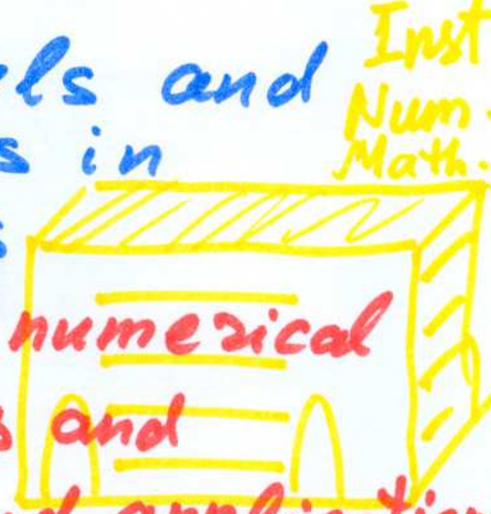
7 th conference on mathematical models and numerical methods in biomathematics



Special Session on numerical methods for viscous and elastic media and applications to Biomathematics

Moscow, Russia, 30 oct. 2015

G. Panasenko (Univ St Etienne)

Asymptotic partial decomposition for multistructures

ICJ UMR CNRS 5208

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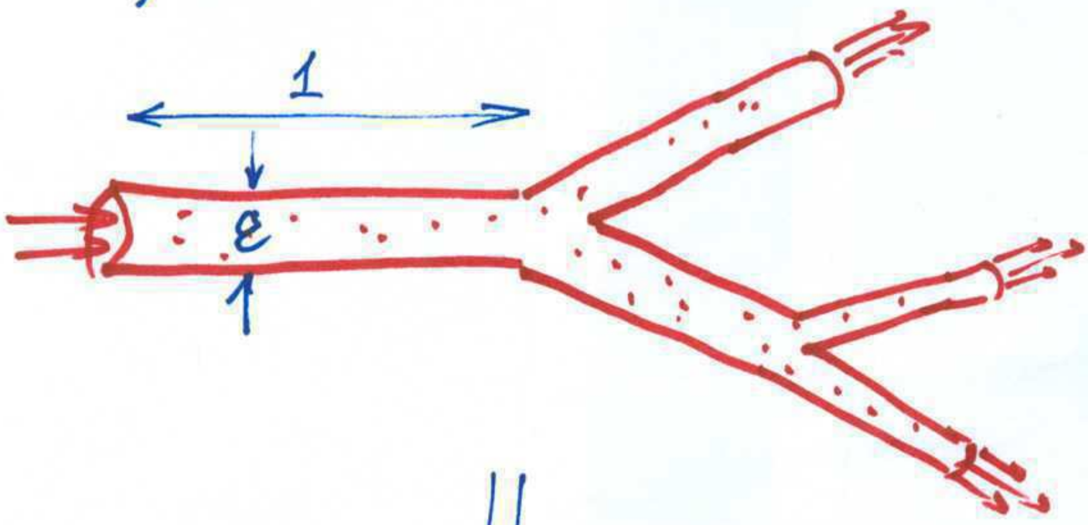
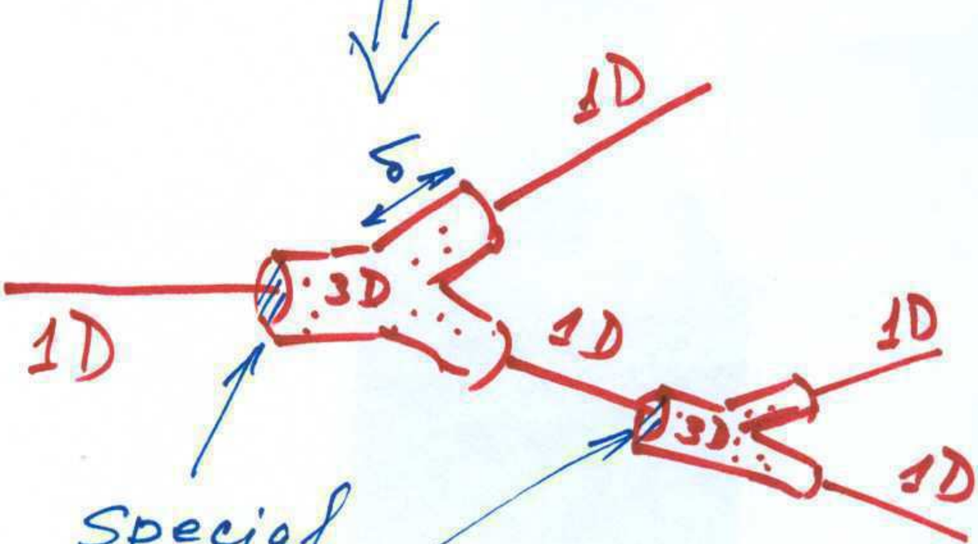
Outline

- Motivation
- Thin Structures
- Steady Navier-Stokes
  - existence
  - asymptotic exp.
  - eq. on the graph
  - estimate
- Non-steady Navier-Stokes
  - existence
  - as. expansion
  - eq. on the graph
- Partial dimension reduction.

Motivation: partial dimension reduction:

- blood circulation
- hydraulic installations





Special junction conditions:

$\delta = ?$

$\forall N$ error $O(\varepsilon^N)$

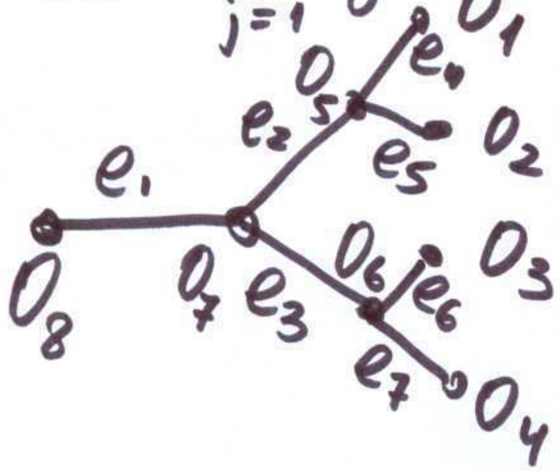
Definition of a thin structure

1. Graph. $O_1, \dots, O_N \in \mathbb{R}^n$, $n \in \{2,3\}$

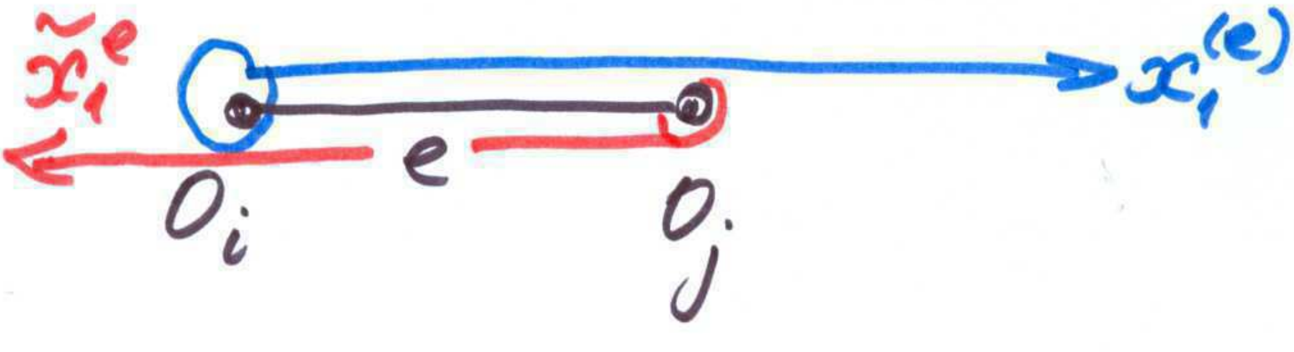
$e_1, \dots, e_M$ closed segments

$e_j = \overline{O_{i_j} O_{k_j}}$, $e_{j_1} \cap e_{j_2} \in \{O_1, \dots, O_N\}$.

$\mathcal{B} = \bigcup_{j=1}^{M} e_j$



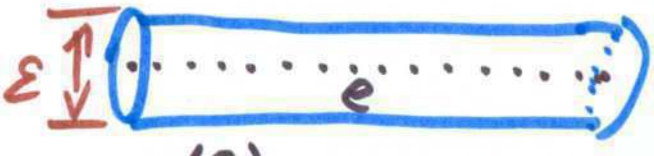
2. Local coordinate systems



3. Thin structure

$e_j = e$

$\sigma^j = \sigma^{(e)}$



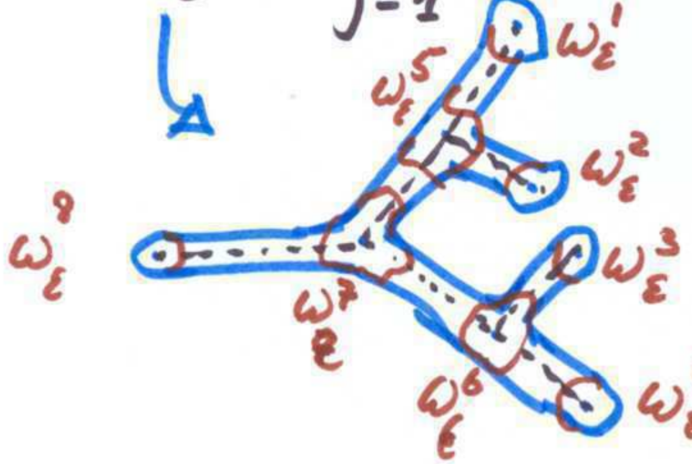
$$B_\varepsilon^{(e)} = \left\{ x^{(e)} \;\middle|\; x_1^{(e)} \in (0, |e|),\ \frac{x^{(e)\prime}}{\varepsilon} \in \sigma^{(e)} \right\}$$

ω^i bounded domains in $\mathbb{R}^n$

$i = 1, \dots, N$

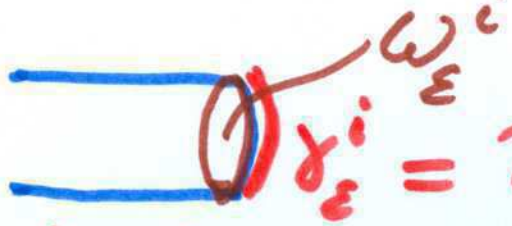
$$\omega_\varepsilon^i = \left\{ x \in \mathbb{R}^n \;\middle|\; \frac{x - O_i}{\varepsilon} \in \omega^i \right\}$$

$$B_\varepsilon = \left(\bigcup_{j=1}^{M} B_\varepsilon^{(e_j)} \right) \cup \left(\bigcup_{j=1}^{N} \omega_\varepsilon^j \right)$$



connected

$\partial B_\varepsilon \in C^2$



$\gamma_\varepsilon^i = \partial\omega_\varepsilon^i \cap \partial B_\varepsilon$,

$i = N_1+1, \dots, N$: O_i vertices.

