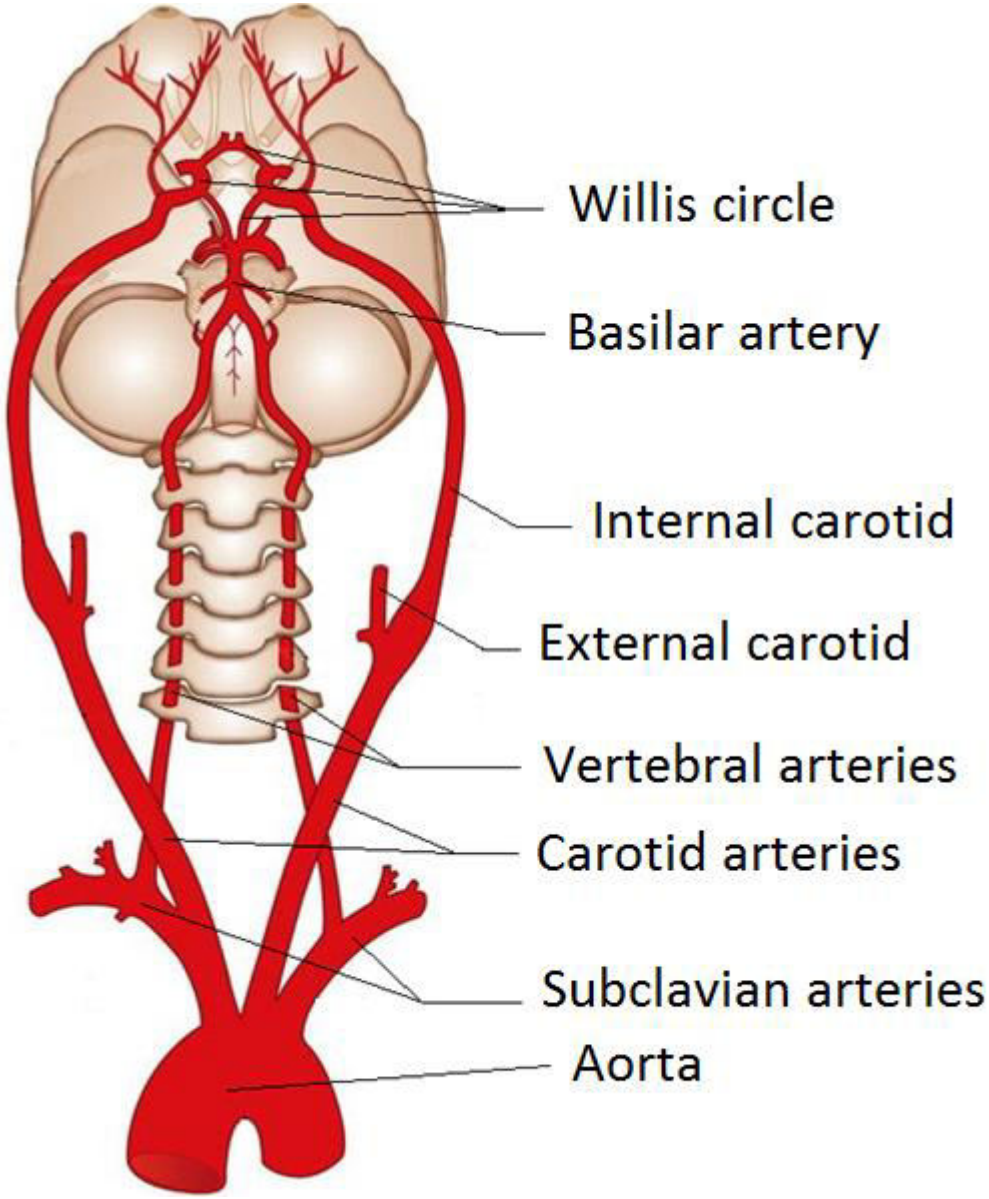
COMPUTATIONAL MODELLING OF MULTIPLE STENOSES IN CAROTID AND VERTEBRAL ARTERIES

Timur Gamilov[1,2], Sergey Simakov[1,2], Philip Kopylov[3]

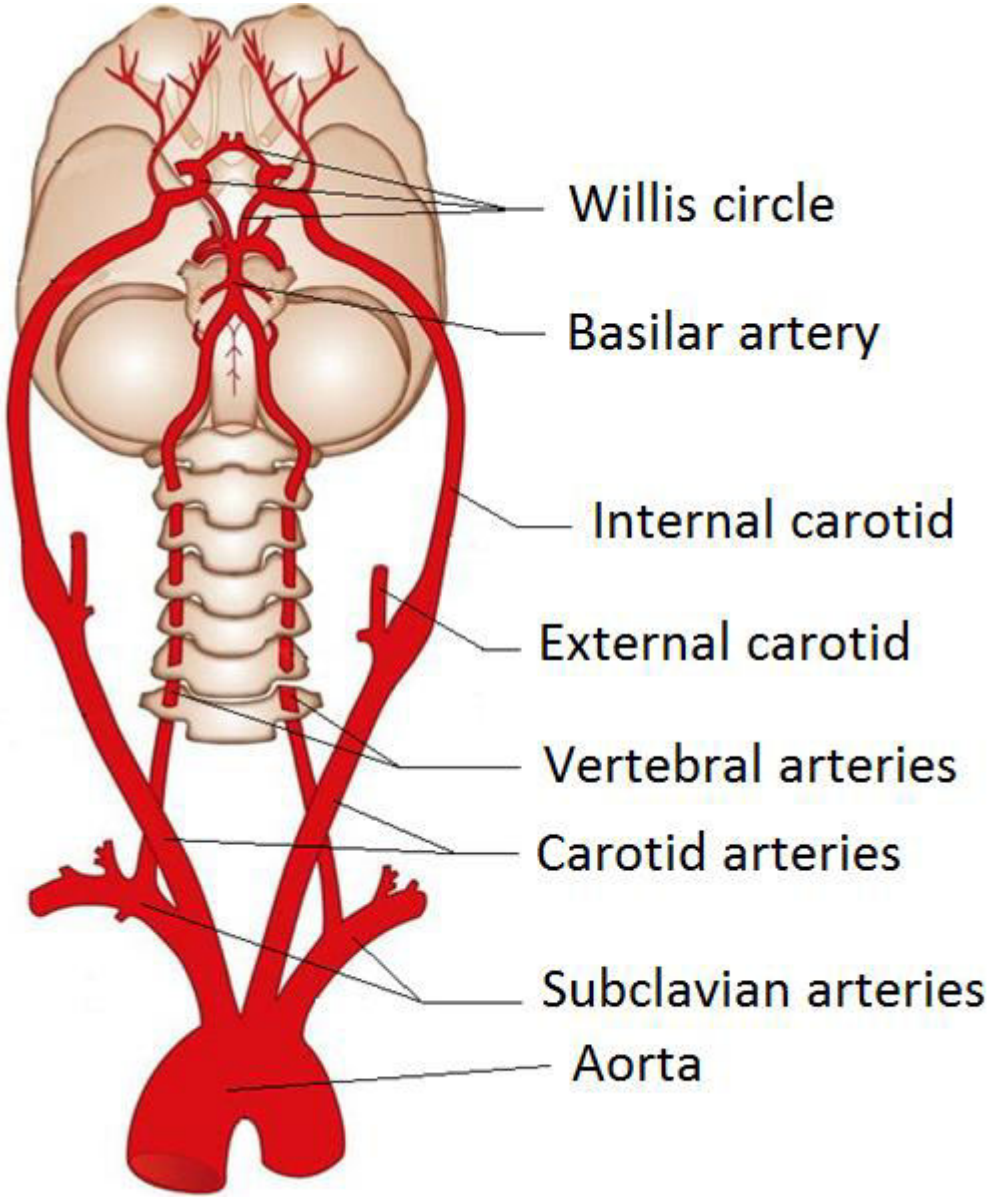
[1] Moscow Institute of Physics and Technology
[2] Institute of Numerial Mathematics of the Russian Academy of Sciences
[3] I.M. Sechenov First Moscow State Medical University

17th International Symposium on Mathematical and Computational Biology
30.10-03.11.2017

Blood vessels of head and neck

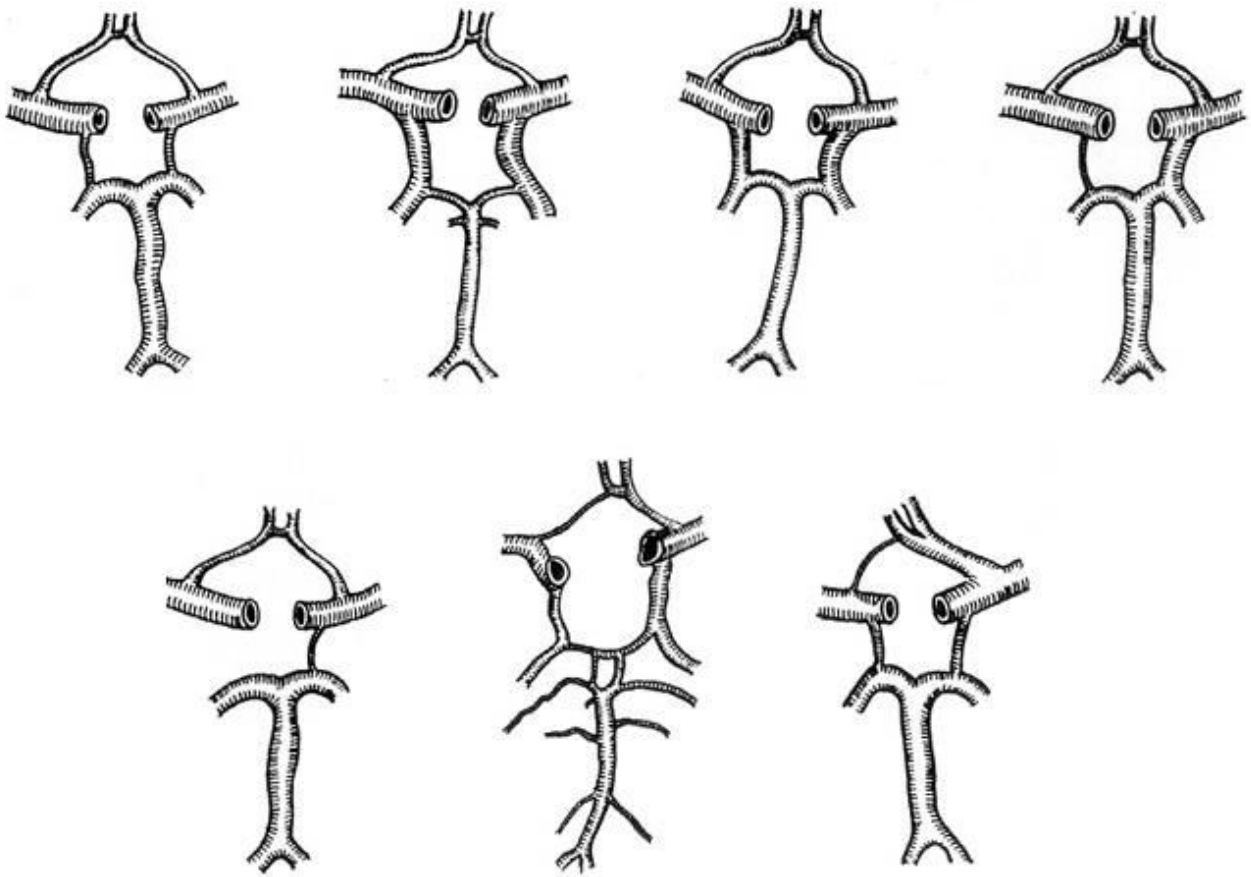


Blood vessels of head and neck



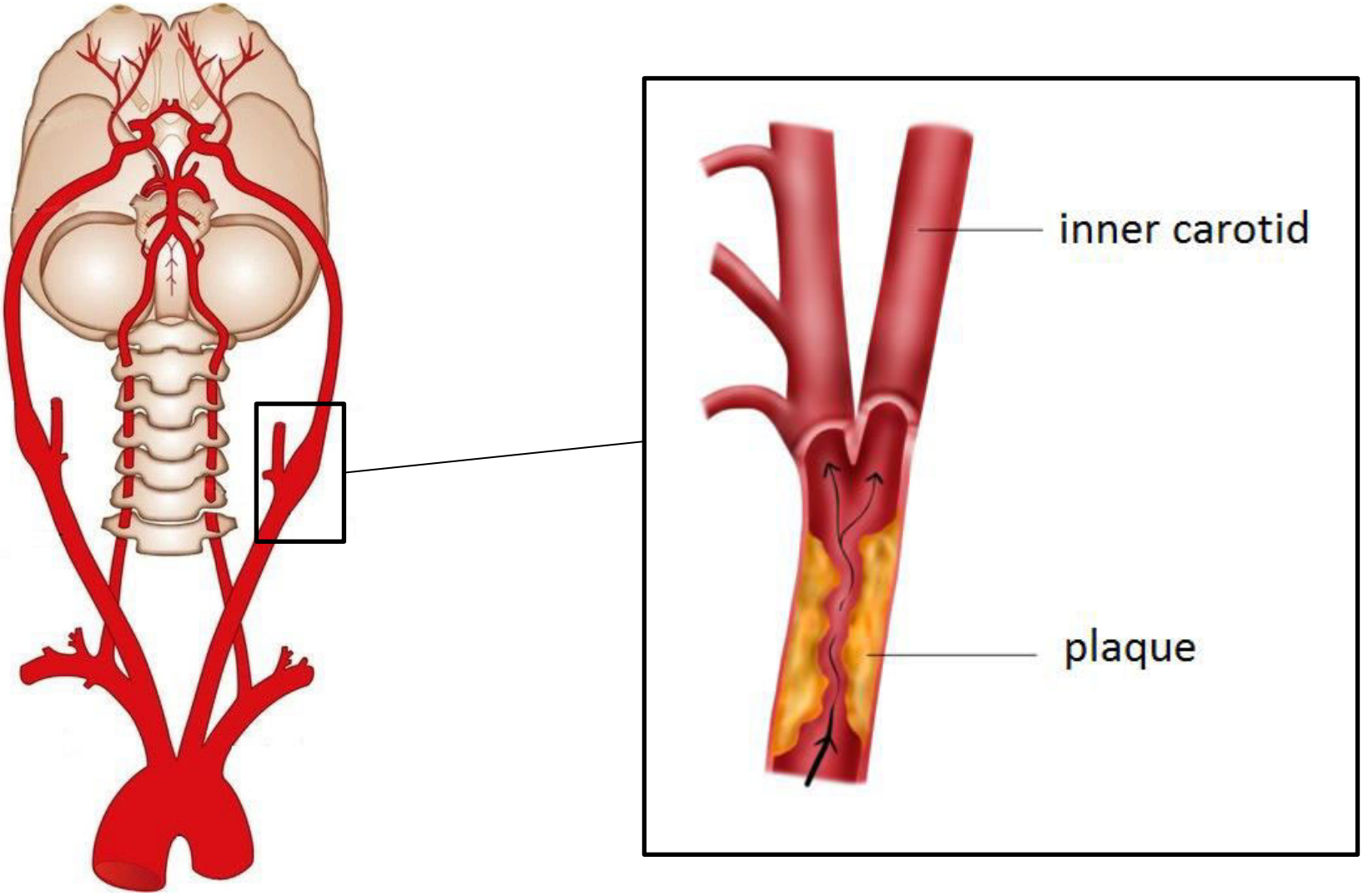
- 20 % blood volume
- high density
- regulatory mechanisms
- collateral paths
- individual

