

## Modelling of cerebral aneurysm parameters under stent installation

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**Abstract** — A computer simulation of a cerebral aneurysm is performed. Based on real surgery data, 3D geometry of the anomaly is reconstructed, hydrodynamic parameters of the blood flow (velocity, pressure, gradient) are calculated, strains and stress are obtained on the vascular walls in the case of the aneurysm and after stent installation changing the geometry of a vessel. It is shown that successful surgery is characterized by lowering the flow velocity, pressure, its gradient, and stresses on the vascular walls.

A cerebral arterial aneurysm is a local distension, i.e., a local protrusion of a vascular wall. An arterial wall has several layers, an aneurysm generally consists of an inner layer called intima. It has no muscular structure, is less tensile than the wall of a healthy vessel and hence can be broken under a load which a healthy vessel can withstand well. In most cases the aneurysm occurs under anatomical variations or pathological changes in the structure of cerebral vessels, as well as at the points of vessel bifurcations or under arteriovenous malformations. This anomaly is one of the most frequent and dangerous diseases of cerebral arteries. The treatment of aneurysms is very difficult, their appearance and development proceeds for a long time without any symptoms up to the moment of breaking. If the presence of an aneurysm is determined, neurosurgeons need to estimate its growth rate and the risk of its breaking. This is necessary for correct determination of the time and technology of surgery. The medical aspects of this problem are discussed in detail in [4].

The problem of aneurysm modelling is very complicated and has many parameters. The formation and dynamics of an aneurysm is explained by many factors, including histological, genetic and hemodynamic ones. There are several theories of the formation and growth of aneurysms. In [8, 14], the distribution of shear stresses

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induced by the blood flow in a vessel is considered as the main factor causing the formation and growth of aneurysms. It is assumed that the endothelium is sensible to this tension and can change the vessel structure subject to it. Thus, the cross-section of a vessel depends on the parameters of the blood flow, in particular, on shear tension [12, 14]. If the shear tension increases, the cross-section of the vessel uniformly increases over a certain length. If the shear tension increases locally, then a local distension may appear in a vessel, which is an aneurysm. At the same time, there are no data unambiguously associating the formation, growth, and breaking of an aneurysm with the growth of shear tension. There are two alternative theories of ‘low’ and ‘high’ flows. The first theory assumes that lower flow velocities and lower shear tensions corresponding to them lead to a blood congestion, which results in the disfunction of nitric oxide (NO) release on the wall. This implies weakening of the vascular wall, its distension and a local protrusion, i.e., an aneurysm. In the ‘high’ flow theory, large flow velocities and shear tensions lead to the destruction of endothelium and an excess release of NO, which also weakens the wall of the vessel. The protrusion of the wall is caused by the hemodynamic shock of the blood flow. There are arguments pro and contra each of these theories, therefore, at the moment we cannot reject any of them with confidence. Review [14] also presents the results of computer simulations of an aneurysm and associated vessels, including 3D models.

The treatment of aneurysms is performed surgically. The endovascular method consists in the exclusion of the aneurysm from the blood flow by intravascular deployment of agents causing blood clot formation inside the aneurysm, the so-called embolization of an aneurysm [4, 5, 8, 14]. For example, such agents are spirals or stents. A stent is a special structure having the form of a cylindrical frame. It is installed into a vessel and can implement several functions, i.e., to decrease the blood flow in the aneurysm, which causes the blood clot formation in it, to change the geometry of the vessel and (or) the flow in order to optimize the hemodynamic parameters in it. So-called wide-neck aneurysms consisting actually of a single cupula are difficult for treatment. It is hard to fill those aneurysms with an embolizing agent, therefore, a stent is installed in those cases, which changes either the flow in the aneurysm, or the vessel geometry and redirects the flow.

In this paper we propose a mathematical model of aneurysm surgery by stent installation changing both the vessel geometry and the flow in it. In order to describe the blood flow, we use the stationary Navier–Stokes equations for a viscous, incompressible, Newtonian fluid. The behaviour of vascular walls is described by the linear elasticity equations. Numerical experiments have been performed with the use of the ANSYS package at the Information-Computing Center of the Novosibirsk State University.