4. Navier-Stokes (steady)

$$\begin{cases} -\nu\Delta\vec{u}_\varepsilon + (\vec{u}_\varepsilon, \nabla)\vec{u}_\varepsilon + \nabla P_\varepsilon = f_\varepsilon, \\ \operatorname{div}\vec{u}_\varepsilon = 0, \quad x \in B_\varepsilon \\ \vec{u}_\varepsilon = \varepsilon^2 \vec{g}_i\left(\frac{x-O_i}{\varepsilon}\right), \quad x \in \gamma_\varepsilon^i \\ \vec{u}_\varepsilon = 0, \quad x \in \partial B_\varepsilon \setminus \gamma_\varepsilon, \\ \gamma_\varepsilon = \bigcup_{i=N_1+1}^{N} \gamma_\varepsilon^i . \end{cases}$$

$$f_\varepsilon(x) = f_i\left(\frac{x-O_i}{\varepsilon}\right), \quad i = 1, \dots, N_1$$

f_i, g_i compact support functions,

g_i vanishes at the bound. of γ_ε^i

smooth $\sum_i \int_{\gamma_\varepsilon^i} \vec{g}_i \cdot \vec{n} = 0$.

5. Th.1. $\exists!$ sol. (for suff. small ε).

6. Asymptotic expansion.

$$u_\varepsilon^{(J)} = \sum_{l=2}^{J} \varepsilon^{l} \left\{ \sum_{j=1}^{M} u_l^{e_j}\left(\frac{x^{(e_j)\prime}}{\varepsilon}\right) \zeta_1 \zeta_2 + \sum_{i=1}^{N} u_l^{BLO_i}\left(\frac{x-O_i}{\varepsilon}\right)(1-\zeta_3) \right\}$$

Boundary layers

$$p_\varepsilon^{(J)} = \sum_{l=2}^{J} \varepsilon^{l-2} \sum_{j=1}^{M} p_l^{e_j}(x_1^{(e_j)}) \zeta_1 \zeta_2 + \sum_{l=1}^{J} \varepsilon^{l-1} \sum_{i=1}^{N} p_l^{BLO_i}\left(\frac{x-O_i}{\varepsilon}\right)(1-\zeta_3)$$

$\zeta_1, \zeta_2, \zeta_3$ are cut-off functions

$(u_l^{(e)}, p_l^{(e)})$ Poiseuille flow in infinite tube $\mathbb{R} \times \sigma^{(e)}$

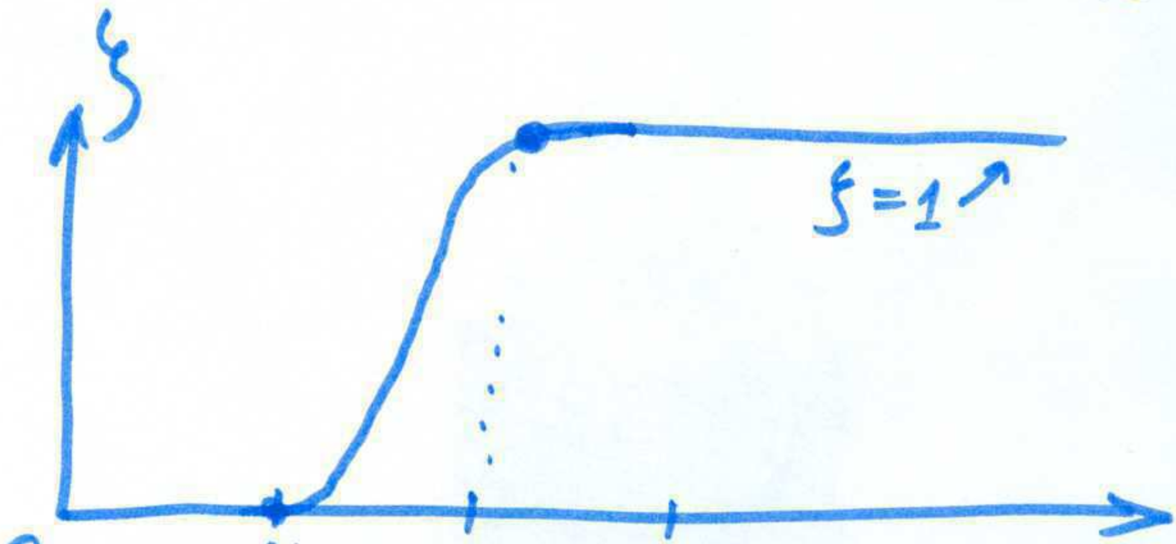
Namely

$$\zeta_1 = \zeta\left(\frac{x_1^{(e_j)}}{3r\varepsilon}\right), \quad \zeta_2 = \zeta\left(\frac{|e_j| - x_1^{(e_j)}}{3r\varepsilon}\right),$$

$$\zeta_3 = \zeta\left(\frac{x - O_i}{e_{min}}\right),$$ where

$$\zeta(\tau) = \begin{cases} 0, & \tau \le 1/3 \\ 1, & \tau \ge 2/3 \end{cases}, \quad 0 \le \zeta \le 1$$

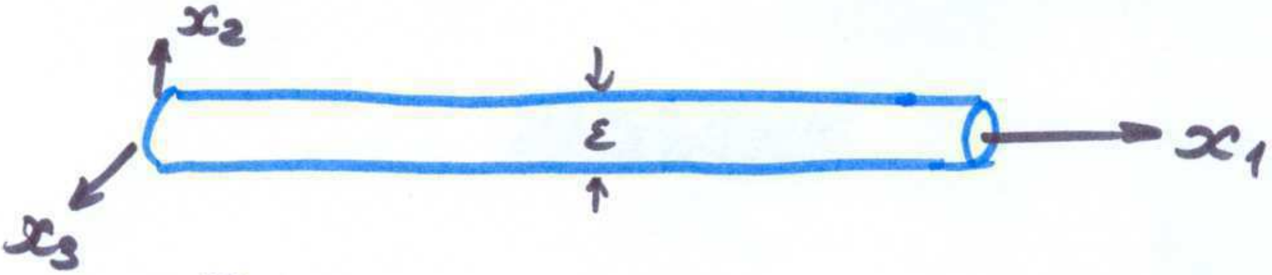
smooth

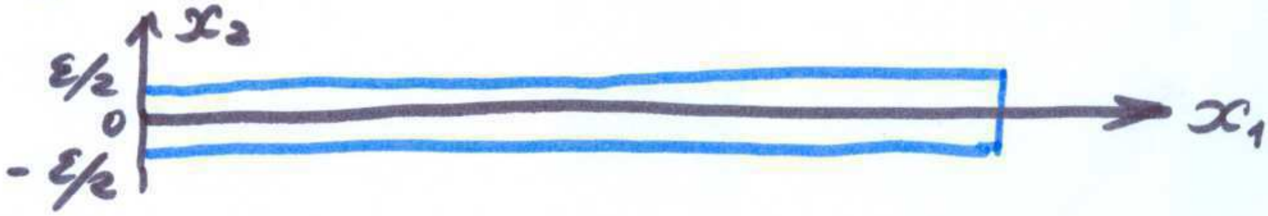


$r = \max \operatorname{diam} \sigma^j$

$e_{min} = \min |e_j|$

Poiseuille flow
in an infinite tube (channel)





$$\vec{V}_p(x) = \begin{pmatrix} u_p(x_2, x_3) \\ 0 \\ 0 \end{pmatrix} \qquad (n=3)$$

$$\vec{V}_p(x) = \begin{pmatrix} u_p(x_2) \\ 0 \end{pmatrix} \qquad (n=2)$$

$$u_p(x_2) = \frac{a}{2\nu}\left(x_2^2 - \left(\frac{\varepsilon}{2}\right)^2\right) \qquad (n=2)$$

$$u_p(r) = \frac{a}{4\nu}\left(r^2 - \left(\frac{\varepsilon}{2}\right)^2\right) \qquad (n=3)$$

or

$$\nu \Delta' u_p = a \text{ on } \sigma, \quad u_p|_{\partial\sigma} = 0$$

$$P_p = a x_1 + b$$

In local var. the 1st comp. U of $u_\ell^{(e)}$:

$$\begin{cases} -\nu \Delta' U(y') + \boxed{c_\ell^{(e)}} = 0, \quad y' \in \sigma^{(e)} \\ U|_{\partial\sigma^{(e)}} = 0, \quad \boxed{c_\ell^{(e)} = \text{const}} \end{cases}$$