Circle of Willis

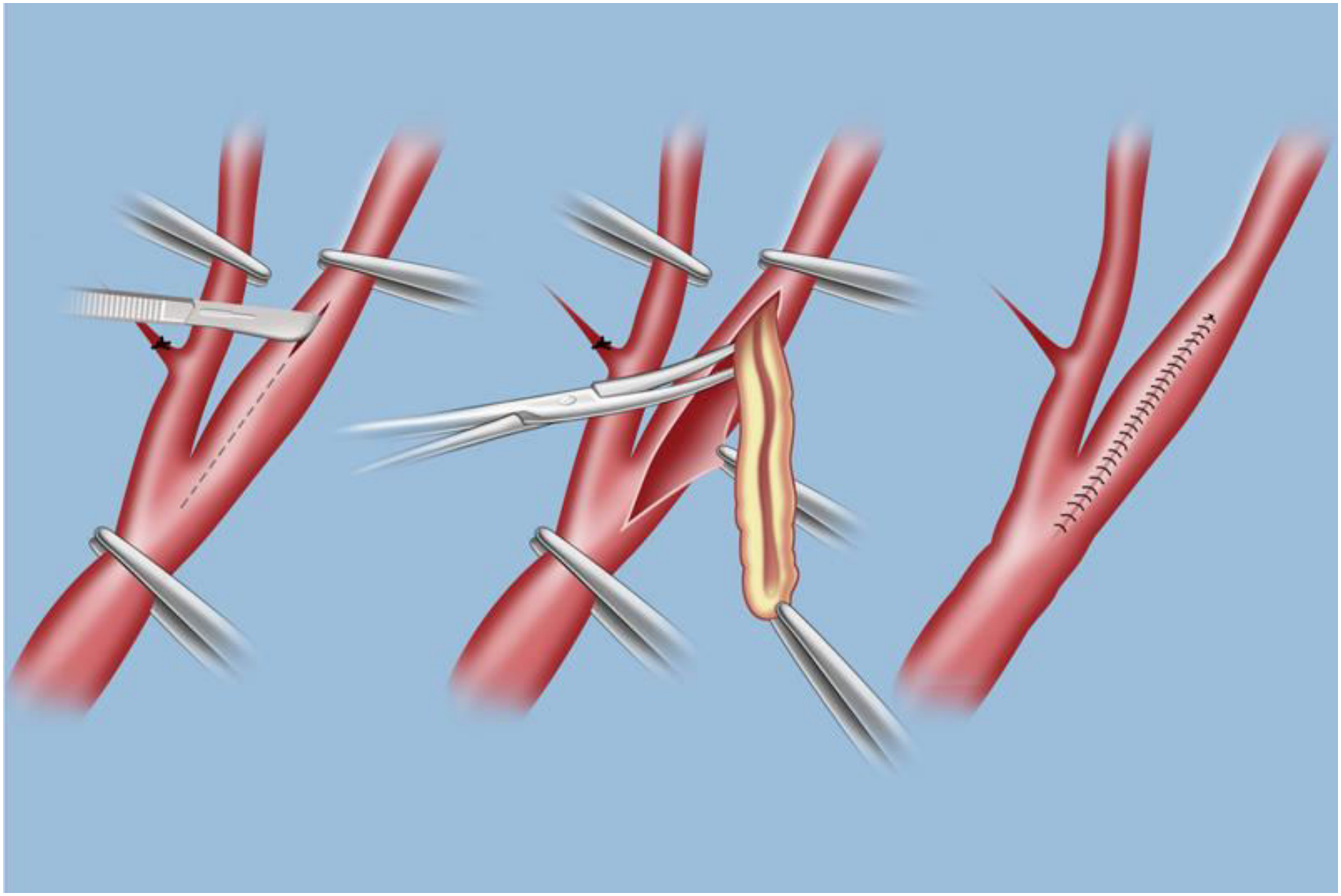


Full circle:
20-25 %

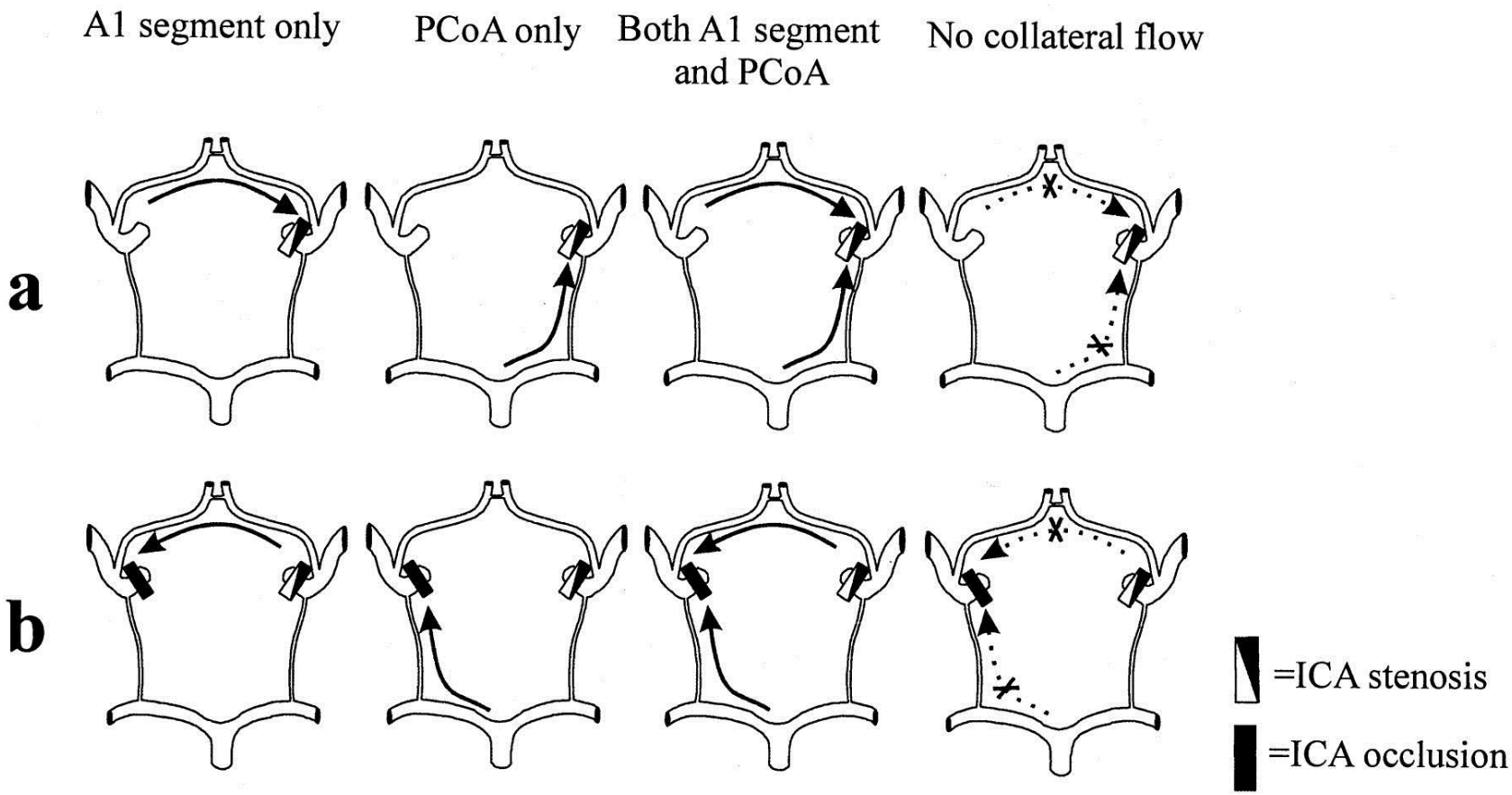
Stenosis



Carotid endarterectomy

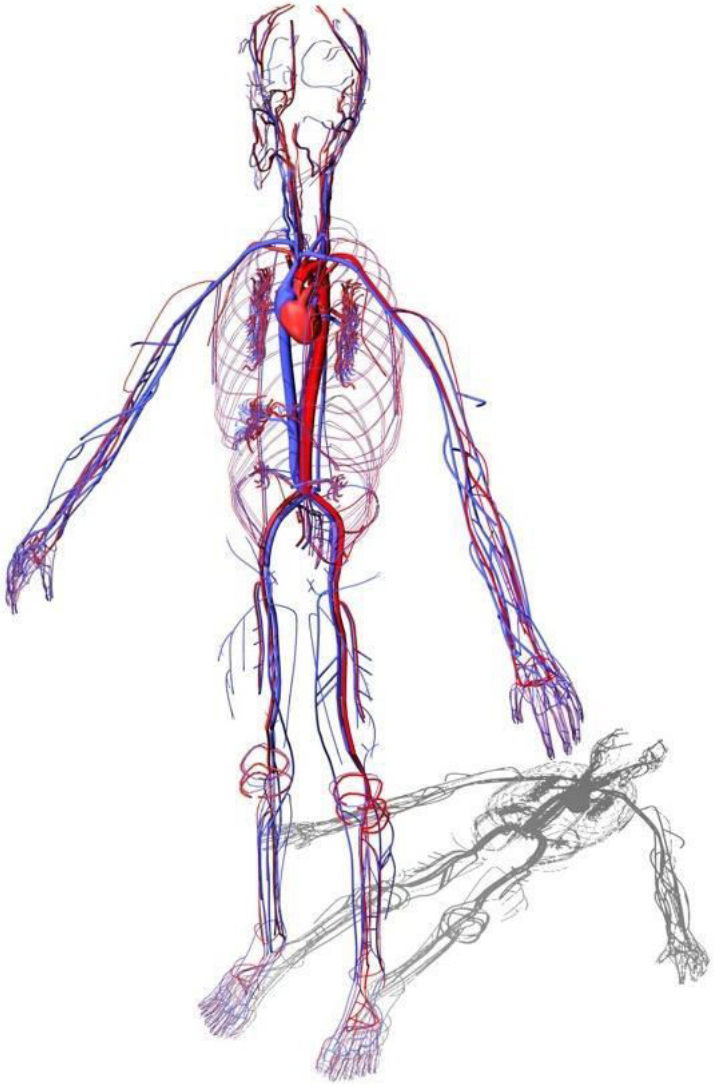


Blood flow patterns



Effect of Carotid Endarterectomy on Primary Collateral Blood Flow in Patients With Severe Carotid Artery Lesions
J. Hendrikse, D.R. Rutgers, C.J.M. Klijn, et.al. 2003

Blood flow model



Ashmetkov, Bunicheva, Mukhin, et.al. (2005)
Alastruey, Parker, Peiro, et.al. (2007)
Muller, Toro (2014)

Blood flow circulation model

1) Mass balance $$\frac{\partial A}{\partial t}+\frac{\partial (uA)}{\partial x}=0$$

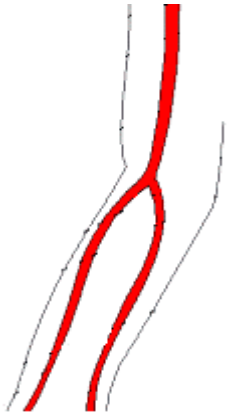
2) Momentum balance

$$\frac{\partial u}{\partial t}+\frac{\partial}{\partial x}\left(\frac{u^2}{2}+\frac{P}{\rho}\right)=-16\mu u\frac{\eta(A)}{Ad^2}+\psi(\ldots),\ \eta(A)=\begin{cases}2, A>A_0\\ \frac{A}{A_0}+\frac{A_0}{A}, A\le A_0\end{cases}$$

3) Bifurcations

3.1 $$\sum_{k=k_1,\ldots,k_M}\varepsilon_k^m Q_k=0,\ \varepsilon_k^m=\pm 1,\ Q_k=u_k A_k$$