We are simulating the actual surgery performed by neurosurgeons of the Novosibirsk Research Institute of Blood Circulation Pathology, Akad. E. N. Meshalkin Clinic. We are using the values of pressure and velocity of the blood flow measured directly in cerebral vessels during the operation. Those measurements have been taken with the use of a Volcano ComboMap sensor. The sensor has the diameter

0.36 mm and allows one to measure simultaneously the pressure and the blood flow velocity in vessels with a diameter greater than 1.5 mm. The velocity is measured by the ultrasonic Doppler method (12 MHz), the pressure is measured by a piezoelectric manometer [5, 10]. The aim of the paper is to determine the hemodynamic blood flow parameters in a vessel and the strength properties of the vascular walls in the presence of an aneurysm before and after the stent installation changing the geometry of the vessel. The operation performed was successful from the medical viewpoint. This makes it possible to express its success by comparison of the natural parameters before and after the operation, these parameters are the flow velocity, streamline vorticity, pressure, shear strains, and stresses on vascular walls.

When simulating such complicated and multifactor objects, we always tend to optimize the complexity of the model to get a simplest possible model taking into account the ‘significant’ factors. In this paper we show that the model proposed here describes the improvements in hydrodynamic and mechanical parameters of the blood flow and the vessel and hence can be used in pre-surgery modelling, which is an important and promising approach in modern medicine.

## 1. Mathematical model

The blood flow is described by the Navier–Stokes equations for a three-dimensional stationary motion of an incompressible viscous Newtonian fluid

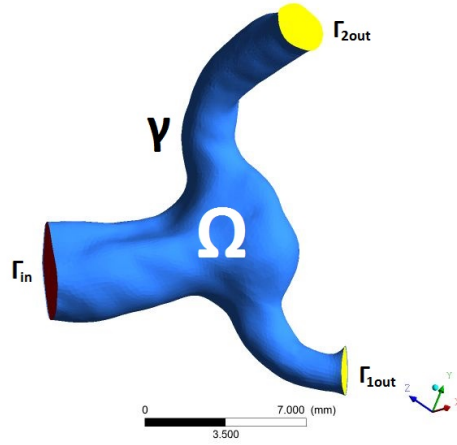
$$\left. \begin{aligned} \operatorname{div} \mathbf{v} &= 0 \\ \mathbf{v}_t + (\mathbf{v} \cdot \nabla) \mathbf{v} + \nabla p &= \nu \Delta \mathbf{v} \end{aligned} \right\} \text{ in } \Omega \quad (1.1)$$

where  $\mathbf{v}$  is the velocity,  $p$  is the pressure,  $\nu$  is the kinematic viscosity coefficient,  $\Omega$  is the internal volume of the computational domain including the T-shaped configuration of the vessels and the aneurysm placed at the point of a bifurcation;  $\gamma = \partial\Omega$  is the boundary, i.e., the vascular wall,  $\Gamma_{\text{in}}$  is the cross-section of the parental vessel of the tee,  $\Gamma_{1\text{out}}$  and  $\Gamma_{2\text{out}}$  are the cross-sections of the child vessels (exits of the tee).

The nonslipping condition  $\mathbf{v} = 0$  is posed on  $\gamma$  for Navier–Stokes equations (1.1), at the entrance  $\Gamma_{\text{in}}$  the constant velocity  $v_{\text{real}} = 20$  cm/s is specified across the cross-section, the constant pressure values  $p_1 = 67$  mm Hg and  $p_2 = 68$  mm Hg are given at the exits  $\Gamma_{1\text{out}}$  and  $\Gamma_{2\text{out}}$ . These values were measured in the course of an operation [5, 10]. Navier–Stokes equations (1.1) were solved numerically by the SIMPLE method [11], the least squares method was used for discretization of the spatial derivative (the values at the centers of the cells were used), the momentum is discretized by a second-order upwind scheme.

The stressed state of the vascular walls is described by the model of an isotropic linear-elastic material

$$\sum_{j=1}^3 \frac{\partial \sigma_{ji}}{\partial x_j} = 0, \quad \Delta \sigma_{ij} + \frac{1}{1+\nu} \frac{\partial^2 (\sigma_{11} + \sigma_{22} + \sigma_{33})}{\partial x_i \partial x_j} = 0$$



**Figure 1.** Computational domain.

where  $\sigma = (\sigma_{ij})$  is the stress tensor,  $i, j = 1, 2, 3$ ;  $\nu$  is the Poisson coefficient. The first group of equations relates to the equilibrium conditions, the second one represents the Beltrami–Michell relations. Stresses caused by the blood pressure are specified on the inner surfaces of the walls. The external side of the wall is free of load. The inlet and outlet boundaries are fixed.