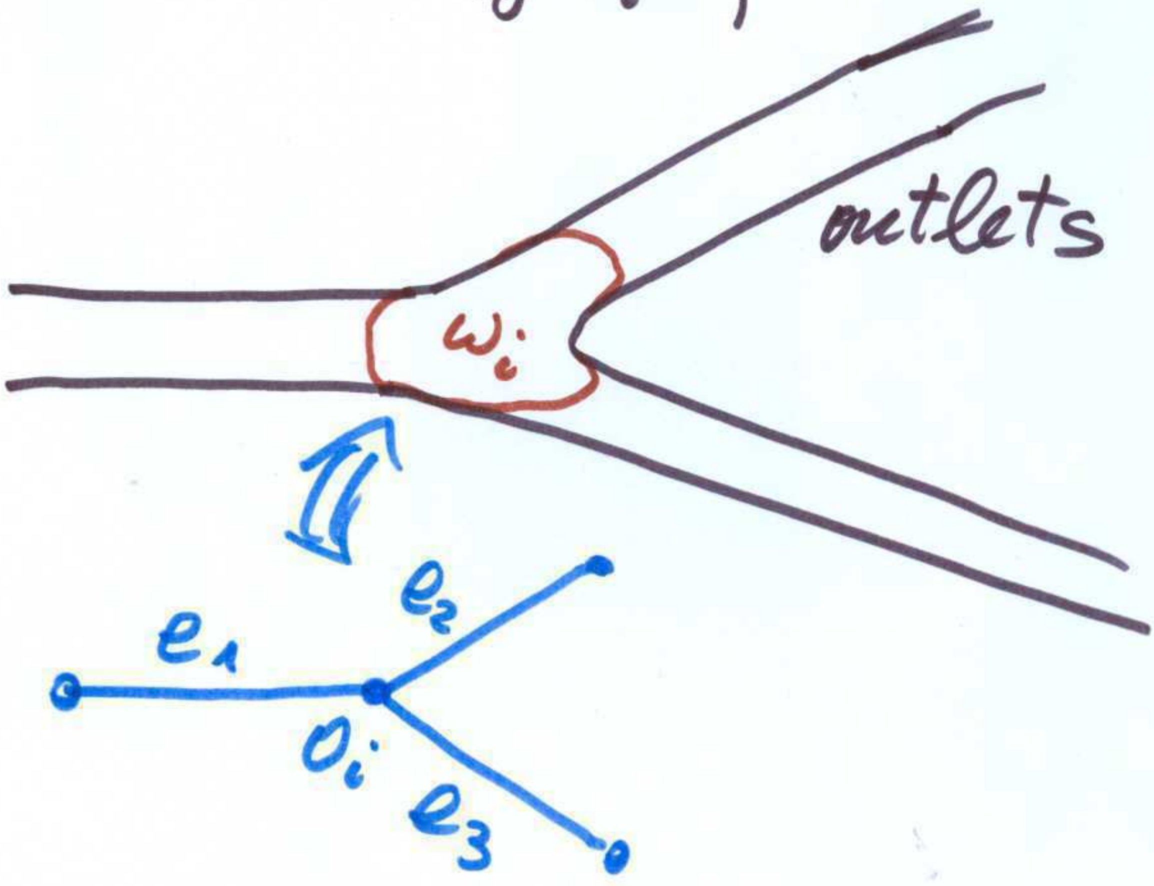
and

$$P_\ell(x_1^{(e)}) = \boxed{c_\ell^{(e)}}\, x_1^{(e)} + \underbrace{\boxed{d_\ell^{(e)}}}_{\text{const}}$$

$$\left(\vec{U}_\ell^{BLO_i}, P_\ell^{BLO_i}\right)$$ Bound. layer solution

to the boundary layer pbl in

$$\Omega_i = \omega_i \cup \left\{ \bigcup_{j: O_i \in e_j} \left\{ \xi^{(e_j)} \in \mathbb{R}_+ \times \sigma^{(e_j)} \right\} \right\}$$



$$\begin{cases} -\Delta u_\ell^{BLO_i}(\xi) + \nabla p_\ell^{BLO_i}(\xi) = F_{i\ell}(\xi), \\ \qquad \xi \in \Omega_i \\ \operatorname{div} u_\ell^{BLO_i} = \Psi_{i\ell}(\xi) \\ u_\ell^{BLO_i} = 0, \quad \xi \in \partial\Omega_i \\ \text{or } = g_i(\xi)\,\delta_{\ell 2} \text{ on } \partial\Omega_i \cap \partial\omega_i \\ \text{for } i = N_1+1, \dots, N. \end{cases}$$

$F_{i\ell}, \Psi_{i\ell} \in L^2(\Omega_i)$, $\exp \searrow 0$.

$\exists!$ solution (up to an additive constant for $p_\ell^{BLO_i}$) such that

$$u_\ell^{BLO_i} \xrightarrow[\exp]{} 0, \quad p_\ell^{BLO_i} \xrightarrow[\exp]{} const = \alpha_{ij} \quad |\xi| \to +\infty$$

iff $\int_\Omega \Psi_{i\ell}\, d\xi = 0$.

$\Leftrightarrow$ condition on $\zeta_l^{(e)}$:

$$-\sum_{e:\,O_i\in e}\left(P_l^e(x_1^{(e)})\right)' \varkappa^e = \widehat{\Psi}_{il}$$

where $\widehat{\Psi}_{il}$ known,

$$\varkappa^e = \int_{\sigma^e} \widetilde{u}^e(y')\,dy',$$

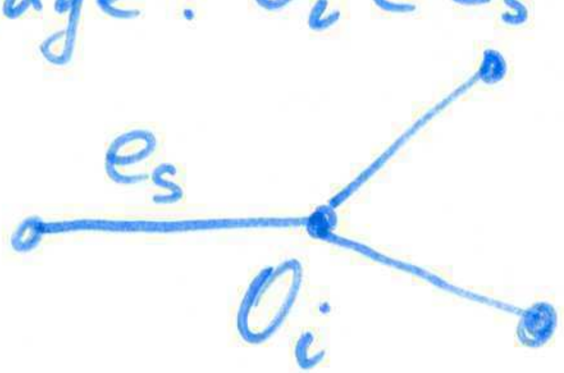
$$\begin{cases} -\Delta \widetilde{u}^e + 1 = 0, & y' \in \sigma^e \\ \widetilde{u}^e\big|_{\partial\sigma^e} = 0 & . \end{cases}$$

Remark. For any l we may chose $d_{l+1}^{(e)}$: $P_l^{BLO_i} \xrightarrow{\exp} 0$

Solve an auxiliary pbl. on the graph $\mathcal{B}$:

$$\begin{cases} -\varkappa^e {p_l^e}'' = 0 \quad \forall e \subset \mathcal{B} \\ -\sum\limits_{e:O_i \in e} \varkappa^e \left(p_l^e(x_1^{(e)})\right)' = \hat{\psi}_{il}, \\ \qquad\qquad i = 1, \dots, N \\ p_l^e(0) = p_l^{e_s}(0) + \hat{\varphi}_{il}^e, \ \forall e : O_i \in e, \\ \qquad\qquad i = 1, \dots, N_1 \end{cases}$$

$\hat{\varphi}_{il}^e$ known, e_s a selected edge : $O_i \in e_s$.



$$\boxed{\sum_i \hat{\psi}_{il} = 0}$$

7. Justification.

- Calculus of the residuals
- Application of the a priori estimate

$\Downarrow$

$$\boxed{\text{Th. 2. } \|\vec{u}_\varepsilon^{(J)} - \vec{u}_\varepsilon\|_{H^1(B_\varepsilon)} \leq C \varepsilon^J \sqrt{\operatorname{mes} B_\varepsilon}}$$

Conclusion: structure of as. exp. is Poiseuille + exp. decaying Boundary layers

→ G.P. CRAS, 326, II b, 1998, 867

→ G.P. CRAS, 326, II b, 1998, 893

→ F. Blanc, O. Gipouloux, G.P., A.M. Zine
M3AS, 9, 9, 1999, 1351

→ G.P. "Multiscale Modelling for
Structures and Composites"
Springer, 2005

8. Non-steady Navier–Stokes eqs

G.P., K. Pileckas

$$
\begin{cases}
\dfrac{\partial \vec{u}_\varepsilon}{\partial t} - \nu \Delta \vec{u}_\varepsilon + (\vec{u}_\varepsilon, \nabla) u_\varepsilon + \nabla p_\varepsilon = 0 \\
\operatorname{div} \vec{u}_\varepsilon = 0, \quad x \in B_\varepsilon \\
\vec{u}_\varepsilon = \vec{g}_i\left(\dfrac{x - O_i}{\varepsilon}, t\right), \quad x \in \gamma_\varepsilon^i \\
\vec{u}_\varepsilon = 0, \quad x \in \partial B_\varepsilon \setminus \gamma_\varepsilon, \quad \gamma_\varepsilon = \bigcup\limits_{i=N_1+1}^{N} \gamma_\varepsilon^i \\
\vec{u}_\varepsilon \text{ is } T\text{-periodic in } t
\end{cases}
$$

$$\vec{g}_i \in C^{\left[\frac{3+1}{2}\right]+1}\left([0,T]; W_2^2\right),$$

$$\operatorname{div}_{x'} \vec{g}_i = 0, \quad \sum_{i=N_1+1}^{N} \int_{\gamma_\varepsilon^i} \vec{g}^i \cdot \vec{n} = 0,$$

$\nu > 0$.

Th 3. $\exists!$ sol. (for suff. small ε).