3.2 $$p_k+\frac{\rho u_k^2}{2}=p_j+\frac{\rho u_j^2}{2},\quad \forall j,k$$



3.3 Compatibility conditions

Wall state equation

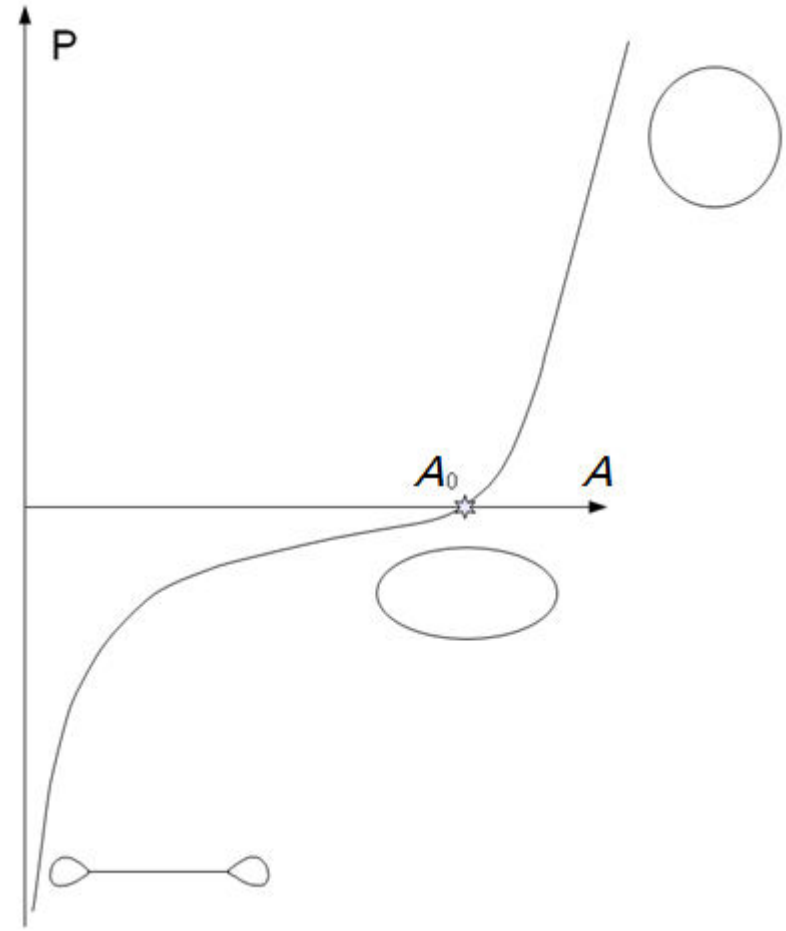
4) Vessel wall elasticity

Analytic approximation

$$P(A) = P^{ext}(t,x) + \rho c^2 f(A)$$

$$f(A) = \begin{cases} \exp(A/A_0 - 1) - 1, A > A_0 \\ \ln(A/A_0), A \le A_0 \end{cases}$$

$P^{ext}(t,x)$ – external pressure



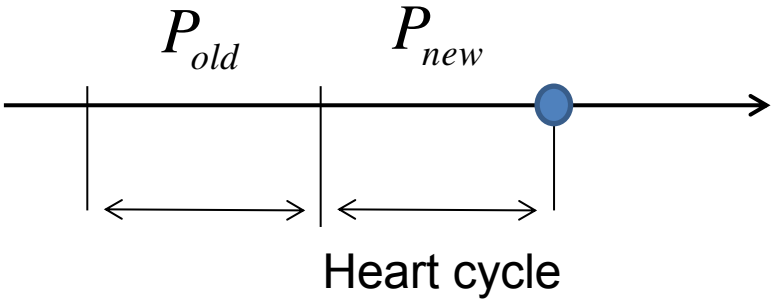
Pedley, Luo, 1998

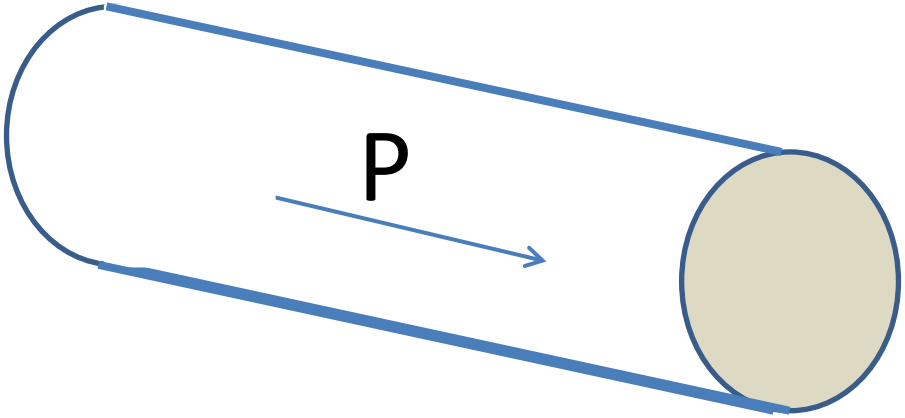
Autoregulation

Response to external pressure:

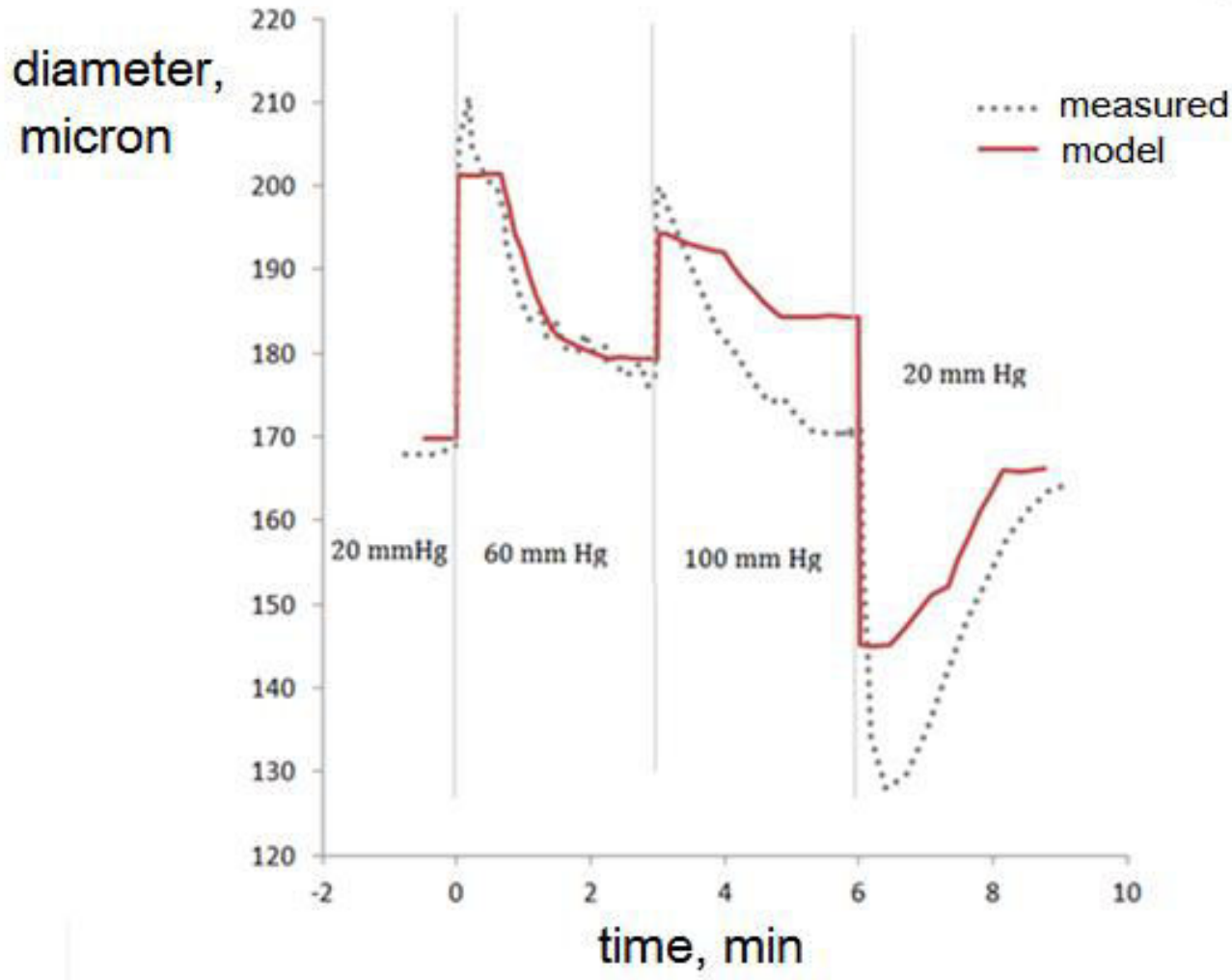
$$P = \rho c^2 \left(\exp\left(\frac{A}{A_0} - 1 \right) - 1 \right)$$

$$\frac{c_{new}}{c_{old}} = \left(\frac{P_{new}}{P_{old}} \right)^{1/2}$$



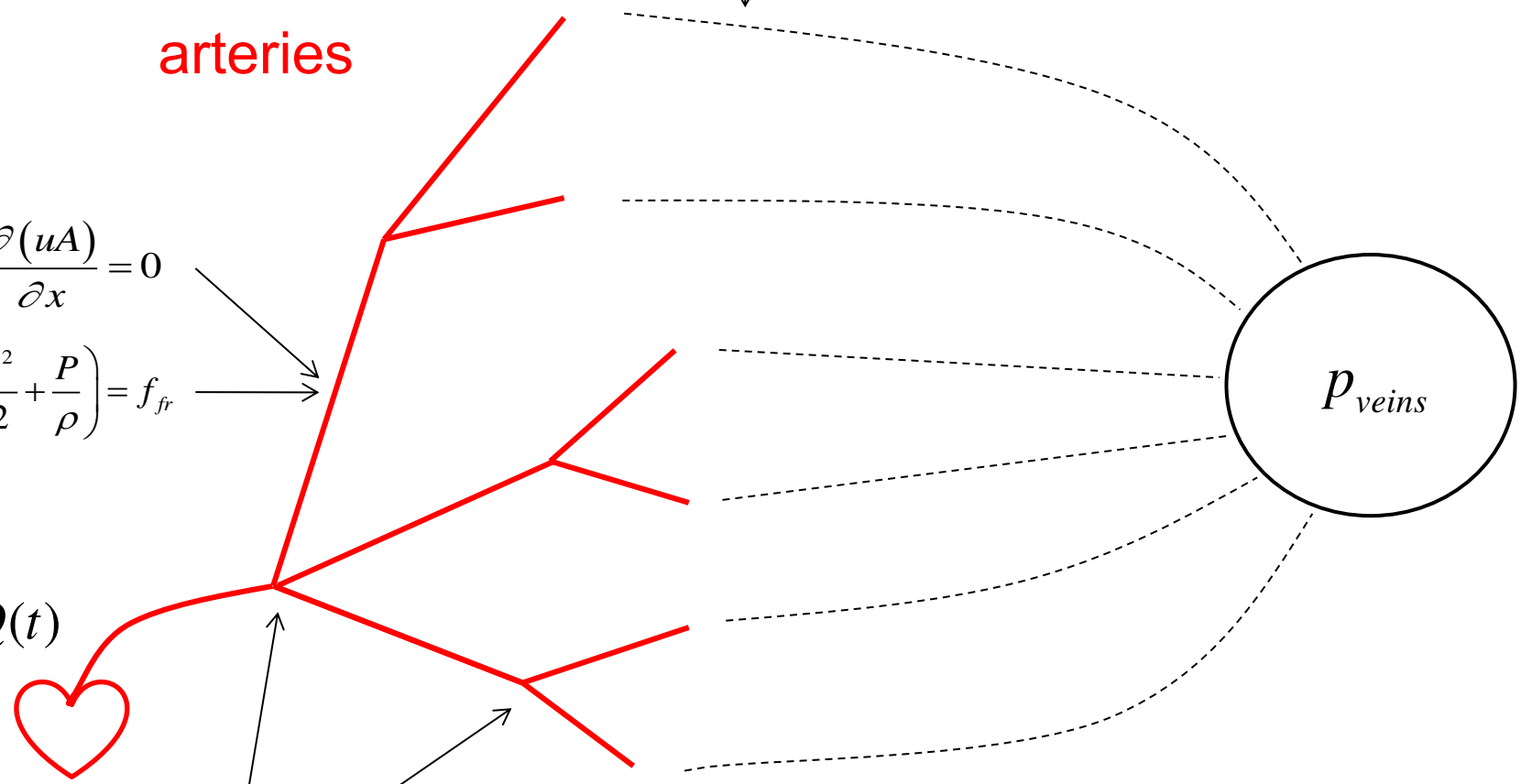


Autoregulation. Rat artery



Bifurcations. Resistance

$$p_k - p_{veins} = R_k Q_k$$



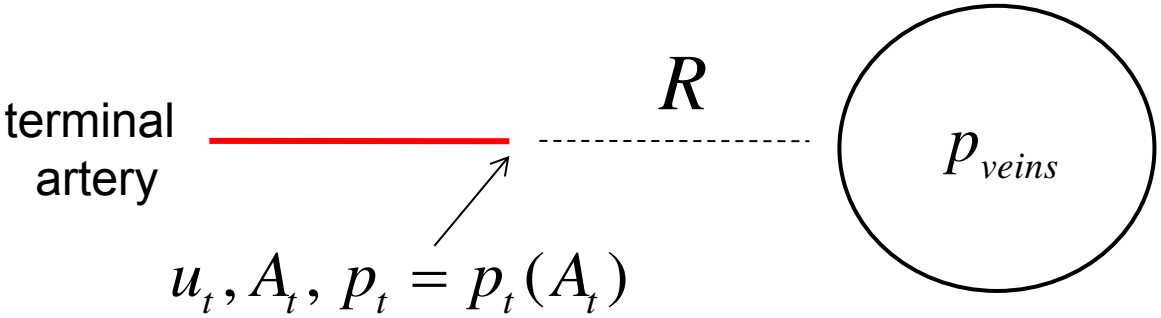
$$\frac{\partial A}{\partial t} + \frac{\partial (uA)}{\partial x} = 0$$

$$\frac{\partial u}{\partial t} + \frac{\partial}{\partial x}\left(\frac{u^2}{2} + \frac{P}{\rho}\right) = f_{fr}$$

