Embolization of Arteriovenous Malformations: Model and Optimization

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Moscow, 2014

The object of the issue.

Arteriovenous malformations (AVM) are abnormal connections between arteries and veins, bypassing the capillary system.

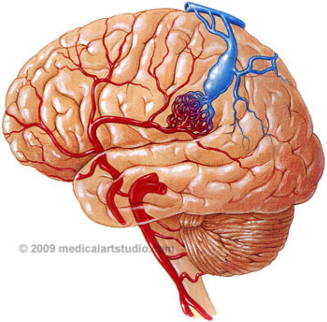
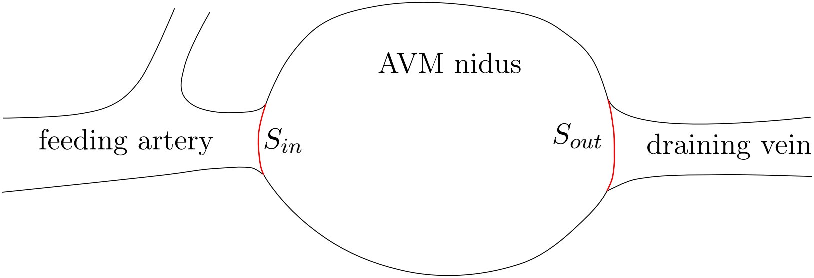


Figure : Image source: http://www.medicalartstudio.com

Model sketch.



Initial conditions.

Darcy's law and continuity equation inside AVM:

$$\vec{v} = -\frac{K}{\mu_b}\overrightarrow{\nabla p},$$

$$\mathrm{div}\vec{v} = 0.$$

Boundary conditions:

$$\begin{cases} \vec{v}(M) = \vec{v}_{in}(M), & \forall M \in S_{in}, \\ p(M) = p_{out}, & \forall M \in S_{out}, \\ (\vec{v}(M), \vec{n}(M)) = 0, & \forall M \in S_b. \end{cases}$$

Embolic agent mass concentration $C(M,0) = 0$.

Embolization.

Coefficient of dynamic viscosity:

$$\mu = \mu_b + \mu_b[\mu]C + \mu_b k[\mu]^2 C^2,$$

Density of embolic agent solution:

$$\rho = \rho_b + C\left(1 - \frac{\nu_{ef}}{M_e}\rho_b\right).$$

Embolic agent solution flow.

Embolic agent mass concentration dynamics:

$$m\frac{\partial C}{\partial t} = \mathrm{div}\left(D\overrightarrow{\nabla C} - C\vec{v}\right).$$

Darcy's law and continuity equation for solution:

$$\vec{v} = -\frac{K}{\mu}\overrightarrow{\nabla p},$$

$$m\frac{\partial \rho}{\partial t} + \mathrm{div}\left(\rho\vec{v}\right) = 0.$$

Boundary conditions and control.

For all $t > 0$:

$$\begin{cases} \vec{v}(M) = \vec{v}_{in}(M), & \forall M \in S_{in}, \\ p(M) = p_{out}, & \forall M \in S_{out}, \\ (\vec{v}(M), \vec{n}(M)) = 0, & \forall M \in S_b. \end{cases}$$

$$\begin{cases} C(M,t) = \frac{\rho_e u(t)}{(M_b(0) - F(\|\mu(t)\|))\left(1 - \frac{\nu_{ef}}{M_e}\rho_b\right)}, & \forall M \in S_{in}, \\ C(M,t) = 0, & \forall M \in S_{out}, \\ \left(\overrightarrow{\nabla C}(M,t), \vec{n}(M)\right) = 0, & \forall M \in S_b. \end{cases}$$

Constraint and optimization problem.

Unit load:

$$E(t) = \frac{\int\limits_{\Omega} \operatorname{div}\left(\left(p + \frac{\alpha}{2}\rho|\vec{v}|^2\right)\vec{v}\right) dV}{m \int\limits_{\Omega} \left(1 - \frac{\nu_{ef}}{M_e} C\right) dV} \leqslant \overline{E}.$$

Functional:

$$J(\mu, u) = \int\limits_{\Omega} (\mu - \mu_e)^2 \, dV + \lambda \int\limits_0^T \int\limits_{S_{in}} u(t) dV dt. \to \min$$

Thank you for your attention!