regulated maturation of proliferating cells	$\alpha_{12}$ (d <sup>-1</sup> ·cell <sup>-1</sup> )	0.012	[9 × 10 <sup>-4</sup> , 90]
Constitutive differentiation rate of proliferating cells into nonproliferating stage	$\alpha_2$ (d <sup>-1</sup> )	0.001*	[10 <sup>-3</sup> *, 0.35]
Rate constant of the feedback-regulated differentiation of proliferating cells into nonproliferating cells	$\alpha_{22} (d^{-1} \cdot cell^{-1})$	0.045	$[4 \times 10^{-3}, 0.25]$
Constitutive terminal differentiation rate of nonproliferating cells	β <sub>1</sub> (d <sup>-1</sup> )	0.52	[0.15, 1.0*]
Threshold nonproliferating cell number for the onset of the saturation in the maturation rate of proliferating cells	$\theta_{x1}$ (cell)	842	$[0.05, 5.2 \times 10^3]$
Death/immigration rate of terminally differentiated cells	δ (d <sup>-1</sup> )	3.45	[0.51, 17.2]

\*Lower or upper bound constrained estimation.

#### Quantitative relationship between the FE & the PN

 $\log_{10}(FactorExpansion) = -0.54 \cdot \log_{10}(PrecursorNumber) + 3.4$ 





- There is a log-linear relation between CD4<sup>+</sup> T-cell precursor number (PN) and factor of expansion (FE), with a slope of ~ -0.5 over a physiological range of 3-30,000 precursors per mouse
- The 'feedback-regulated balance of growth and differentiation' is a valid concept which can explain the kinetics of CD4<sup>+</sup> T-cell responses to antigenic stimulation

## Lymph node center of immune responses



Nat Rev Immunol. 2012 ;12(11):762-73. HEVs, lymphatics and homeostatic immune cell trafficking in lymph nodes. Girard JP<sup>1</sup>, Moussion C, Förster R.







## Imaging and visualization of LN structures



Lymph node (LN) architecture with major structural units: subcapsular sinus (red), efferent lymphatic vessel (red), B cell follicles (cyan) and the FRC network (green). White rectangle indicates representative T cell zone. Scale bar represents 200 µm

**Data from: Burkhard Ludeiwg, Mario Novkovic, Lucas Onder** Institute of Immunobiology, Kantonsspital St. Gallen, St. Gallen

## Topology - geometry - solid model



Voxel-based generation of the FRC network. The network graph model; initial local structure; smoothed solid model of the local structure.

# 3D anatomically correct constructive solid geometry model of the lymph node

#### Experimental data1



FRC network topology parameters: the edges per node- and the edge length for the T cell zone FRC network. Data obtained from n = 7 mice from two independent experiments

## Modeling FRC network topology



# Computation of the effective permeability of the FRC network



## 3D geometric model of the LN



The FRC network inside



B cell follicles, trabecular sinuses medulla

## Blood microvascular network of LN

С



SCIENTIFIC REPORTS

OPEN Organ-wide 3D-imaging and topological analysis of the continuous microvascular network in a murine lymph node https://www.areautor.com/areautor/areauto

# $40 \,\mu m$ section of the integrated vascular networks



### **Research directions**

- Modelling the structure-function relationships for lymphoid organs
- Anatomically based mathematical models integrating the regulatory processes across multiple scales (multi-physics)
- Translational studies on data-driven application of models for predicting the immune function in virus infections (HIV, LCMV)

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# Thank you very much for your attention!