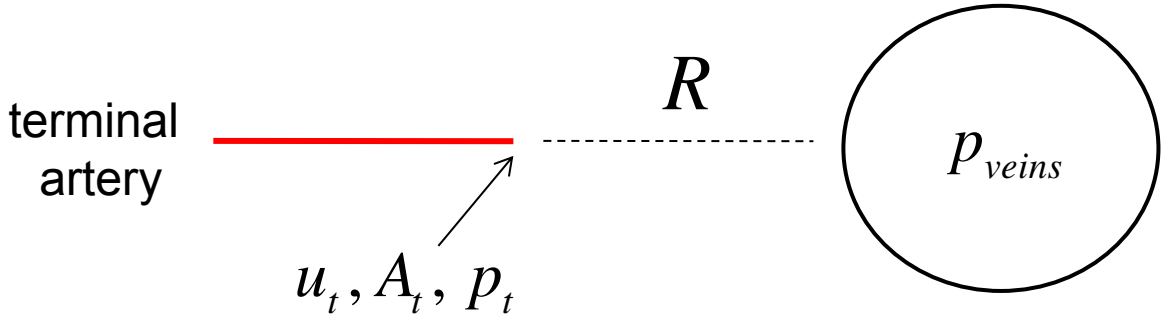
$$u_{in} A_{in} = Q(t)$$

$$p_k + \frac{\rho u_k^2}{2} = p_j + \frac{\rho u_j^2}{2}, \quad \forall j,k$$

Bifurcations. Terminal vessel

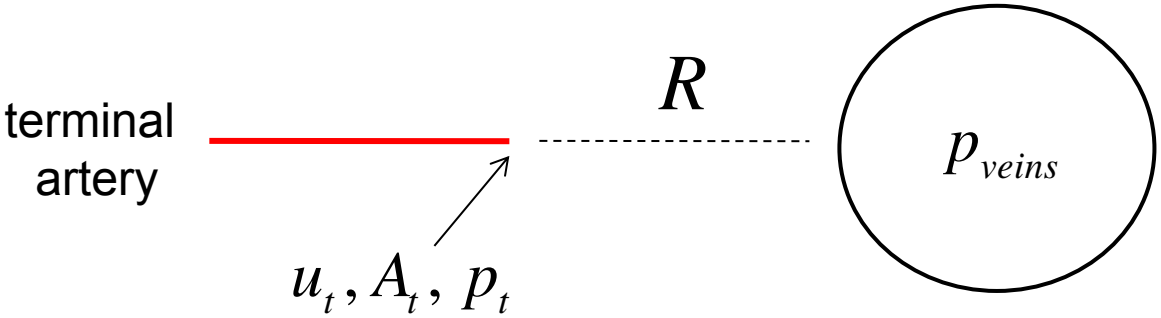


Bifurcations. Terminal vessel

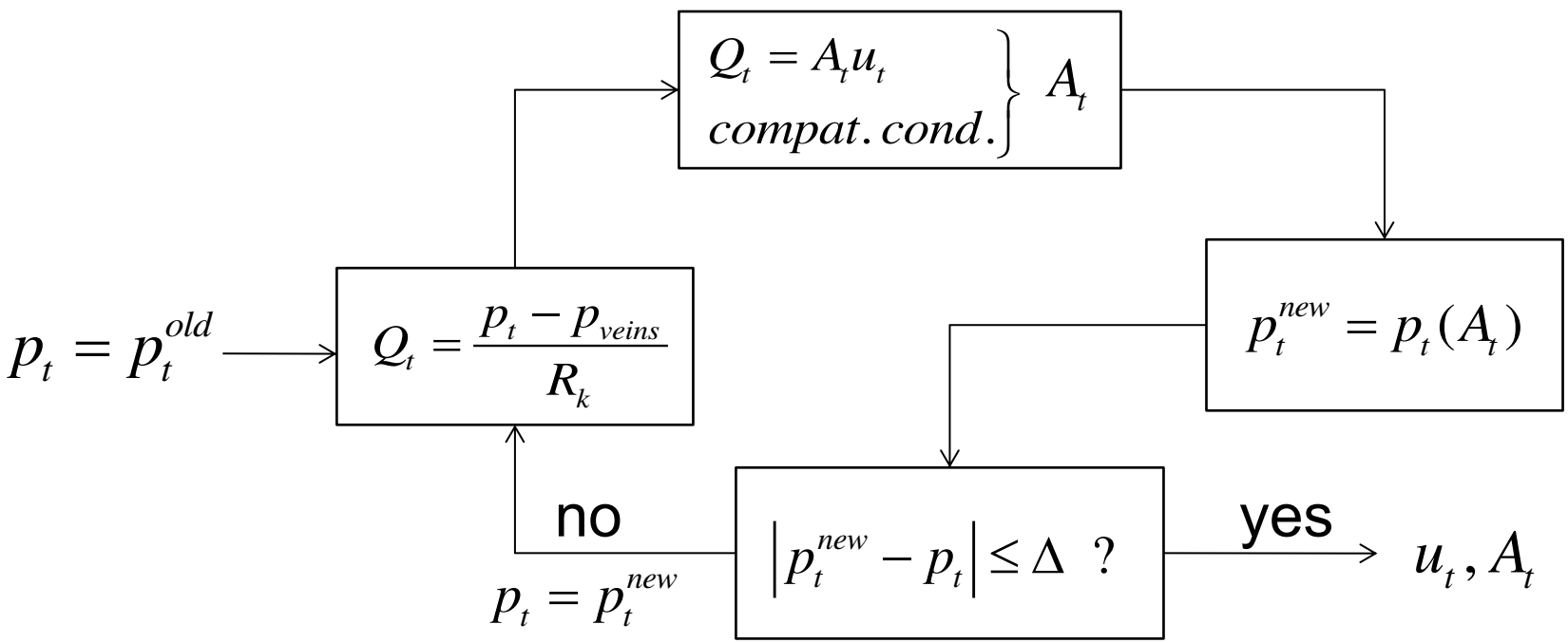


$$\begin{cases} p_t - p_{veins} = RQ_t, \\ p_t = p_t(A_t), \\ compat.\,cond. \rightarrow u_t = \alpha A_t + \beta. \end{cases}$$

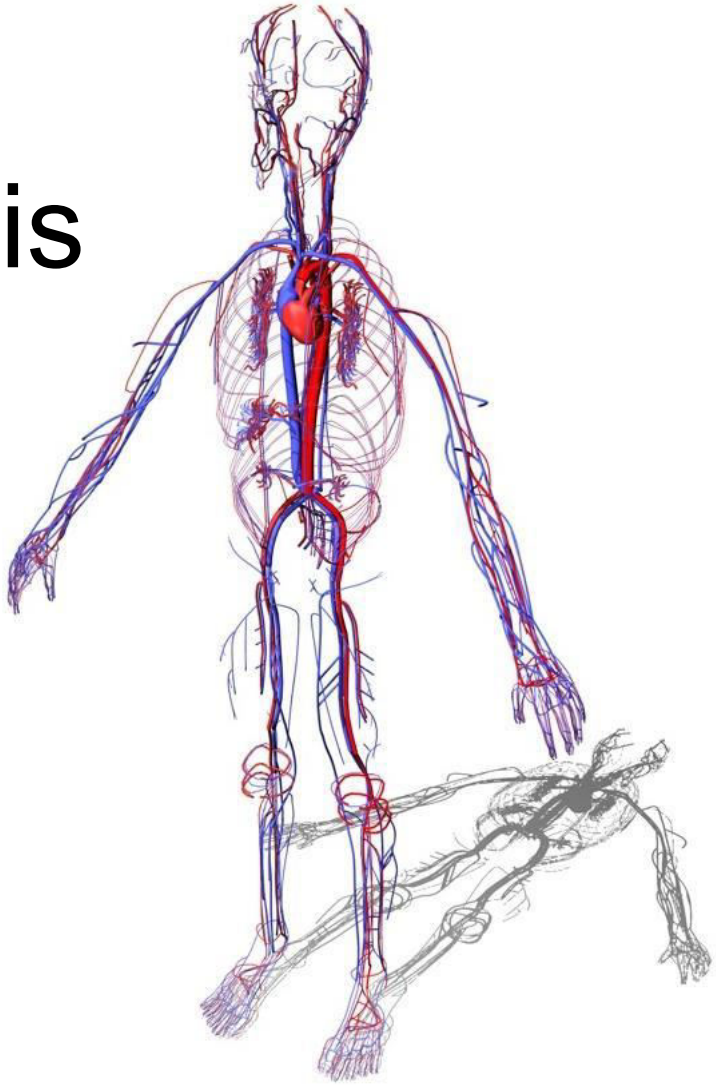
Bifurcations. Terminal vessel



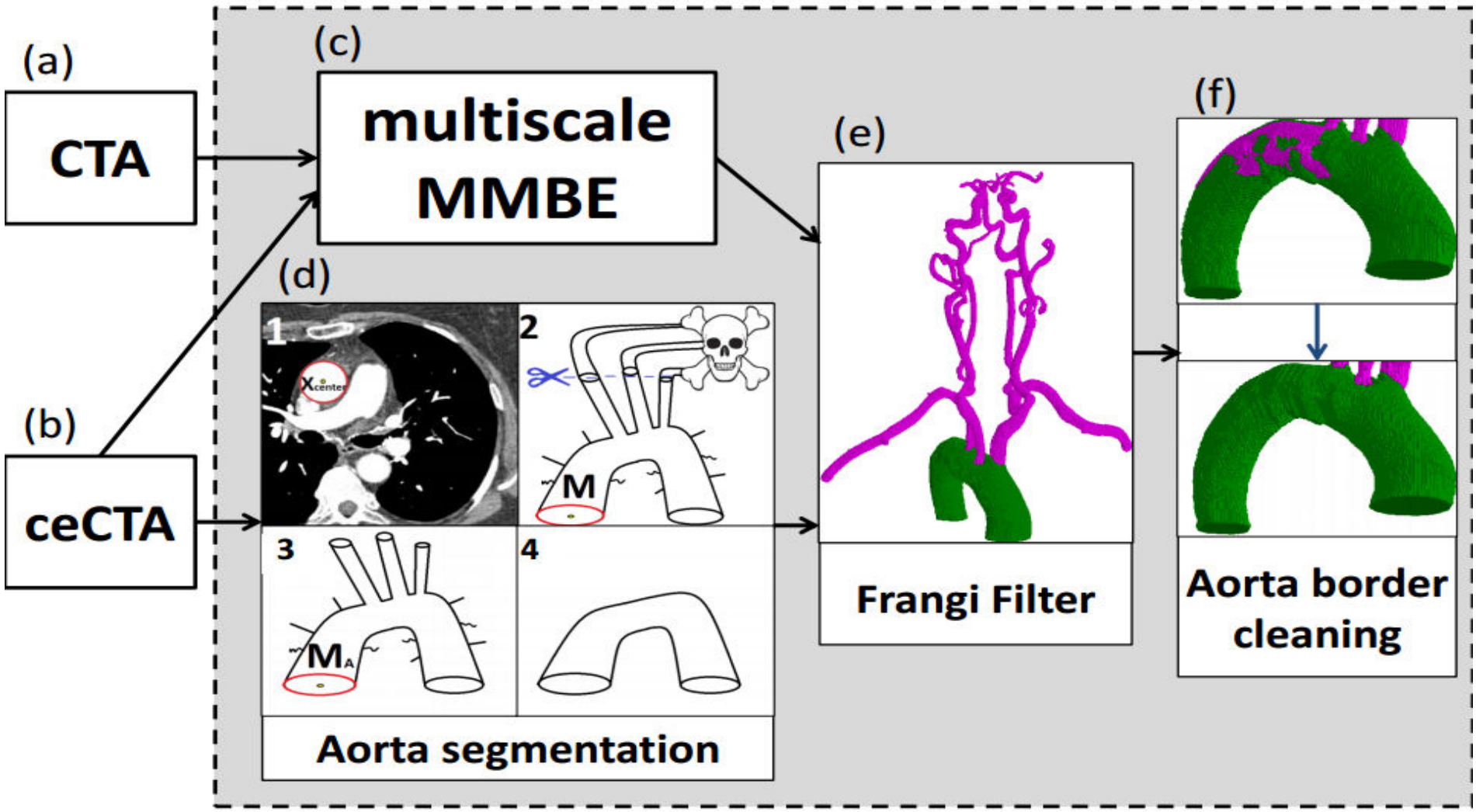
$$\begin{cases} p_t - p_{veins} = RQ_t, \\ p_t = p_t(A_t), \\ compat.\,cond. \rightarrow u_t = \alpha A_t + \beta. \end{cases}$$