Definition of a solution:

Let g be an extension of g_i to B_ε Such that $\forall\, t \in [0,T]$

$$\begin{cases} \|g\|_{L^2} + \|g_t\|_{L^2} + \|g_{tt}\|_{L^2} \le c\varepsilon^{\frac{n-1}{2}} \\ \|\nabla g\|_{L^2} + \|\nabla g_t\|_{L^2} \le c\varepsilon^{\frac{n-3}{2}} \\ \|\Delta g\|_{L^2} \le c\varepsilon^{\frac{n-3}{2}}, \quad n=2,3, \quad \operatorname{div} g = 0 \end{cases}$$

(we prove that such an extension $\exists$).

$(L^2 = L^2(B_\varepsilon))$. Then $u_\varepsilon = w + g$,

$\operatorname{div} w = 0$, $w \in L^2(0,T; H_0^1(B_\varepsilon)) \cap$
$\underset{T\text{-per.}}{\cap}\, L^\infty(0,T; H_0^1(B_\varepsilon))$, $w_t \in L^2(0,T; L^2(B_\varepsilon))$

$$\int_{B_\varepsilon} \big(w_t \cdot \eta + \nu \nabla w \cdot \nabla \eta - ((w+g)\cdot\nabla)\eta \cdot w -$$

$$- (w\cdot\nabla)\eta \cdot g\, dx = -\nu \int_{B_\varepsilon} \nabla g \cdot \nabla \eta\, dx$$

$$- \int_{B_\varepsilon} ((g\cdot\nabla) g + g_t)\cdot \eta\, ds \qquad \text{(N-S)}$$

$\forall \eta \in H_0^1(B_\varepsilon)$, $\operatorname{div} \eta = 0$

Main steps: as. expansion

- construction of the time-dep. Poiseuille flow for given flux $F(t)$ (G.P., K.Pileckas, Applic. An., 2011)

$$V_{long} = \sum_{k=0}^{J/2} \varepsilon^{2k} \widetilde{V}_{2k}\left(\frac{x'}{\varepsilon}, t\right)$$

$$V_\perp = 0; \; p = -s(t) x_n + a(t)$$

$$s(t) = \sum_{k=0}^{J/2} \varepsilon^{2k-2} s_{2k}(t), \; J = 2K$$

$$V_{2k}(y,t) = s_{2k}(t) U_0(y) + \frac{d s_{2k-2}}{dt} U_2(y) + \\ + \dots + \frac{d^k s_0(t)}{dt^k} U_{2k}(y)$$

$$s_0(t) = \frac{1}{\varkappa_0} F(t)$$

$$s_{2k} = -\varkappa_0^{-1} \varkappa_2 \frac{d s_{2k-2}}{dt} - \dots - \varkappa_0^{-1} \varkappa_{2k} \frac{d^k s_0}{dt^k}$$

$$\begin{cases} -\nu \Delta' U_0(y) = 1, & y \in \sigma \\ U_0\big|_{\partial\sigma} = 0 \end{cases}$$

$$\begin{cases} -\nu \Delta' U_{2r}(y) = -U_{2r-2}, & y \in \sigma \\ U_{2r}\big|_{\partial\sigma} = 0 \end{cases}$$

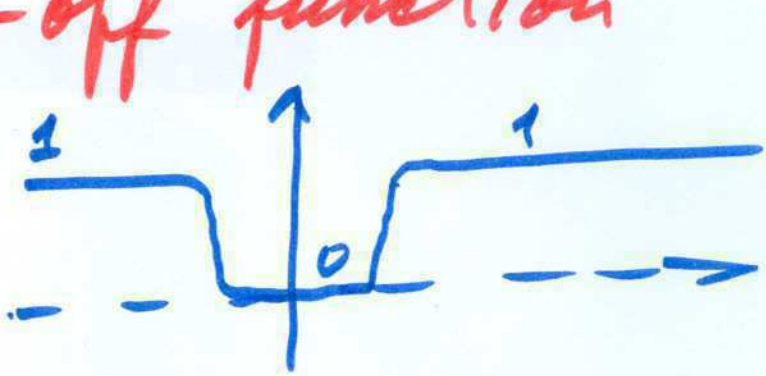
$$\varkappa_{2s} = \int_\sigma U_{2s}(y)\,dy \neq 0, \quad s \geq 0$$

How we calculate flows H?

Problem on the graph for macroscopic pressure.

G.P., K.Pileckas JMP, 2014

- Multiplication of the Poiseuille by a cut-off function



- Construction of the boundary layers-in-space (near the nodes)

- Construction of the boundary layers-in-time (small t, dep. on t/ε^2).

- Evaluation of the residual

- Estimate $\boxed{\|u^a - v\|_{H^{1,0}} = O(\varepsilon^J)}$

Th 4.

Equation on the graph

$H^1(\mathcal{B}) = \{$ continuous on $\mathcal{B}$ functions, $\forall e_j$ they $\in H^1(0, |e_j|)\}$,

$$(p, q)_{H^1(\mathcal{B})} = \sum_{j=1}^{M} \int_0^{|e_j|} \left(p^{(e_j)} q^{(e_j)} + \frac{\partial p^{(e_j)}}{\partial l} \frac{\partial q^{(e_j)}}{\partial l} \right)$$

Let Ψ_l, $l = 1, \dots, N$ real cstes,

$\forall e_j$, $\varkappa_{e_j} > 0$, $F^{(e_j)} \in L_2(\mathcal{B})$.

Find $p \in H^1(\mathcal{B})$:

$$\begin{cases} -\dfrac{\partial}{\partial l}\left(\varkappa_{e_j} \dfrac{\partial p}{\partial l}(l)\right) = F^{(e)}, \ x \in e_j \\ -\displaystyle\sum_{e : O_i \in e} \left(\varkappa_e \dfrac{\partial p}{\partial l}\right) = \Psi_i, \ i = 1, \dots, N \end{cases} \quad (1)$$

Option: $p \in H^1(e_j)$ + prescribed jumps in the nodes

$\Rightarrow$ reduced to (1) by a change.

Let $\boxed{\sum_e \int_0^{|e|} F^{(e)} + \sum_{i=1}^{N} \Psi_i = 0}$ NSC

Theorem 1. $\exists$ a unique (up to an additive constant) solution p to (1). iff NSC holds.

Proof: Lax-Milgram arguments.

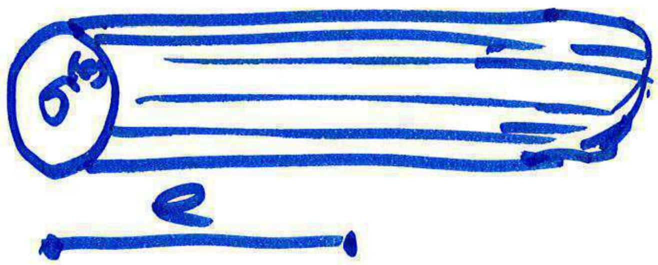
Time dependent equation on the graph

Operator $L^{(e)}$ relating the pressure drop and the flux in an infinite tube.

Given $S \in L_2(0,+\infty)$ find

$V \in L_2(0,+\infty; H_0^1(\sigma^{(e)}))$ with

$\frac{\partial V}{\partial t} \in L_2(0,+\infty; L_2(\sigma^{(e)}))$



satsfying the heat eq.

$$\begin{cases} \frac{\partial V}{\partial \tau}(y',\tau) - \nu \Delta' V(y',\tau) = S(\tau),\ y' \in \sigma^{(e)},\ \tau>0 \\ V(y',\tau)\big|_{\partial\sigma^{(e)}} = 0,\ \tau>0, \\ V(y',0) = 0,\ y' \in \sigma^{(e)} \end{cases}$$

pressure drop

$y' = (y_1, \dots, y_{n-1})$.

flux

Denote

$$L^{(e)} S = \int_{\sigma^{(e)}} V(y',\tau)\, dy'.$$

Given $\psi_l(\tau) \in H_0^1(0,+\infty)$, $l = 1,\dots,N$,

$F^{(e_j)} \in H_0^1(0,+\infty; L_2(\mathcal{B}))$, $j = 1,\dots M$,

find $p \in L_2(0,T; H^1(\mathcal{B}))$:

$$
\left\{
\begin{aligned}
& -\frac{\partial}{\partial \ell}\left(L^{(e)}\frac{\partial p}{\partial \ell}\right) = F^{(e)}, \quad x \in e, \\
& -\sum_{e:\, O_i \in e}\left(L^{(e)}\frac{\partial p}{\partial \ell}\right) = \Psi_i(\tau), \quad i = 1, \dots, N
\end{aligned}
\right. \qquad (2)
$$

Option: prescribed jumps of p in the nodes.

$$
\boxed{\sum_e \int_0^{|e|} F^{(e)} + \sum_{i=1}^{N} \Psi_i = 0} \quad \text{NSC}
$$

for almost all $\tau \in (0, T)$.

Theorem 2. $\exists$ a unique solution p vanishing in O_N to pbl. (2) iff NSC holds.

$\not\mathbb{Z}$

Exponential decay in time.