The relation between the stress tensor ( $\sigma$ ) and the strain tensor ( $\varepsilon$ ) are determined by Hooke's law

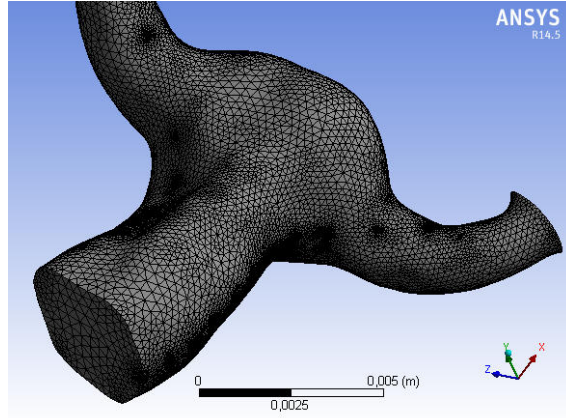
$$\sigma = E\varepsilon.$$

The Young modulus  $E = 1$  MPa,  $\nu = 0.49$ , the width of the wall is 0.4 mm. Such model is rather commonly used [3, 14].

At present, the Navier–Stokes equations are considered for the problem of a viscous incompressible fluid with various inlet and outlet boundary conditions. Along with specification of the velocity vector values, one can specify pressure and one velocity component so that the velocity vector is perpendicular to the inflow-outflow boundary [6, 13]. In a hydroelastic flow problem, it is possible to specify the (total) pressure and to pose the condition that the tangent velocity component equals zero [9]. The description of the Fluent package mentions only one condition for the outlet, i.e., pressure. Such boundary condition is natural in the case when the simulation uses experimental (clinical) data. Our calculations have demonstrated good agreement of the calculated and experimental data. This is an argument in favor of the Fluent package widely used nowadays [14] in hemodynamics calculations.

The vascular wall geometry was constructed on the base of MR tomograms for all three computed cases. The three-dimensional representation of the vessels in the anomaly region was obtained by the tomogram processing code ITK-SNAP. The type of the stent used in the operation is intended for reinforcing the walls of the vessel so that the stent becomes a part of this wall after its installation [1].

Figure 1 shows the configuration of the vessels before and after stenting. The stent installation results in the transformation of the anomalous T-configuration of



**Figure 2.** Computational grid.

the bifurcation with a protrusive aneurysm into a natural Y-shaped one where the aneurysm is almost imperceptible [1, 2].

We use a tetrahedral computational grid (see Fig. 2). Refining the grid with 66482 nodes and 352260 elements to 360570 nodes and 2036619 elements, i.e., five times, we get that the deviation of the pressure values is less than 1%; slightly larger deviations (up to 5%) are observed for the absolute value of the velocity. Further refinement of the grid does not affect the result, which indicates the adequacy of the numerical approach used.

## 2. Results: hydrodynamic parameters

We have calculated the following hydrodynamic parameters before and after the operation: velocity, streamline distribution, pressure, and its gradient.

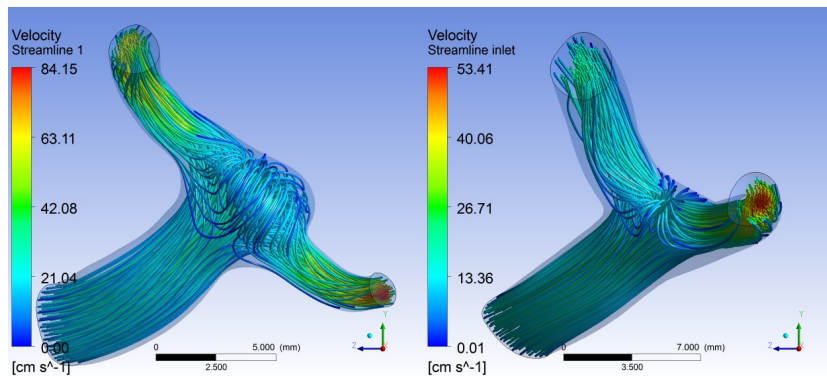
Figure 3 shows the behaviour of the streamlines. As was expected, there is a stagnation point at the bifurcation of the vessel. After the stent installation the vorticity of the streamlines decreases, which is a positive trend for a wide-neck aneurysm.