Carotid artery stenosis



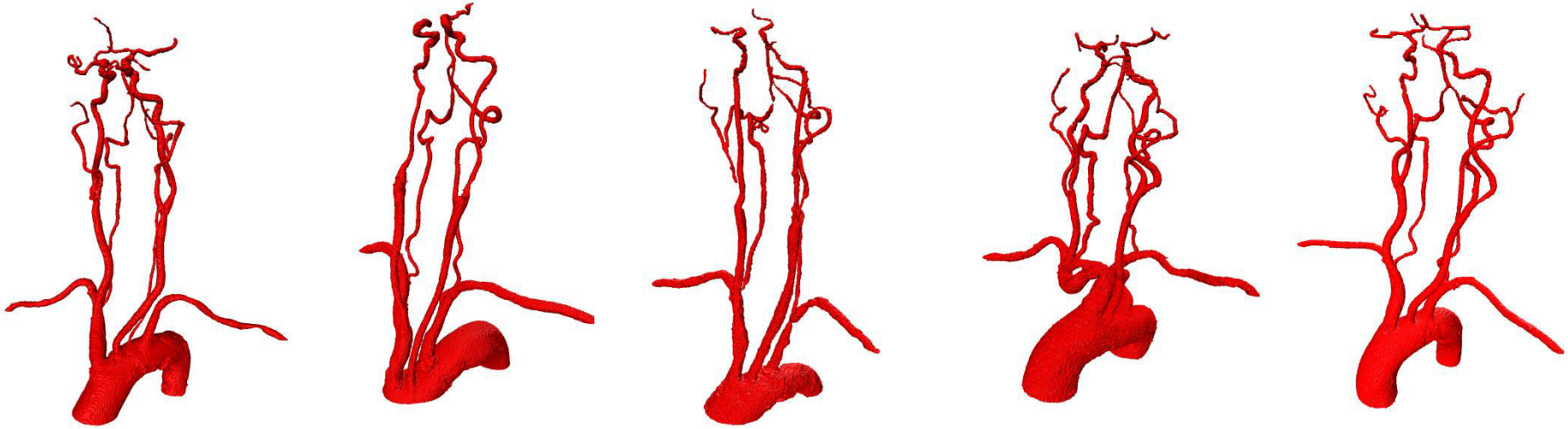
Geometry extraction

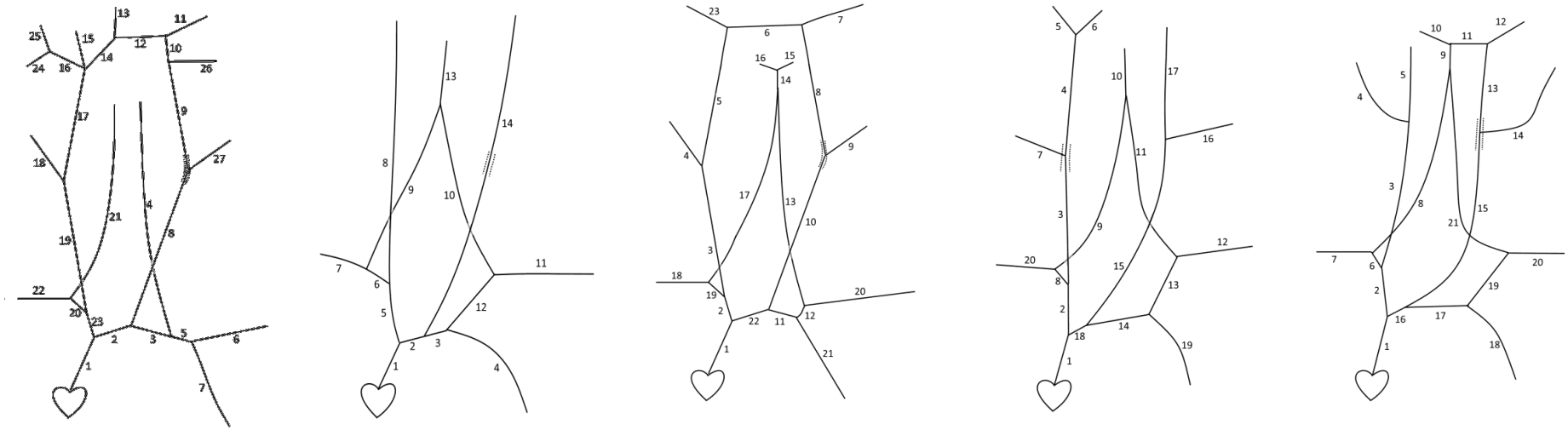


Wednesday (2 Nov),13:20
R.Pryamonosov (INM, Moscow). Patient-oriented graph network reconstruction of coronary arteries