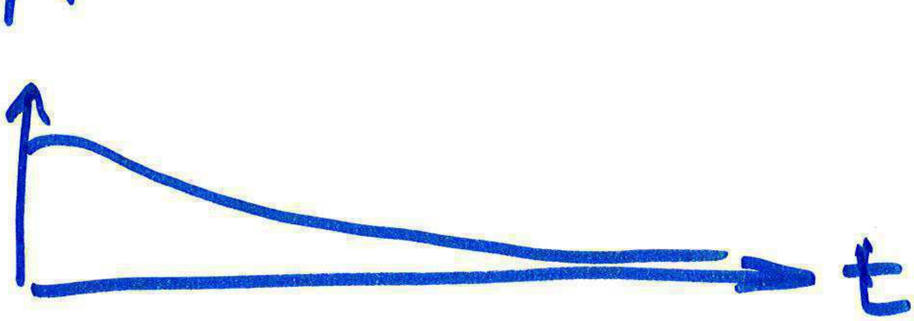
$$\mathcal{L}_{2\beta}(0,+\infty)=\left\{f\in L_2(0,+\infty)\,\Big|\,\int_0^{+\infty}|f(\tau)|^2e^{2\beta\tau}d\tau<+\infty\right\}$$

$$H^1_\beta(0,+\infty)=\left\{f\in H^1(0,+\infty)\,\Big|\,f,f'\in\mathcal{L}_{2,\beta}(0,+\infty)\right\}$$

$$H^1_{0,\beta}(0,+\infty)=H^1_\beta(0,+\infty)\cap\left\{f\big|_{t=0}=0\right\}$$

Theorem 3. Let p be solution to (2) for all $T>0$, $\psi_i\in H^1_{0,\beta}(0,+\infty)$, $i=1,\dots,N$, $F^{(e_j)}\in H^1_{0\beta}(0,+\infty;L_2(e_j))$, $j=1,\dots,M$, $\beta>0$. Then $\exists\,\beta_1\in(0,\beta)$ such that

$$p\in\mathcal{L}_{2\beta_1}(0,+\infty;H^1(\mathcal{B})).$$

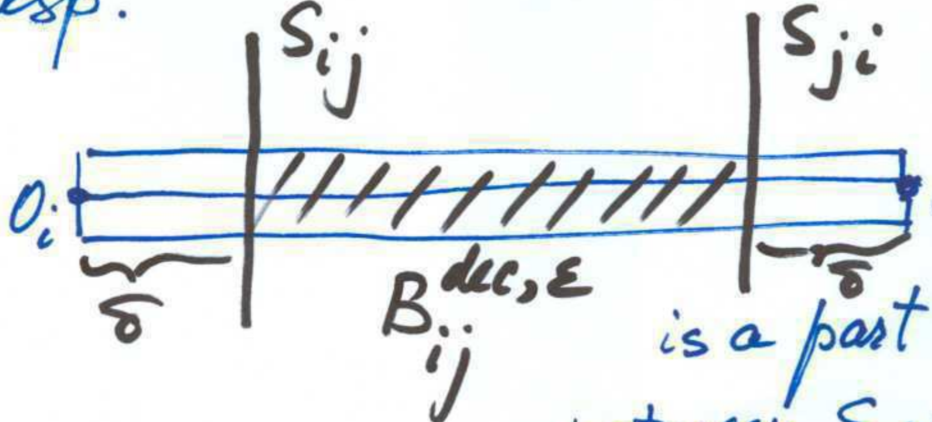


MAPDD for N-S
(suppsing $g=0$ for small t)

- Projection of the variational formulation on the space of H^1 divergence free vector-valued functions equal to the non-steady Poiseuille at some distance $\delta \sim \varepsilon|\ln\varepsilon|$ from the nodes

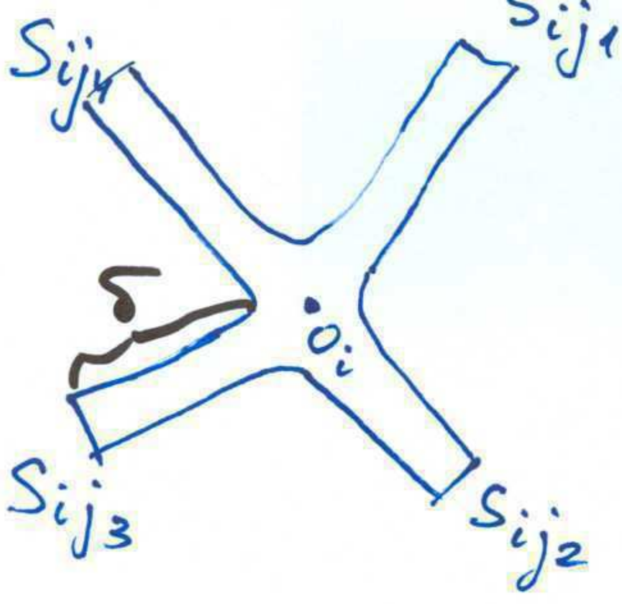
Accuracy: $O(\varepsilon^J)$.

$\forall$ edge $e = \overline{O_i O_j}$ introduce 2 hyperplanes S_{ij} and $S_{ji} \perp e$ at the distance δ from O_i and O_j resp.



is a part of the cyl. between S_{ij} and S_{ji}

Remaining part of B_ε consists of connected sets $B_i^{\varepsilon,\delta} \ni O_i$



Consider the subspace $H^J_{dec}(B_\varepsilon)$ of the space $\{H^1_0(B_\varepsilon) \mid div = 0\}$ such that $\forall\ B^{dec\,\varepsilon}_{ij}$, vector function v has a form $\begin{pmatrix} \sum\limits_{\alpha=0}^{[J/2]} a_{2\alpha} U_{2\alpha}\left(\frac{\tilde{x}'}{\varepsilon}\right) \\ 0 \\ \vdots \\ 0 \end{pmatrix}$ in local coordinates, x' are transversal local coordinates and $U_{2\alpha}$ are solutions of problems:

$$\begin{cases} -\nu \Delta' U_0(y') = 1, \ y' \in \sigma \\ U_0|_{\partial\sigma} = 0 \end{cases}$$

$$\begin{cases} -\nu \Delta' U_{2r}(y') = -U_{2r-2}(y'), \ y' \in \sigma \\ U_{2r}|_{\partial\sigma} = 0, \quad r \geq 1. \end{cases}$$

Define partially decomposed problem:

Find w_d: $\operatorname{div} w_d = 0$, $w_d \in L^2(0,T; H^J_{dec}) \cap L^\infty(0,T; H^J_{dec})$, $w_{dt} \in L^2(0,T; L^2(B_\varepsilon))$ satisfying (NS) $\forall \eta \in H^J_{dec}$.

$u_d := w_d + g$.

<u>Theorem 1</u>. $\exists!$ solution (for sufficiently small ε).

<u>Theorem 2</u>. $\| u_\varepsilon - u_d \| = \| w - w_d \| = $

$$= O(\varepsilon^{J-2}),$$

where $\| \varphi \| = \sup_{0 \le t \le T} \| \varphi \|_{L^2(B_\varepsilon)} + \| \nabla \varphi \|_{L^2(0,T; L^2)}$,

$$\boxed{\delta = \text{const} \cdot J \cdot \varepsilon |\ln \varepsilon|}$$

Tree - like graph.

In this case in every node we can write the flux balance:

$$\sum F_j = 0$$

while for the vertices we get

$$F_j = \int_{\sigma_\varepsilon^j} g_j \vec{n}\, ds$$

This system defines all fluxes F_j.

For any e, on $B_{ij}^{dec\,\varepsilon}$ we can calculate u_d explicitely (see G.P., K. Pileckas, Applic. Analysis, 2011): in local variables the longitudinal component of the velocity

$$v_{long} = \sum_{k=0}^{[J/2]} \varepsilon^{2k} \widetilde{V}_{2k}\left(\frac{x'}{\varepsilon}, t\right),$$

$$V_{2k}(y,t) = s_{2k}(t)\,U_0(y) + \frac{d s_{2k-2}}{dt} V_2(y) + \cdots + \frac{d^k s_0(t)}{dt^k} U_{2k}(y), \qquad (*)$$

where $s_0(t) = \frac{1}{\varkappa_0} F_j(t)$,

$$s_{2k} = -\varkappa_0^{-1}\varkappa_2 \frac{d s_{2k-2}}{dt} - \dots - \varkappa_0^{-1}\varkappa_{2k} \frac{d^k s_0}{dt^k},$$

$$\varkappa_{2s} = \int_\sigma U_{2s}(y)\,dy \neq 0 \quad (s \geq 0)$$

(Moreover, the pressure $p = -s(t)x_n + a(t)$,

$$s(t) = \sum_{k=0}^{[J/2]} \varepsilon^{2k-2} s_{2k}(t) \;).$$

So, in this case the problem is completely decomposed and on every $B_i^{\varepsilon,\delta}$ we get a standard N-S problem with Dirichlet's conditions given by (*). Computations may be parallelised!