Figures 4 and 5 illustrate the distributions of the pressure and the absolute value of the pressure gradient on the vascular walls before and after stenting. The regions of high pressure gradients may be dangerous from the medical viewpoint. The stenting promotes a more uniform distribution of the pressure and decreases the absolute value of the pressure gradient.

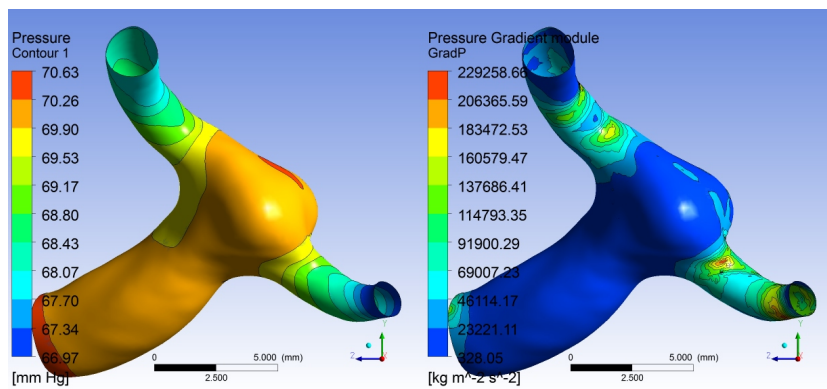
The shear stress distribution

$$\tau = \mu(\nabla \mathbf{v} \cdot \mathbf{n})$$

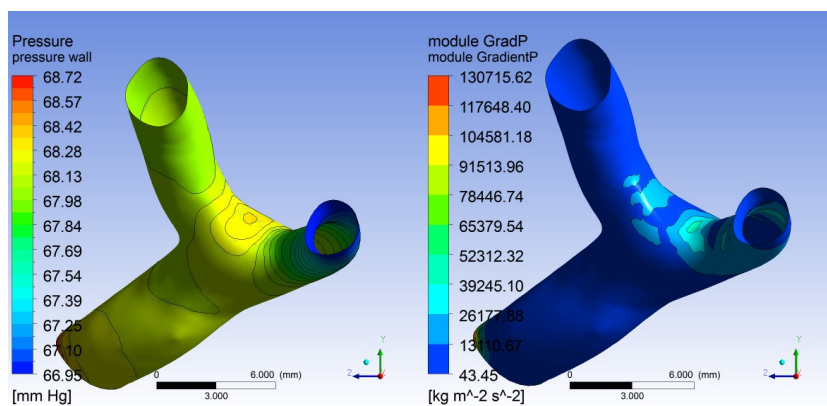
where  $\mu$  is the dynamic viscosity coefficient ( $0.004 \text{ kg}\cdot\text{s}/\text{m}^2$  for blood), and  $\mathbf{n}$  is the normal to the wall, strongly depends on the structure of the incoming vessel, the form of the aneurysm, and the diameter of its neck. In this case the width of the flow is close to the diameter of the cross-section of the aneurysm, therefore, there is no local increase in the shear stress on the cupola of the aneurysm, the stresses



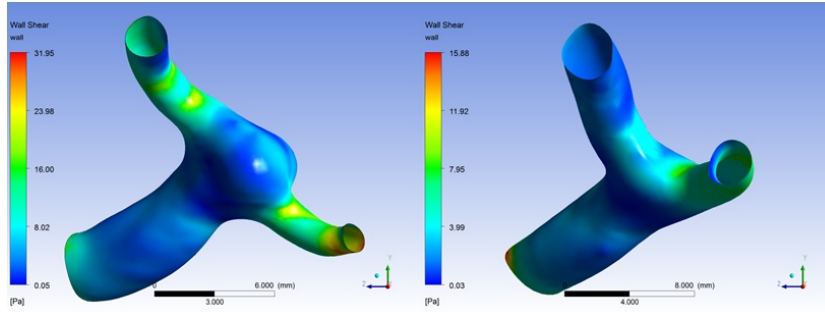
**Figure 3.** Streamlines in the vessel before (left) and after (right) stent installation.