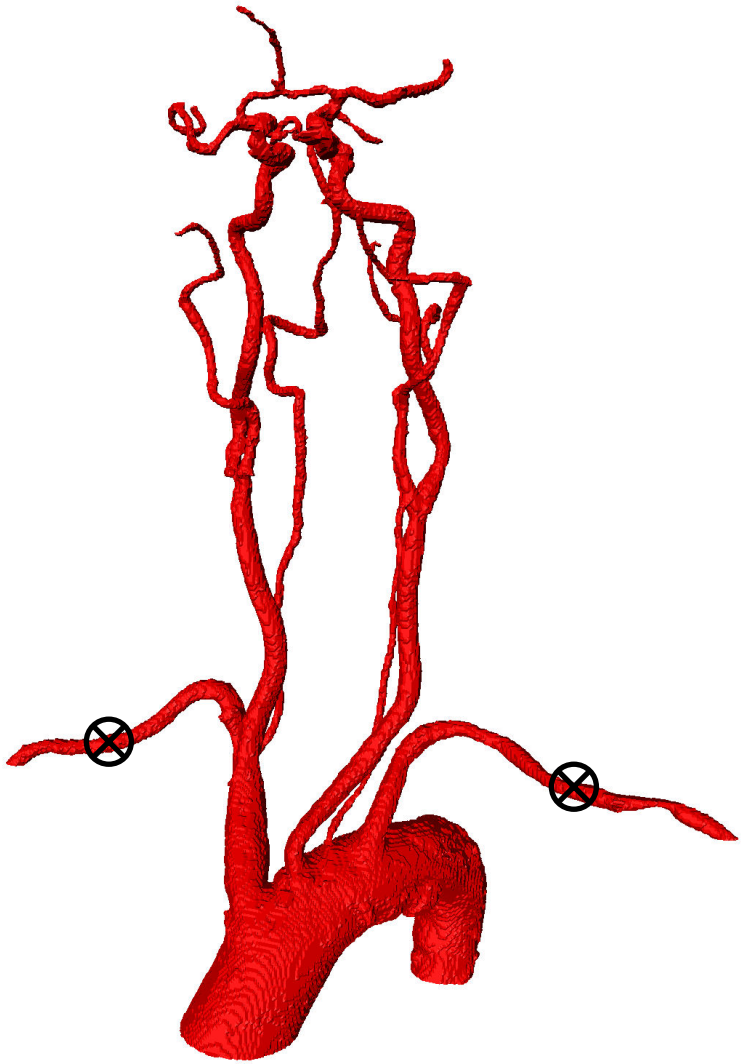
Vessels structure

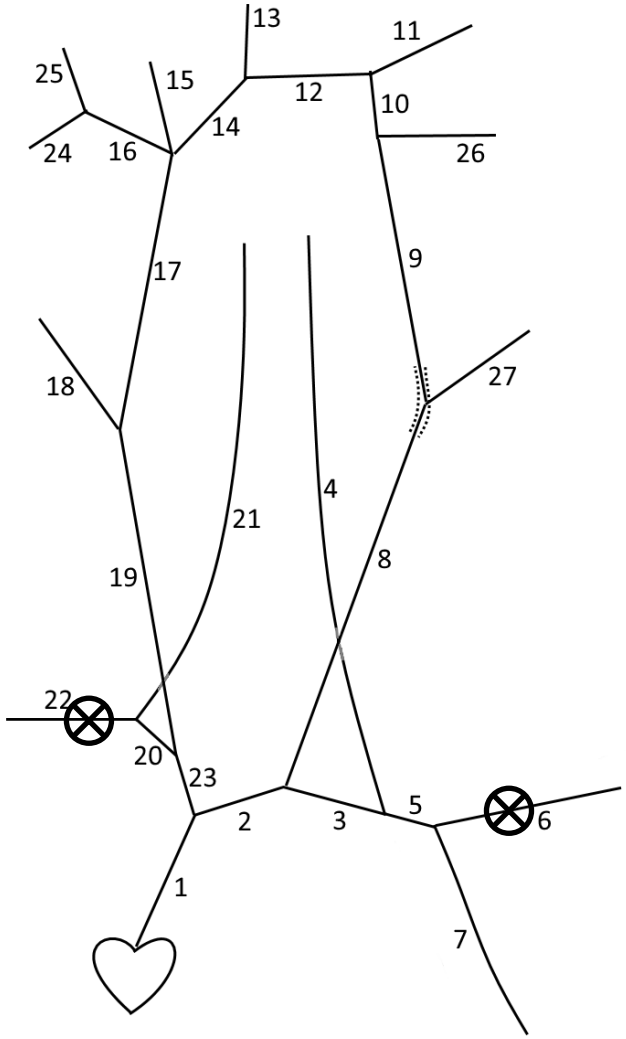
5 patients



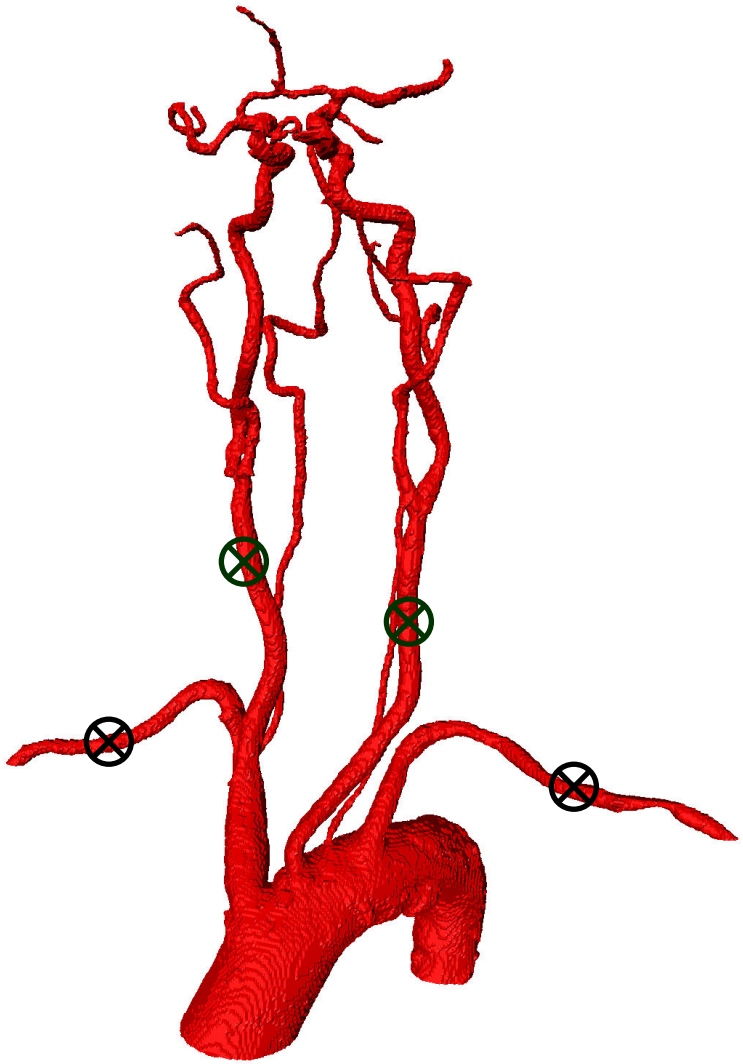


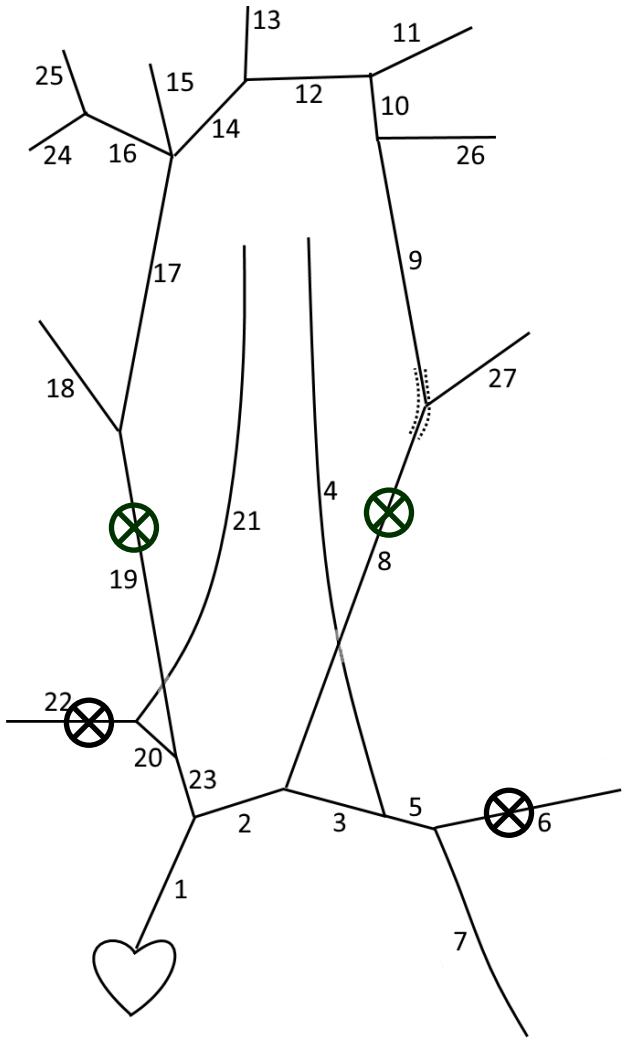
Ultrasound velocity measurements



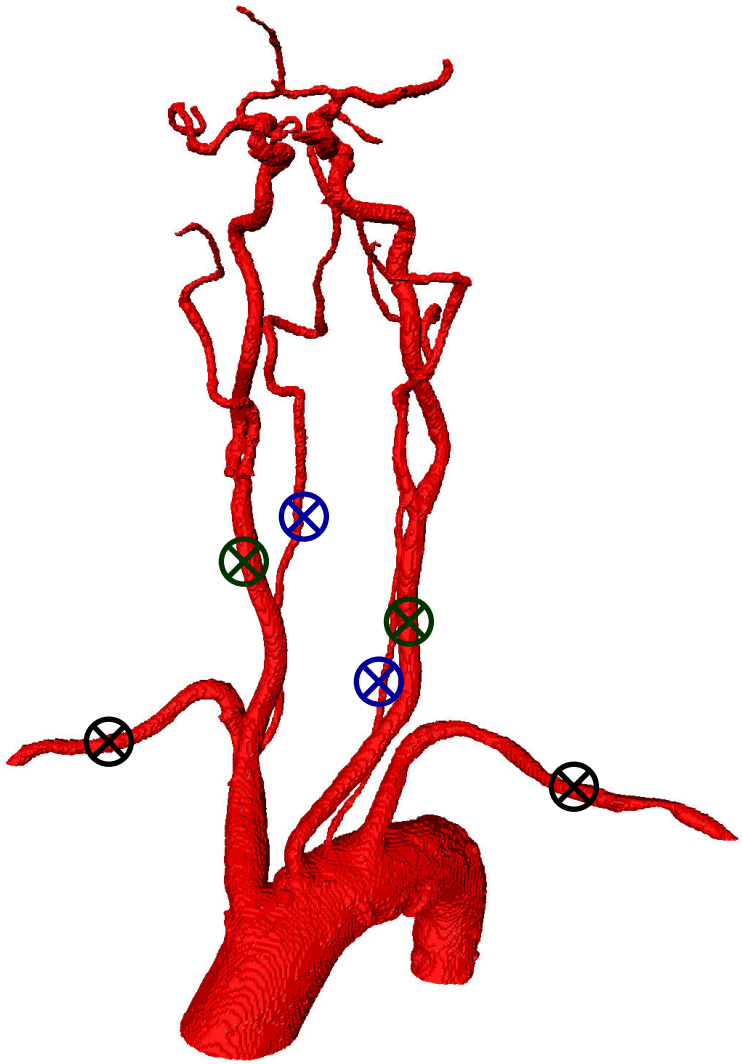


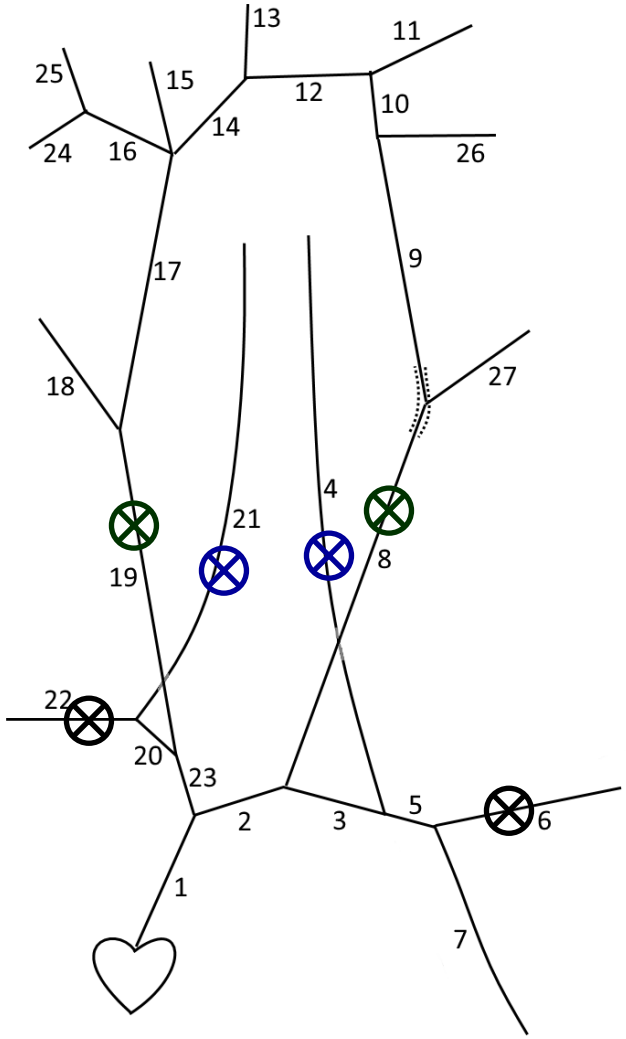
Ultrasound velocity measurements



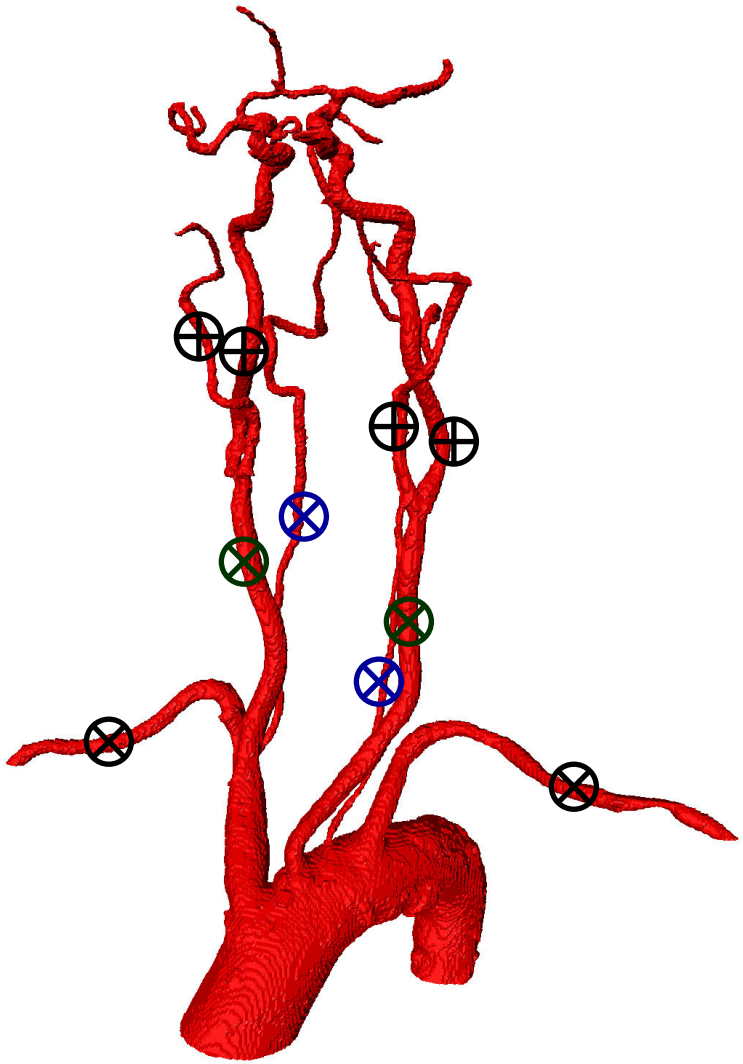


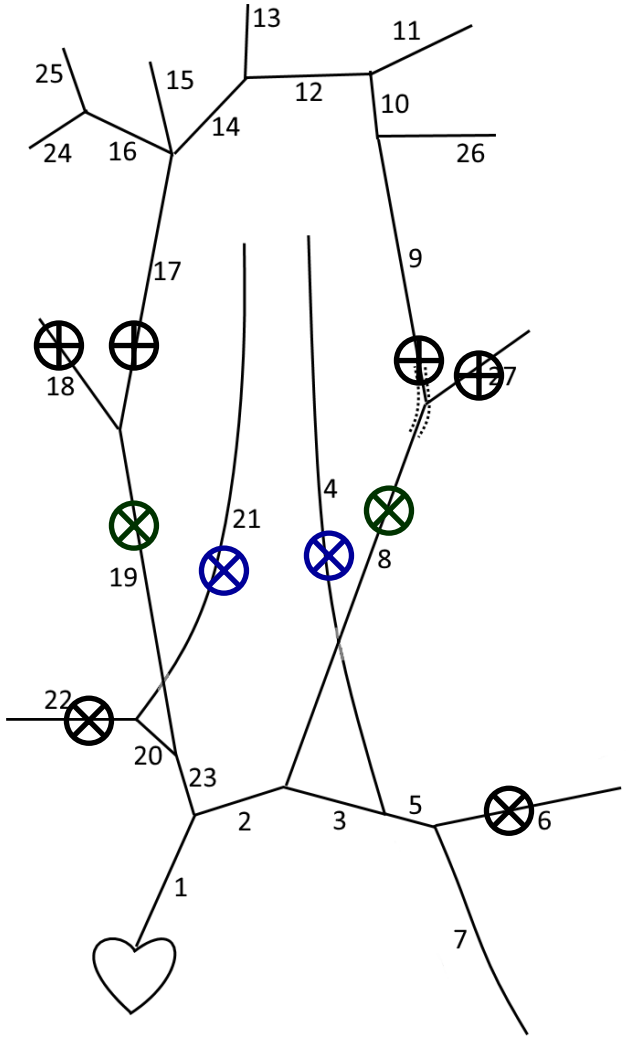
Ultrasound velocity measurements



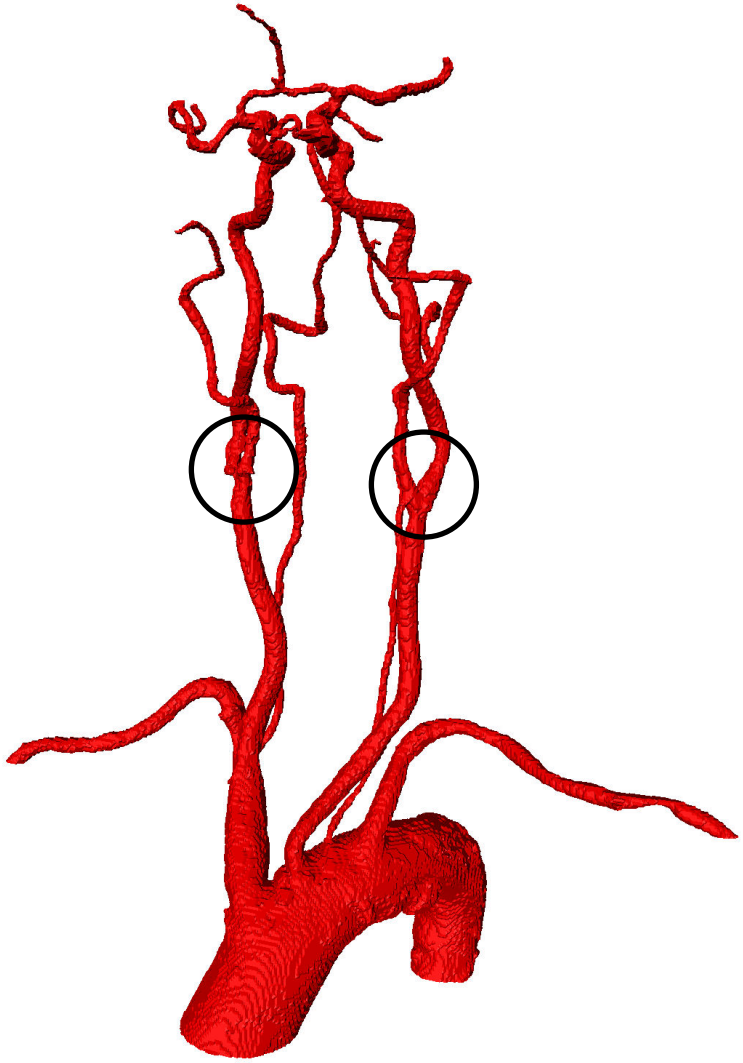


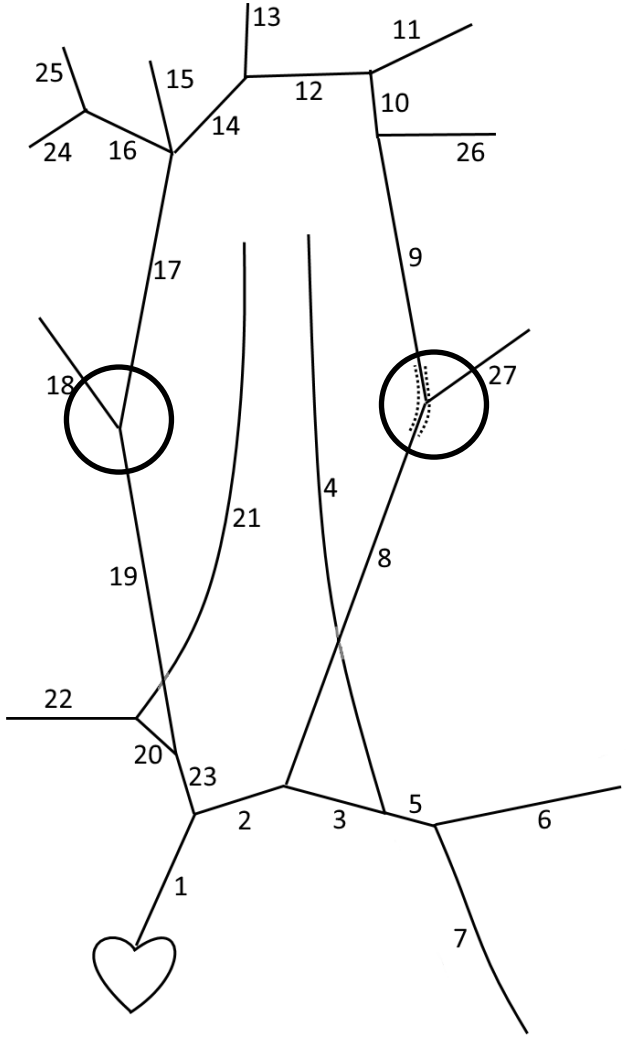
Ultrasound velocity measurements





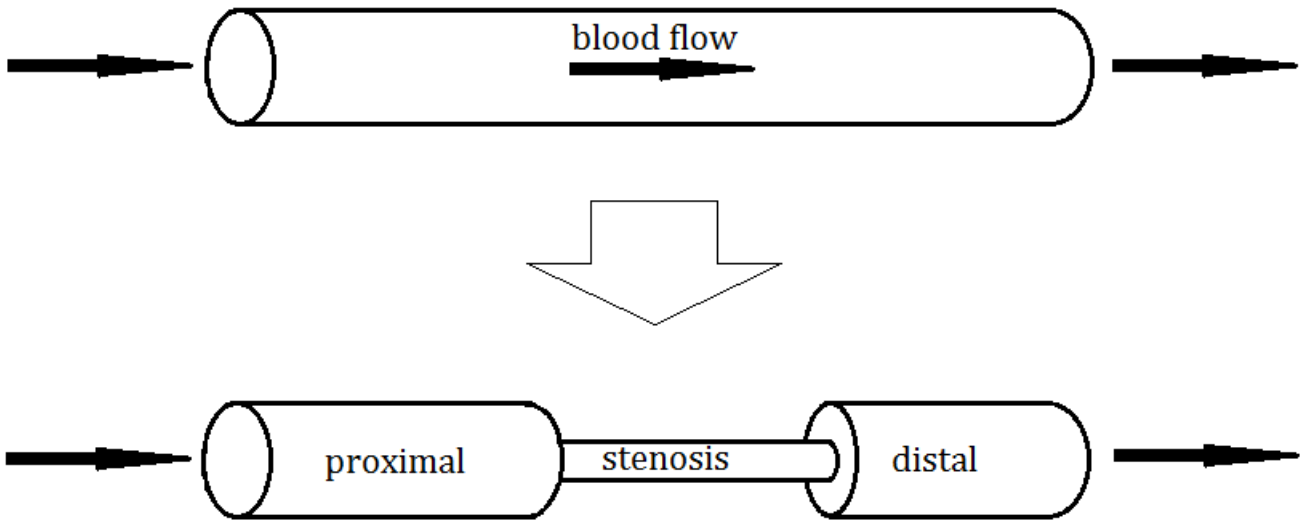
Stenosis





Stenosis

Stenosis - decreased rigidity c_k



Results. Velocity measurements

CT scans

Geometry

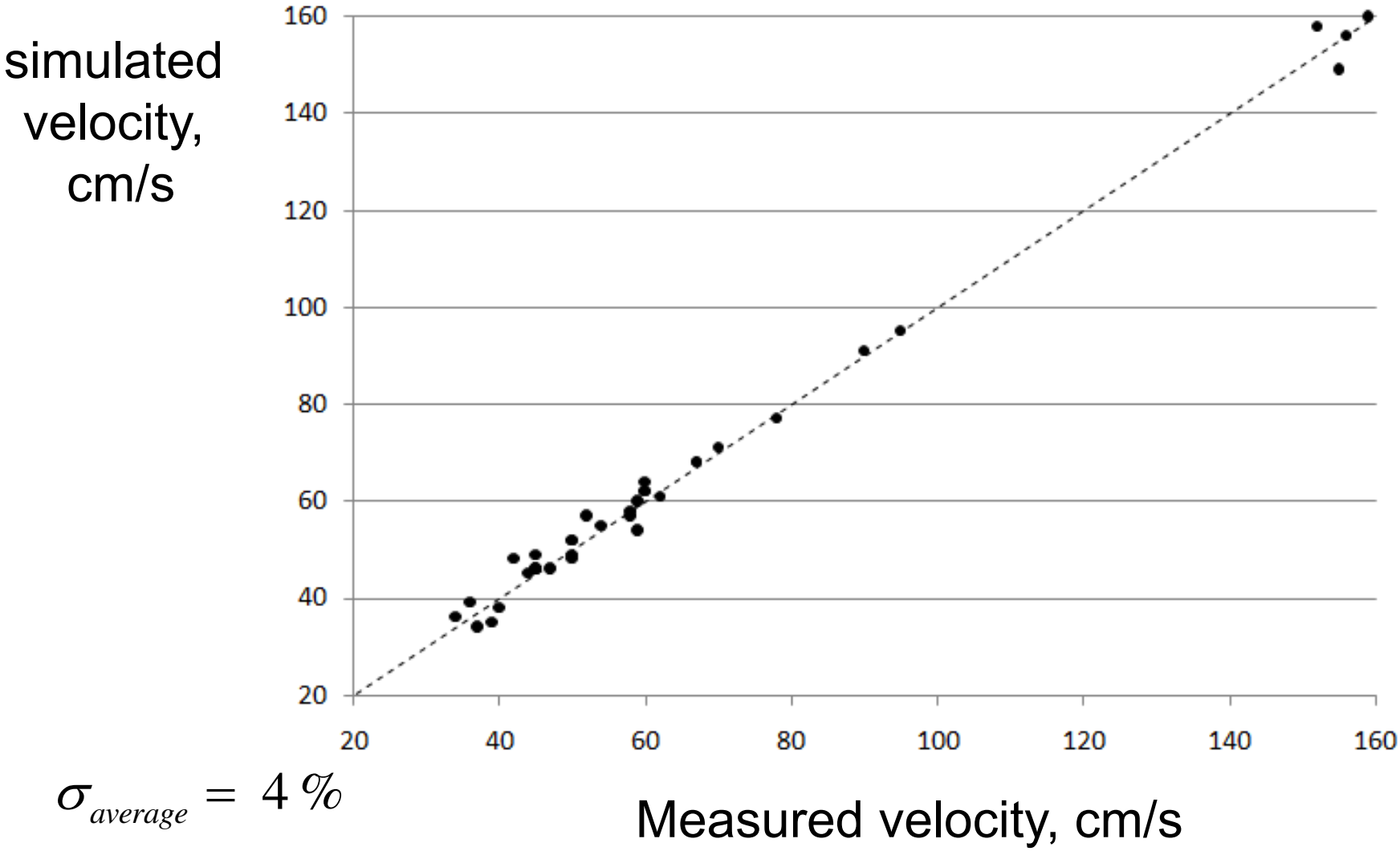