Published in

G.P., K.P. Appliable An. 2012

G.P., K.P. Appliable An. 2014

G.P., K.P. JMP 2014

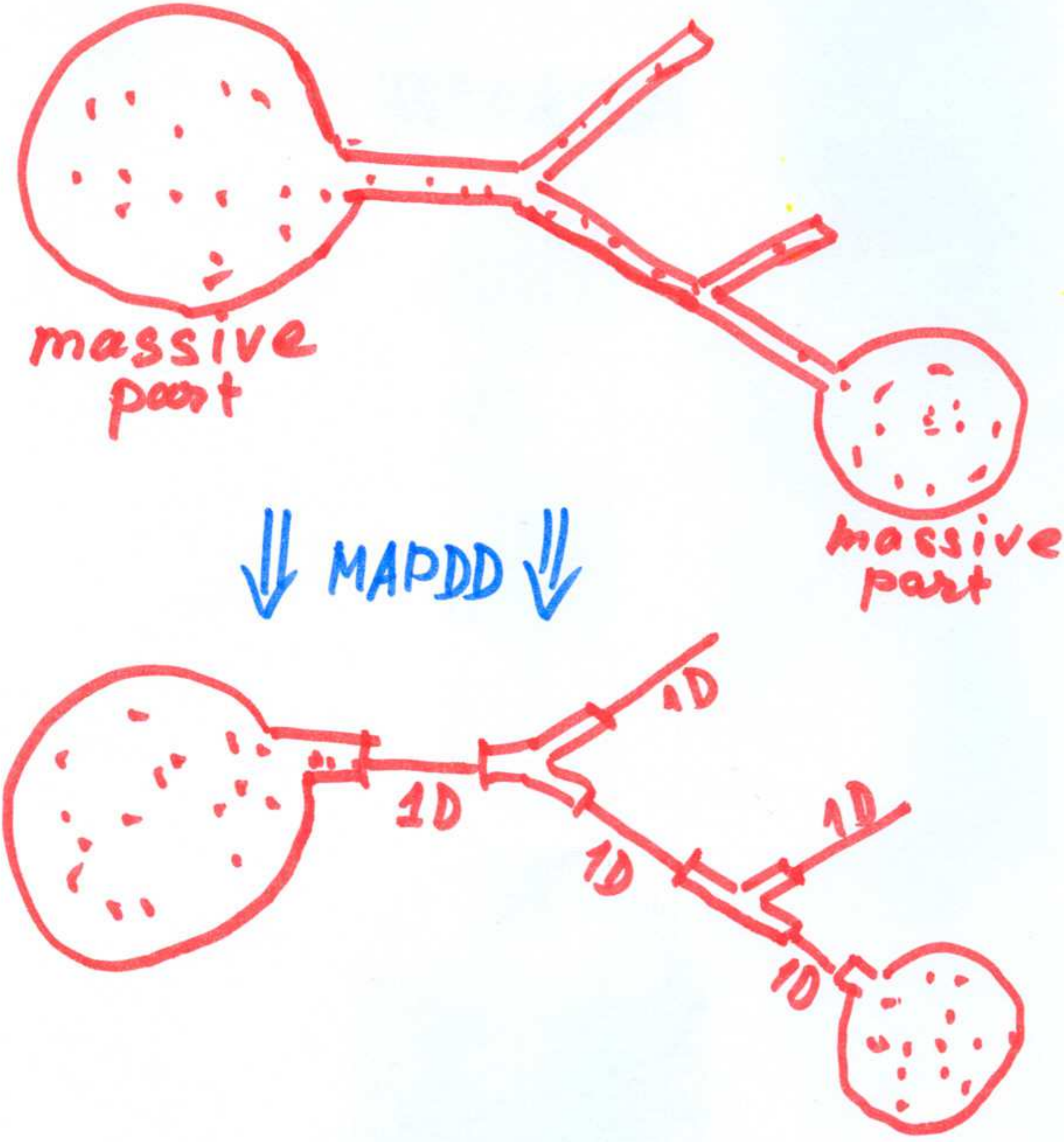
G.P., K.P. NATMA 2015

G.P., K.P. NATMA 2015

(G. Panasenko, K. Pileckas)

Generalizations.

Multistructures



Justification is in progress

NUMERICAL EXPERIMENT: MAPDD for non-Newtonian flows

$$-div(\nu(D\mathbf{v}_\varepsilon)D\mathbf{v}_\varepsilon) + \nabla p = 0,$$

$$div\mathbf{v}_\varepsilon = 0, \quad x \in B_\varepsilon,$$

$$\mathbf{v}_\varepsilon = \mathbf{g}_\varepsilon(x), \quad x \in \partial B_\varepsilon,$$

where $\mathbf{g}_\varepsilon = 0$ at the lateral part of the boundary and = inflow/outflow given function on the remaining part of the boundary, $D\mathbf{v} = \nabla\mathbf{v} + (\nabla\mathbf{v})^t$.

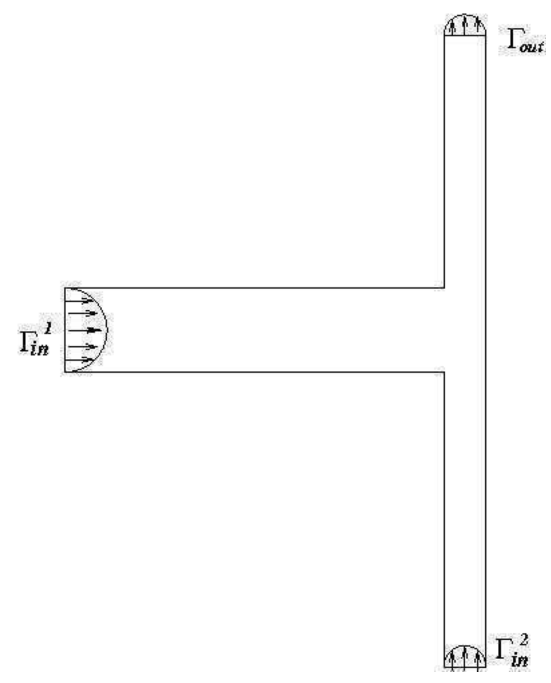
$$\nu(y) = M(1 + (\lambda y_{12})^2)^{\frac{n-1}{2}},$$

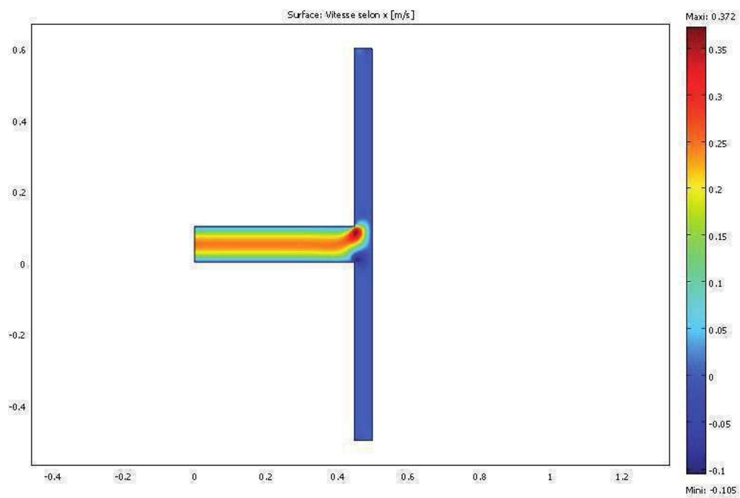
where for the blood we use $n = 0.7, \quad M = 7, \quad \lambda = 0.11$.

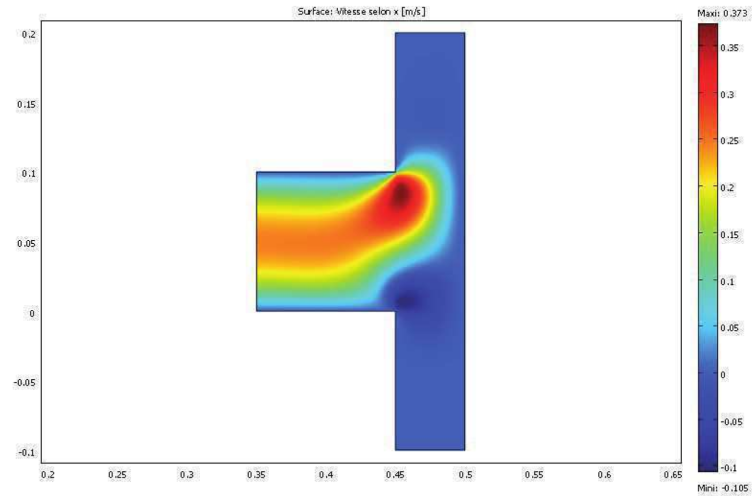
Theoretical estimate: $\delta = O(\varepsilon ln\varepsilon)$, numerical experiment: $\delta = \varepsilon$.

Projection: quasi-Poiseuille function, i.e. the velocity has only one component different from zero (the longitudinal one), and it depends on the transversal variables only, the pressure is linear and the equations are satisfied exactly as well as the no-slip condition at the lateral boundary for every rectangle (cylinder).

Consider three types of the geometry of the domain: the T-shaped one, the Y-shaped one, the YLLY-shaped geometry and compare the direct numerical solution with the MAPDD solution for the reduced geometries.


Γ_{out}
Γ_{in}^{1}
Γ_{in}^{2}


Surface: Vitesse selon x [m/s]
Maxi: 0.372
Mini: -0.105


Surface: Vitesse selon x [m/s]
Maxi: 0.373
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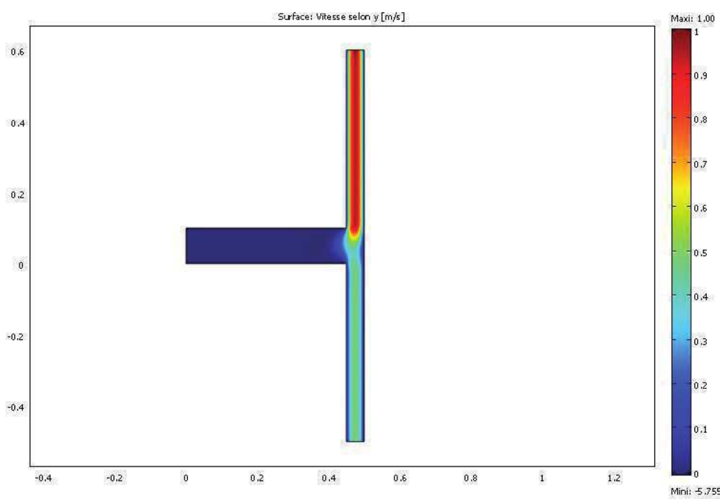
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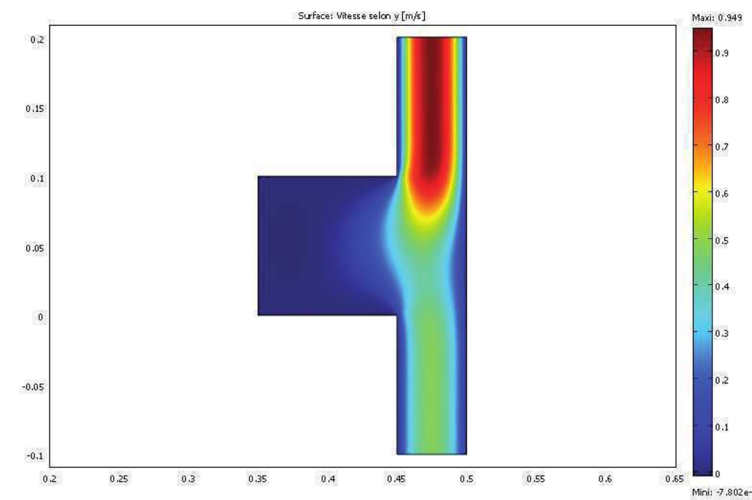
Mini: -5.755e-3