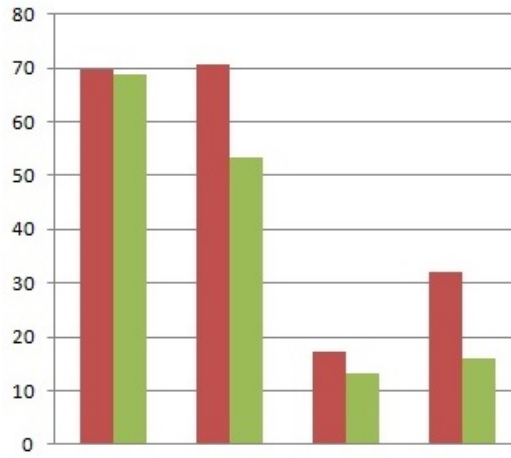
**Figure 4.** Pressure distribution (left) and the absolute value of the pressure gradient (right) on vascular walls before stent installation.



**Figure 5.** Pressure distribution (left) and the absolute value of the pressure gradient (right) on vascular walls after stent installation.



**Figure 6.** Shear stress distribution of vascular walls before (left) and after (right) stent installation.



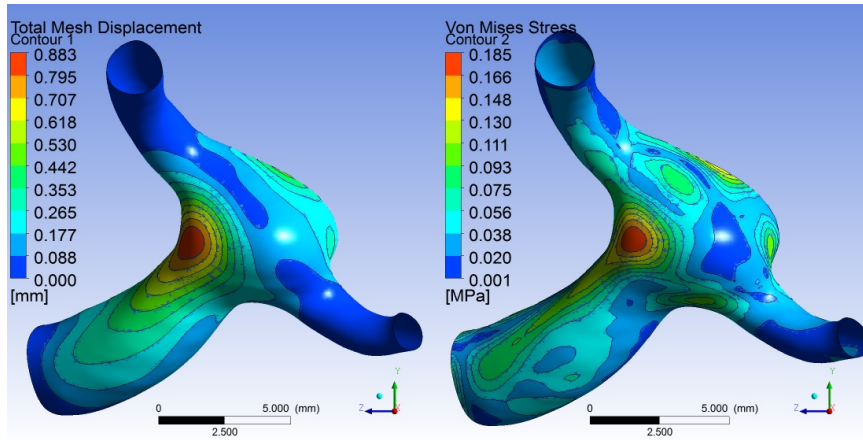
**Figure 7.** Diagram of maximal values of pressure (mmHg), velocity ( $\text{cm}\cdot\text{s}^{-1}$ ), absolute value of the pressure gradient ( $10^4 \text{ kg}\cdot\text{m}^{-2} \text{ s}^{-2}$ ), and maximum of WSS (Pa) before (red) and after (green) stent installation.

are distributed uniformly over the cupola. A local increase in shear stress appears where the flow comes into narrower vessels (see Fig. 6).

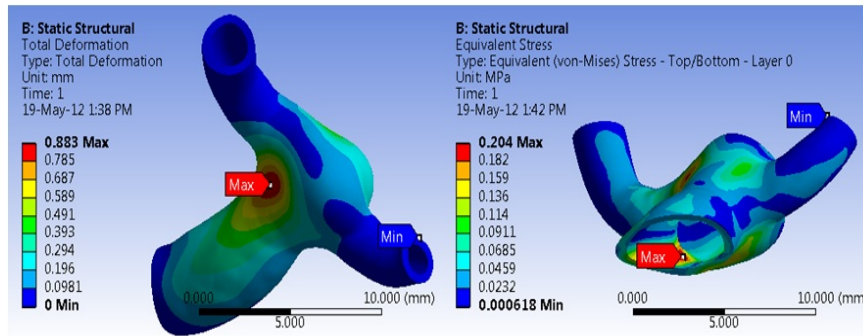
The diagram in Fig. 7 presents the comparison of the parameters before and after surgery. It is seen that stenting results in decreasing the maximal pressure (slightly), the absolute value of the pressure gradient, the maximal velocity, and the maximal shear stress. This indicates that the Y-shaped geometry obtained after surgery is preferable concerning the effect of the flow on the vascular wall [1, 2].