Velocities after treatment

Velocities before treatment

$$c_k, R_k, p_{veins}, Q_{heart}(t)$$

Patient data
Clinical studies

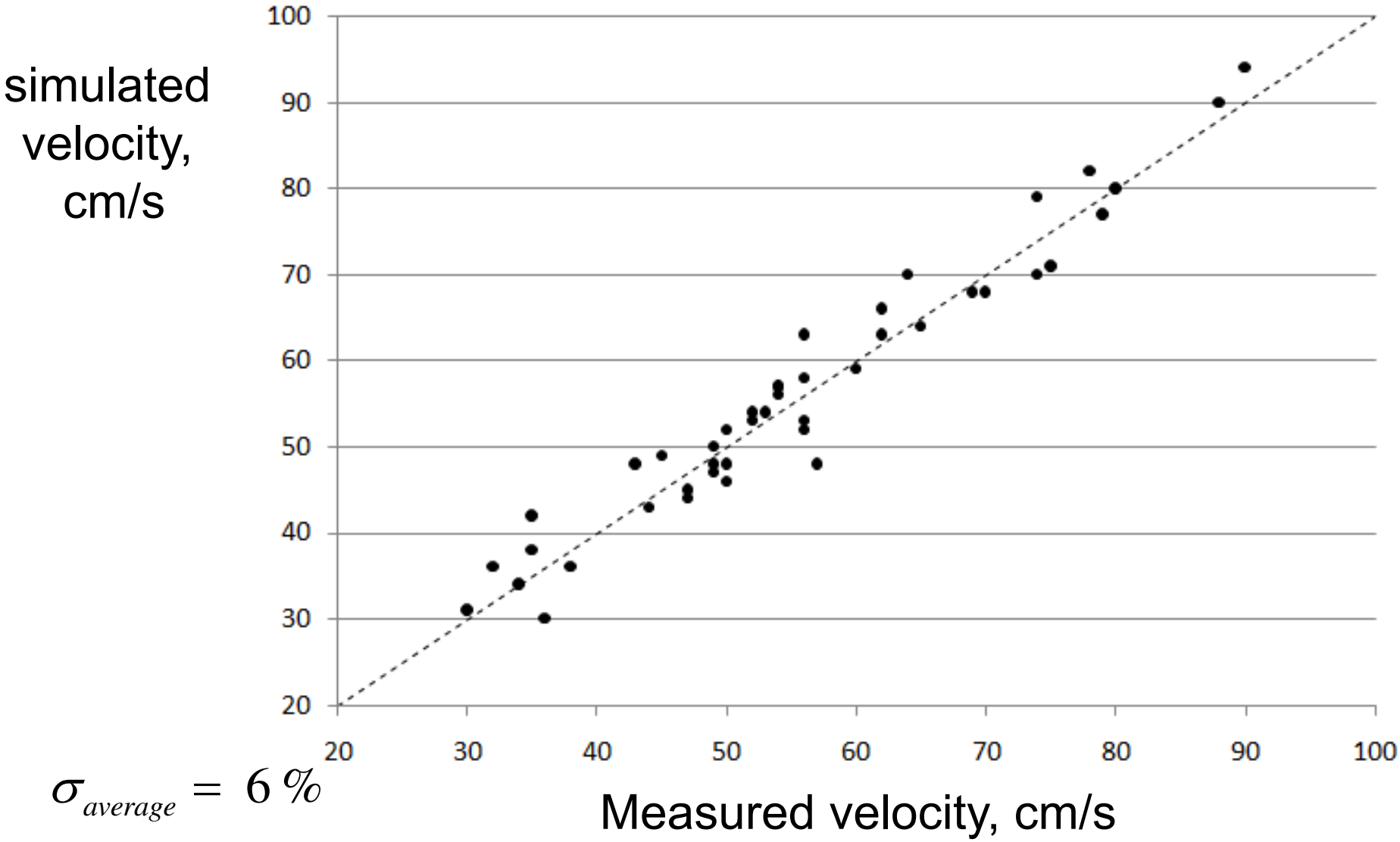
Velocity before treatment



$$\sigma_{average} = 4\,\%$$

$$\sigma_{\max} = 16\,\%$$

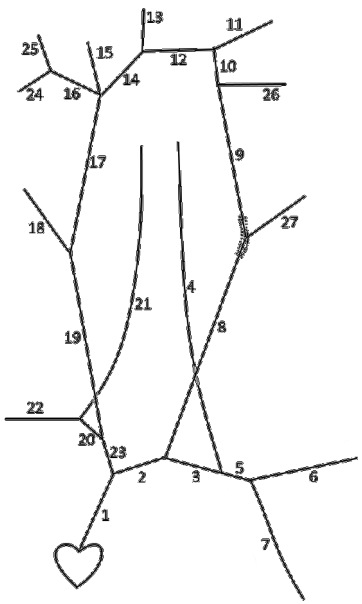
Velocity after treatment

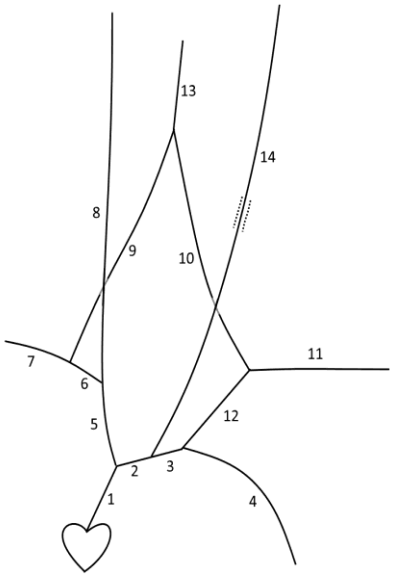


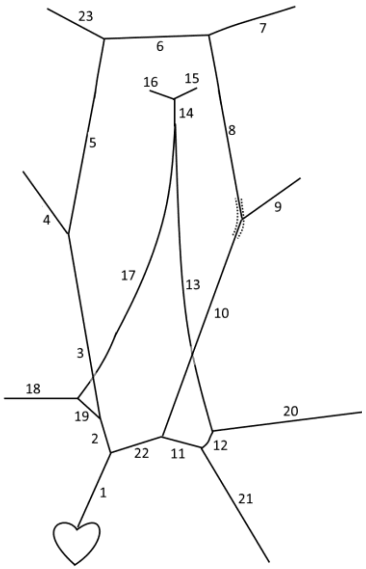
$$\sigma_{average} = 6\ \%$$

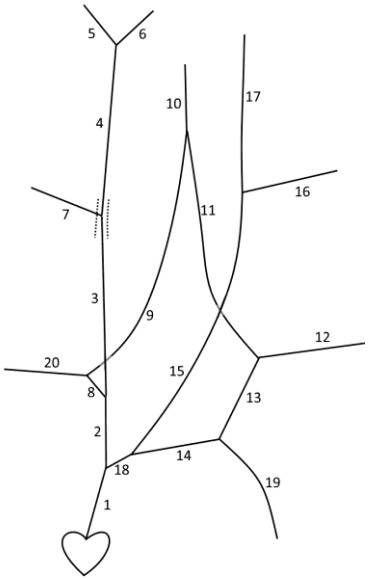
$$\sigma_{\max} = 20\ \%$$

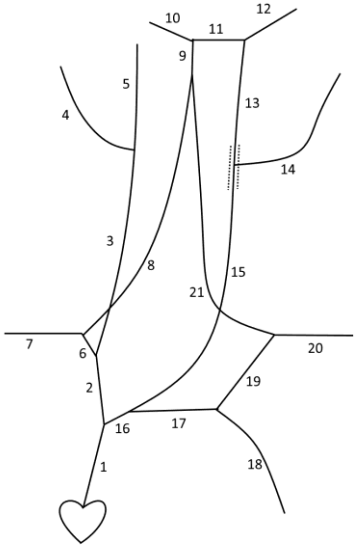
Circle of Willis



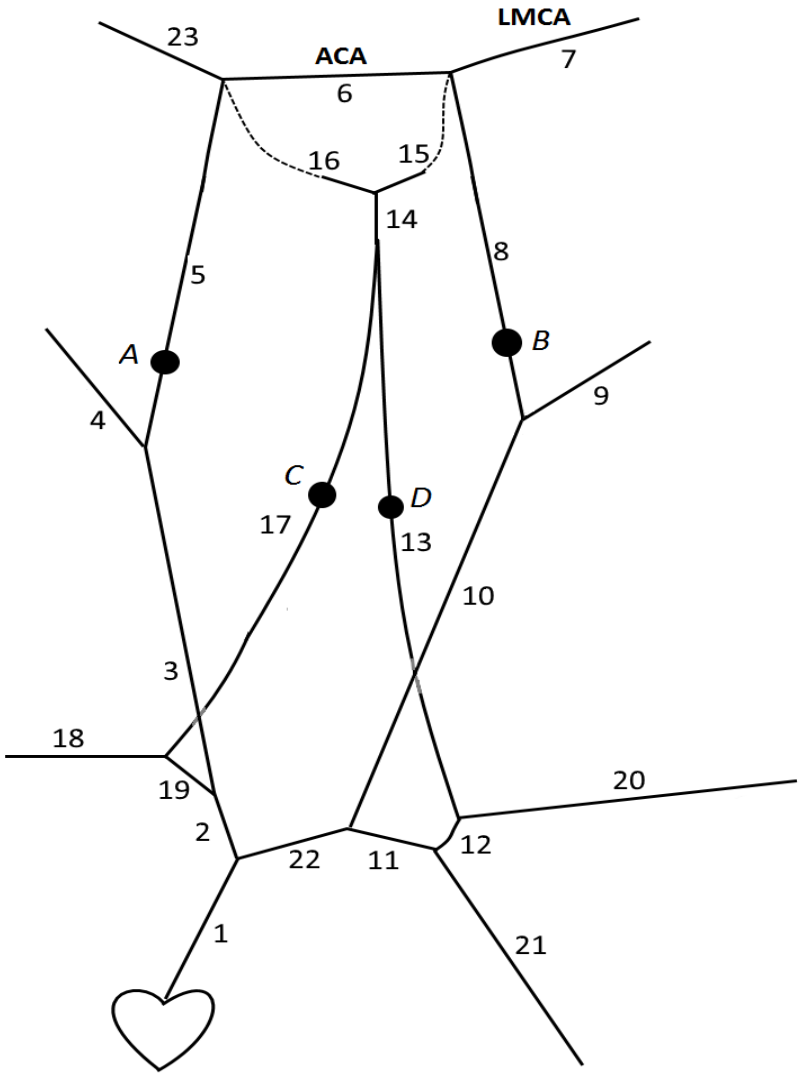


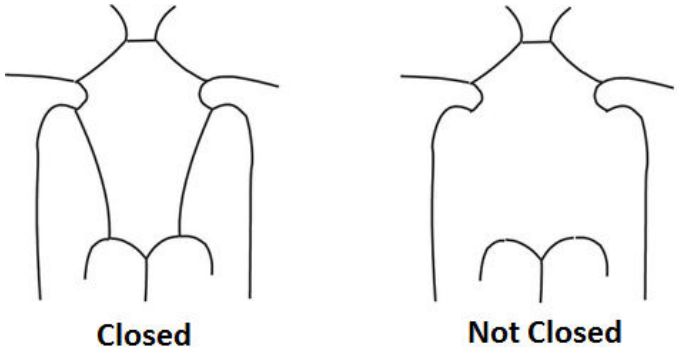






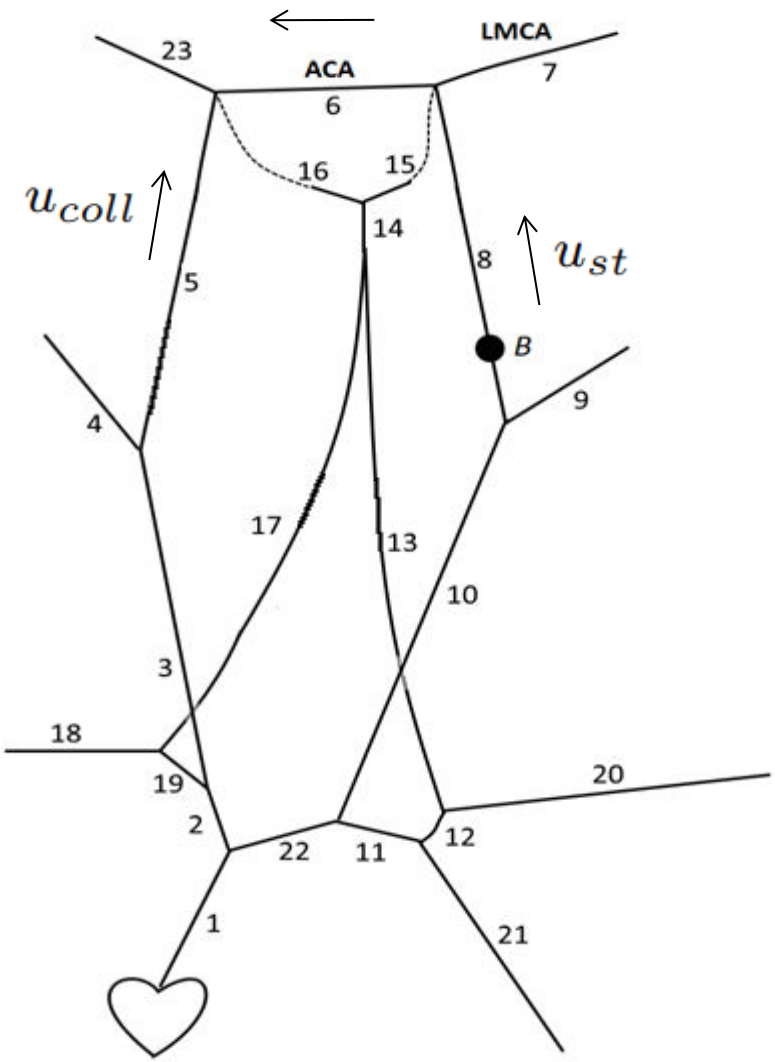
Multiple stenoses





Direction of flow