Surface: Vitesse selon y [m/s]
Maxi: 0.949
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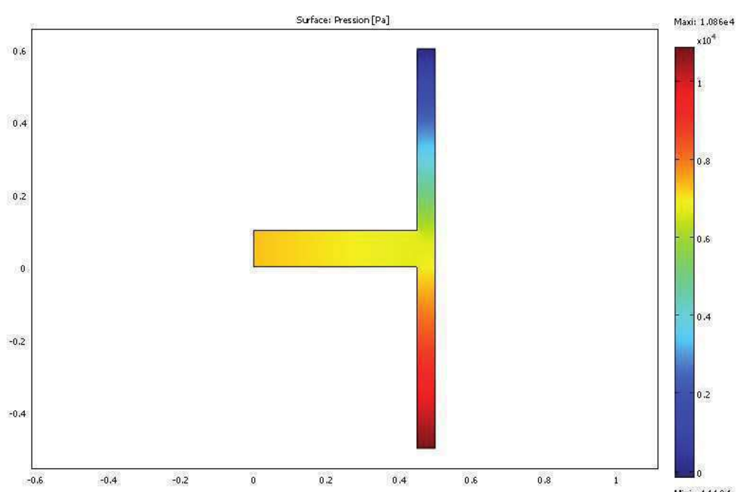
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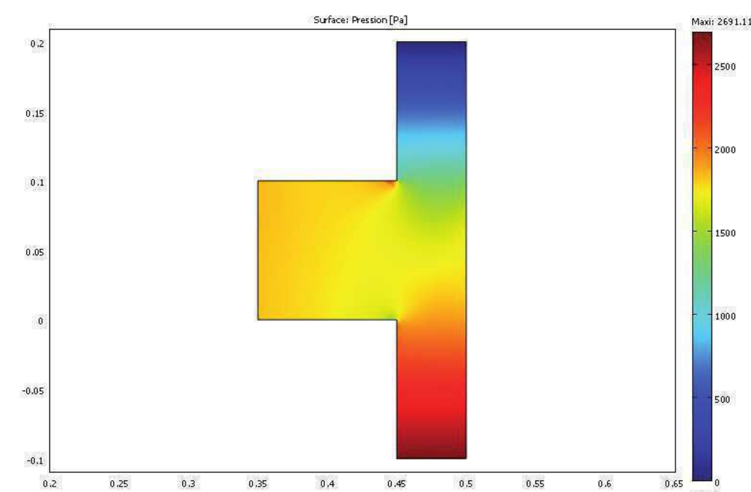
x10^4
Mini: -144.84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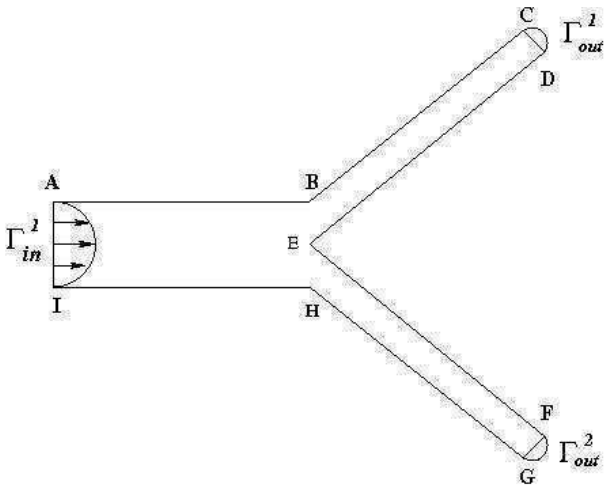
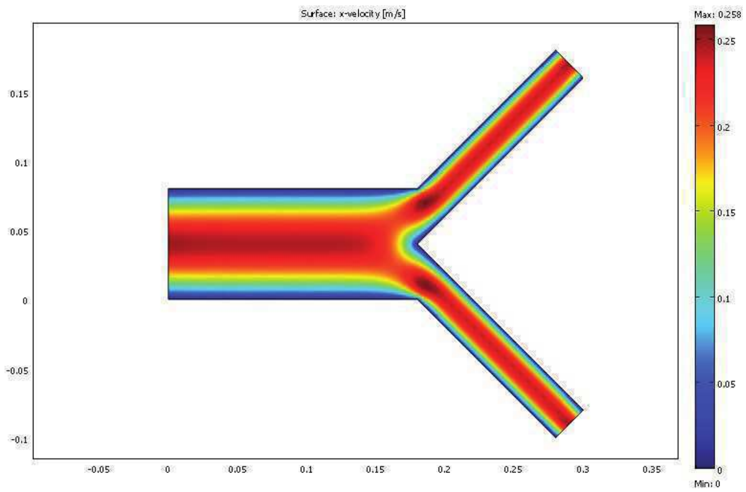
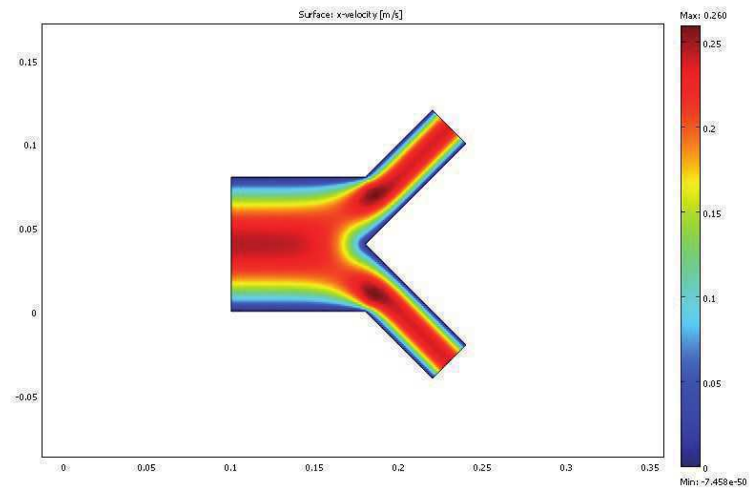
Maxi: 2691.112
Mini: 0


C
Γ_{out}^{1}
D
A
B
Γ_{in}^{1}
E
I
H
F
Γ_{out}^{2}
G


Surface: x-velocity [m/s]
Max: 0.258
Min: 0
0.15 0.1 0.05 0 -0.05 -0.1
-0.05 0 0.05 0.1 0.15 0.2 0.25 0.3 0.35
0.25 0.2 0.15 0.1 0.05 0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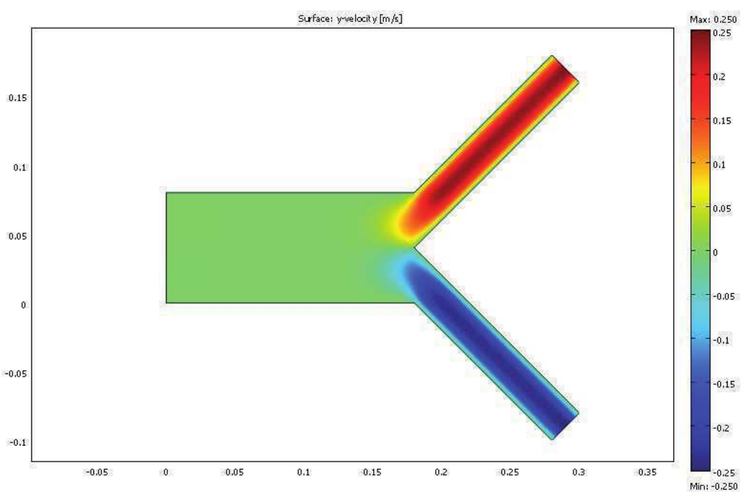
Surface: y-velocity [m/s]
Max: 0.250
Min: -0.250
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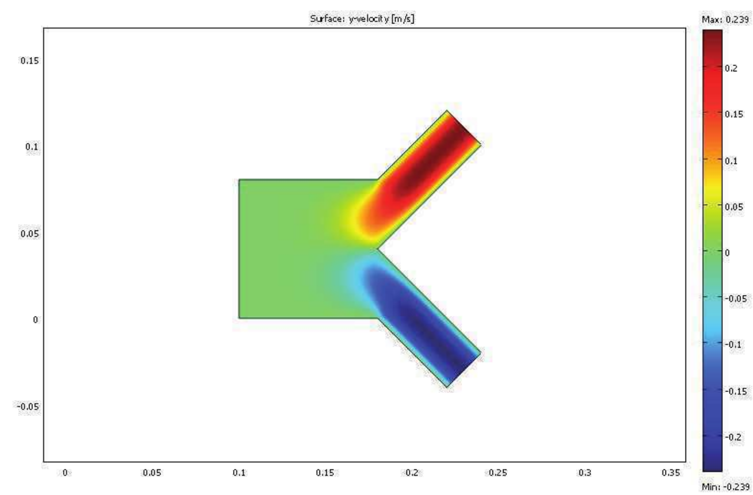
-0.05 0 0.05 0.1 0.15 0.2 0.25 0.3 0.35
0.25 0.2 0.15 0.1 0.05 0 -0.05 -0.1 -0.15 -0.2 -0.25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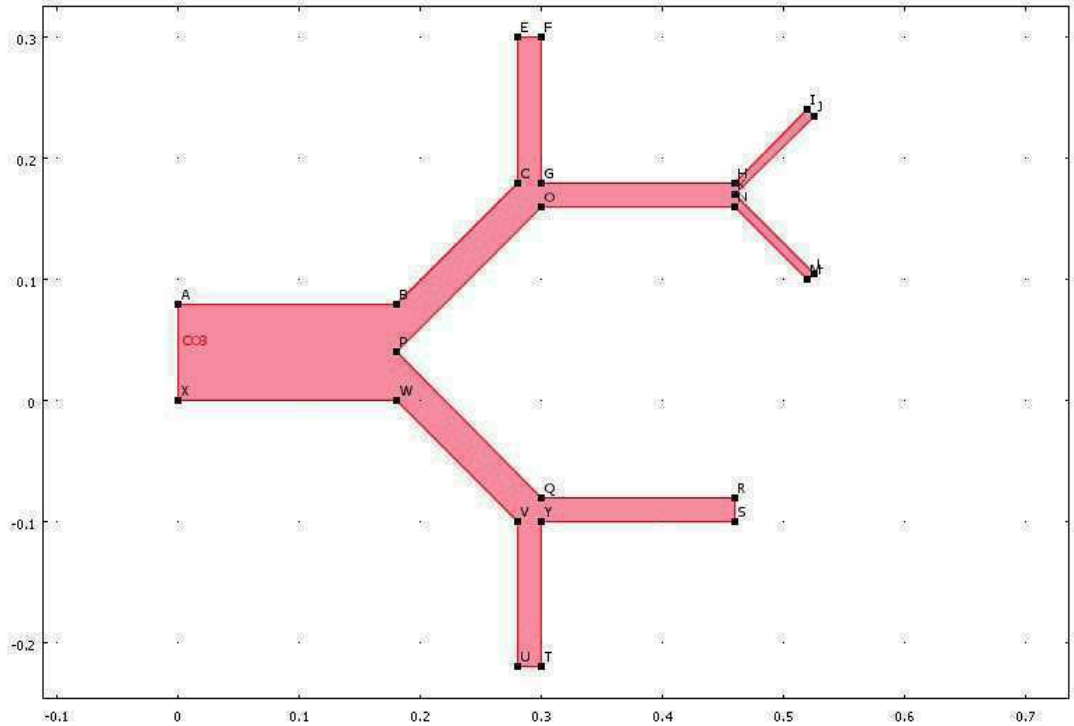
0 0.05 0.1 0.15 0.2 0.25 0.3 0.35
0.2 0.15 0.1 0.05 0 -0.05 -0.1 -0.15 -0.2