### 3. Results: stressed-deformed state parameters of vascular walls

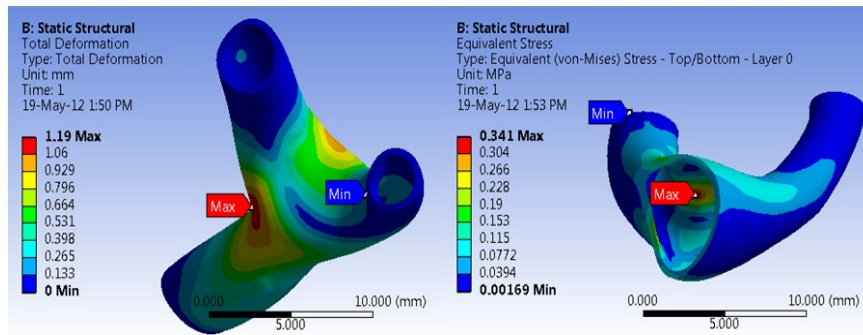
The wall deformation is calculated relative to the initial (not stressed) state. We reckon that the calculation of stresses is necessary, because it is assumed that the break of a material occurs at the point where the stresses reach the critical value (exceed the breaking point of the material). The problem where an aneurysm breaks has no clear solution today [4]. Therefore, the calculation of different parameters



**Figure 8.** Distribution of von Mises deformations (left) and stresses (right) on vascular walls before surgery.

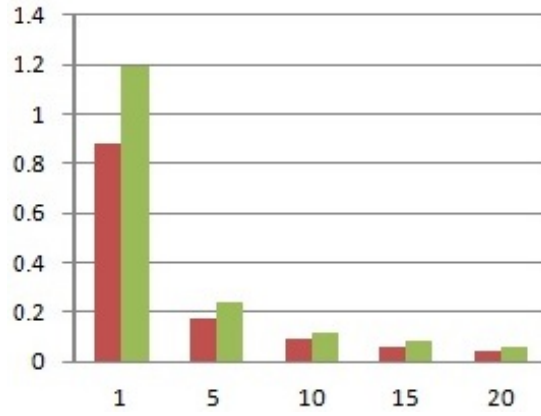


**Figure 9.** Distribution of von Mises deformations (left) and stresses (right) with minimum and maximum points on vascular walls before stent installation.



**Figure 10.** Distribution of von Mises deformations (left) and stresses (right) with minimum and maximum points on vascular walls after stent installation.





**Figure 11.** Diagram of maxima of vascular wall deformations before (red) and after (green) stent installation depending on the Young modulus of the vascular wall.

which may cause such breaking is very important. In the determination of the place of breaking, not only high hydrodynamic parameters, but also the mechanical properties of the wall are of great importance. The physical interpretation of von Mises criterion is that the elastic energy reaches its critical value under deformation. The von Mises stress is calculated from the components of the stress tensor by the formula [7]:

$$\sigma_v = \left[ \frac{1}{2}(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 \right]^{1/2}. \quad (3.1)$$

Figures 8–10 present the distributions of von Mises deformations and stresses before and after surgery. Note that the contours of the deformation and stress distributions are different. At the same time, the contours of the deformations before and after surgery are similar. The deformations have their maximal value at the neck of the aneurysm, but not on the cupola and not at the stagnation point of the flow. It is interesting to note that the stress maximum lies inside the vessel, before the operation it lies in the incoming vessel and after surgery it is at the point of the bifurcation (see Figs. 9 and 10). We can conclude that the aneurysm origin is at the point of bifurcation and its growth occurs on the neck.

There are data [4] indicating that the elasticity modulus changes depending on pressure, and the ratio of those values for the stressed and unstressed states can reach four. We performed numerical calculations for different values of the Young modulus. The results for deformation and stress maxima are presented in the diagram (see Fig. 11).

## Conclusion

In this work we have performed numerical simulation of the cerebral arterial aneurysm before and after stenting. We used a relatively simple model assuming that the vas-

cular walls and the aneurysm are homogeneous. The principal factor in the surgical operation that is successful from the medical viewpoint is the change of the vessel geometry by means of a stent.

It is shown that the surgery decreases the pressure, the absolute value of the pressure gradient on the vascular wall, the flow velocity, and the shear stress.

It is shown that the stress maximum is located on the inner side of the aneurysm, before surgery it is on the wall of the incoming flow, and after surgery it is at the point of the flow stagnation. The points of the maximum of deformations, stresses, and the absolute value of the pressure gradient are different both before and after surgery.

The main peculiarity of this work is the calculation of the actual configuration of the aneurysm and the comparison of hydrodynamic and mechanical parameters of the blood flow and vascular walls before and after stenting. The results obtained here give interesting information for surgery modelling and prognosis of singular points in an aneurysm.

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