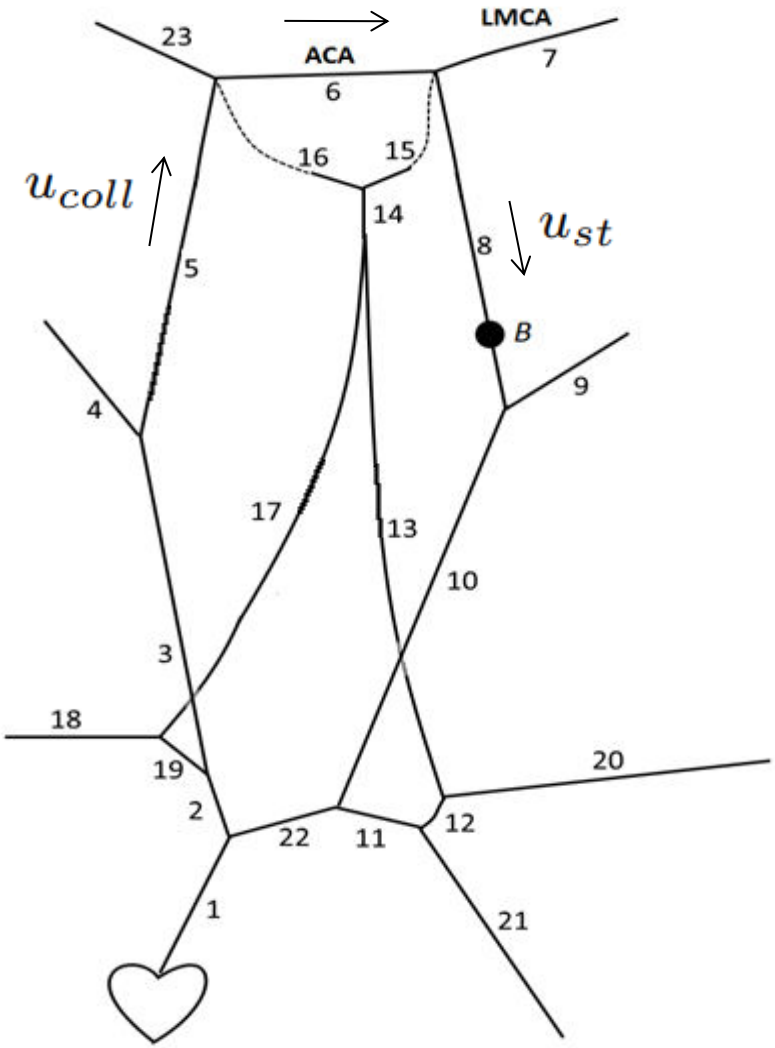
Stenosis B < 50 %



Stenosis	$u_{st}, cm/s$	$u_{coll}, cm/s$	$Q_{ACA}, ml/s$
0 %	90	91	0.21
40 %	81	92	0.08
50 %	73	99	- 0.02
80 %	10	115	- 0.53
95 %	-32	140	-0.57

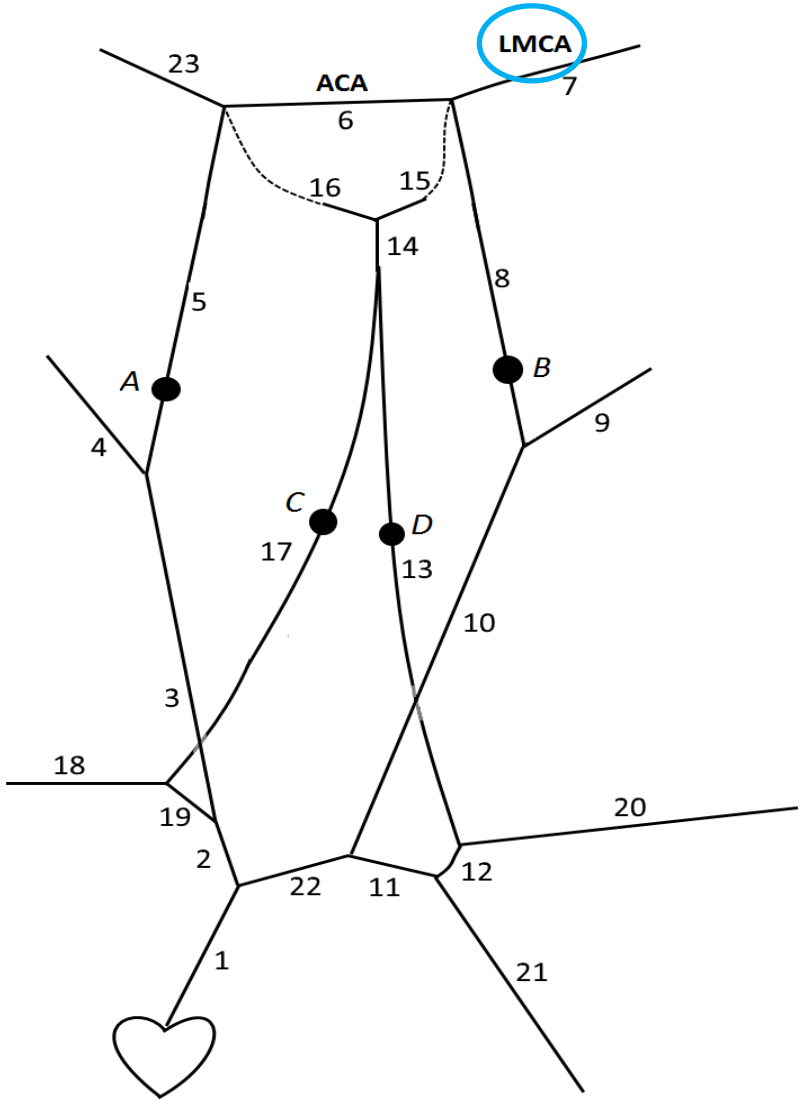
Direction of flow

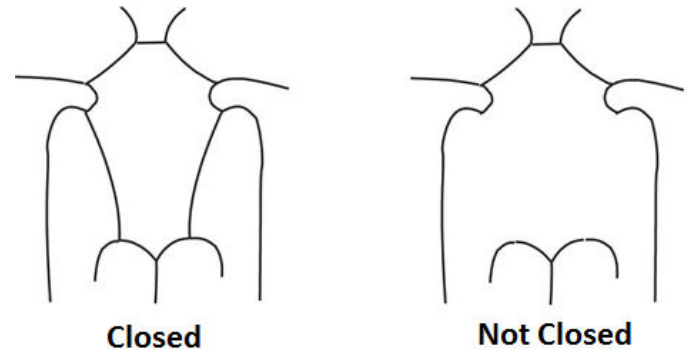
Stenosis B > 80 %



Stenosis	$u_{st}, cm/s$	$u_{coll}, cm/s$	$Q_{ACA}, ml/s$
0 %	90	91	0.21
40 %	81	92	0.08
50 %	73	99	- 0.02
80 %	10	115	- 0.53
95 %	-32	140	-0.57

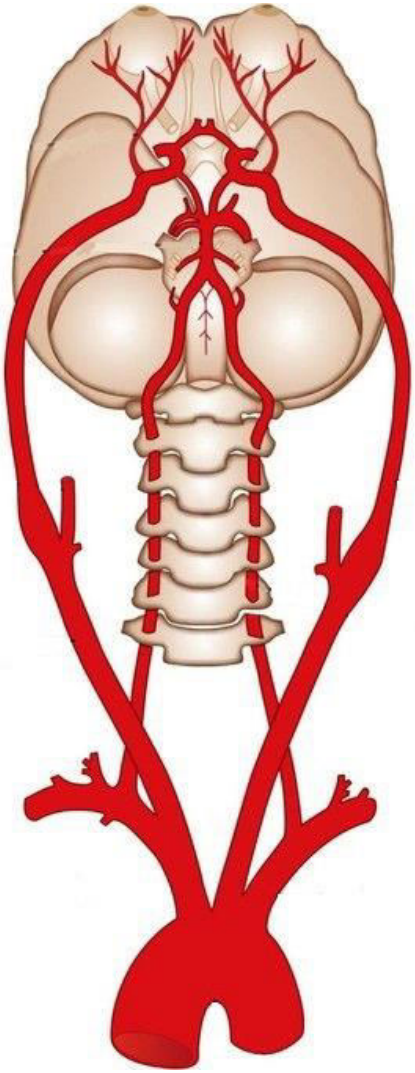
Multiple stenoses





Circle of Willis	LMCA blood flow, ml/s			
	Healthy	ABCD	AB	BCD
Closed	1.21	0.25	1.15	1.19
Not closed	1.02	0.21	0.20	1.01

Conclusion



Model:

- Individual structure
- blood flow pattern
- parameters
- 10 cases -> 5 cases

Conclusion

5 patients