E
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CO3
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-0.1
-0.2
-0.1
0
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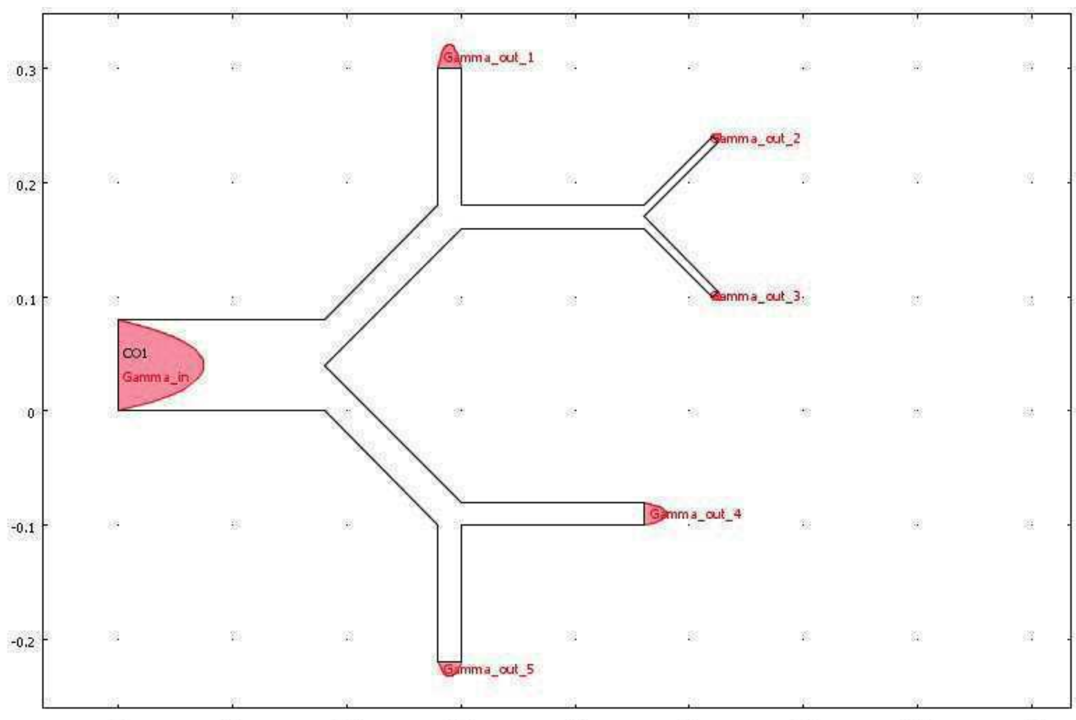
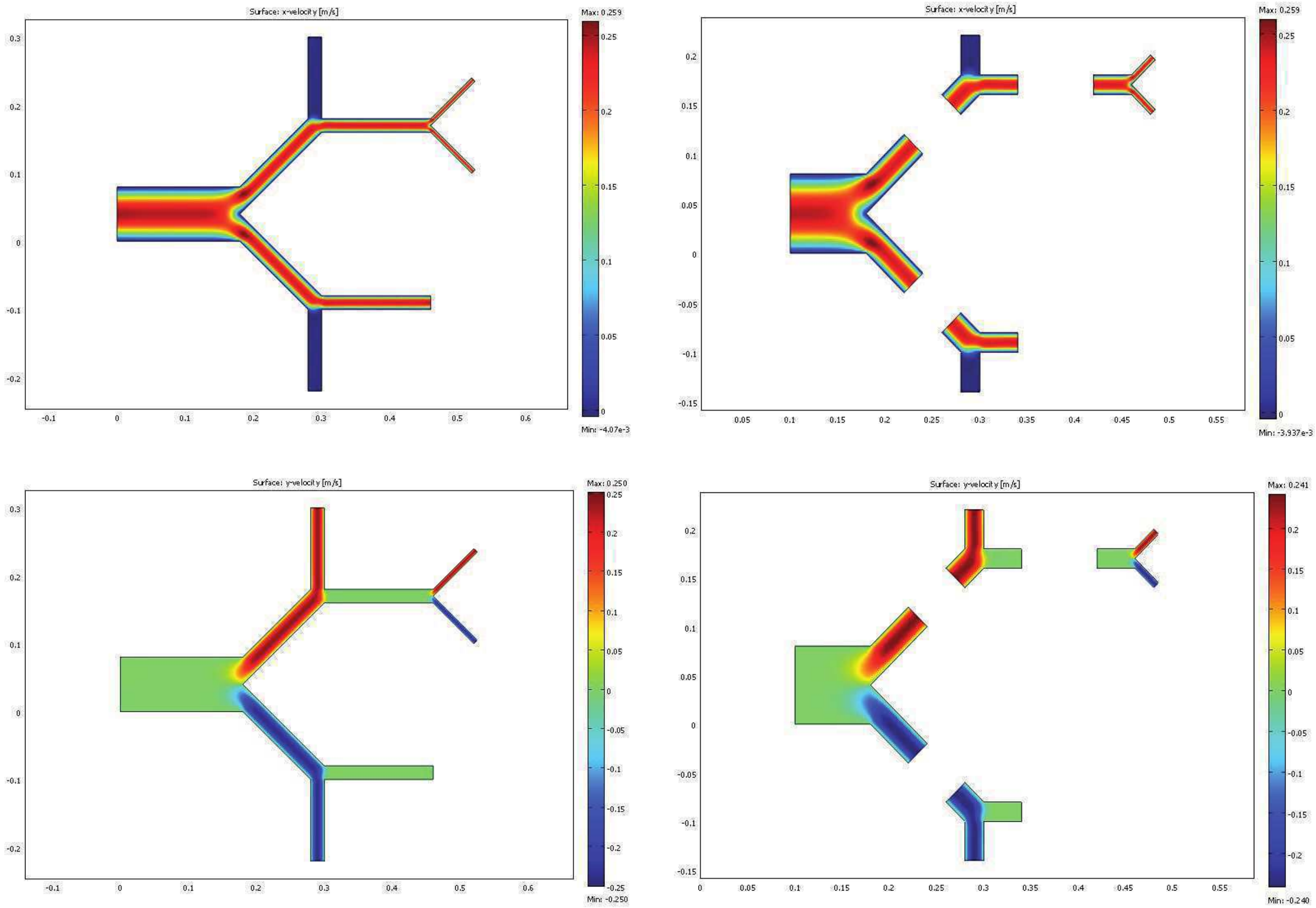
0.3
0.4
0.5
0.6
0.7


Gamma_out_1
Gamma_out_2
Gamma_out_3
CO1
Gamma_in
Gamma_out_4
Gamma_out_5
0.3
0.2
0.1
0
-0.1
-0.2
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8


Surface: x-velocity [m/s]
Max: 0.259
Min: -4.07e-3
Surface: x-velocity [m/s]
Max: 0.259
Min: -3.937e-3
Surface: y-velocity [m/s]
Max: 0.250
Min: -0.250
Surface: y-velocity [m/s]
Max: 0.241
Min: -0.240